CLASSIFICATION OF GENERALIZED SET AND ITS OPERATION RULES

CHENG JINPENG
HEBEI COAL MINING AND CICIL ENGINEERING COLLEGE
HANDAN, HEBEI, CHINA

YUE CHANGAN
HANDNA EDUCATION COLLEGE
HANDAN, HEBEI, CHINA

We have, on the basis of analysing fuzzy set and the examples of grey number, presented the conception of generalized set [1], which is the extention of fuzzy set, while fuzzy set is the specific example, therefore, the generalized is also called fuzzy. And in this paper we shall discuss the classification and operation of generalized set based on the above conception.

1. CLASSIFICATION OF GENERALIZED SET

we present the definition of generalized set in references [2].
Let U be a universe, and if $\forall$ $u \in U$, $\mu(u) \in T$

$$\mu: U \to T$$
$$u \mapsto \mu(u)$$

then a generalized subset of U is defined by μ, which is written as $A(\mu)$. μ is the subordinating function of $A(\mu)$, $\mu(u)$ the subordinating degree of μ to $A(\mu)$, in which R ($\neq \phi$) is the set of real number,T=P(R).

According to the value of $\mu(u)$, the generalized set mamay be divided into two kinds: single value set and complex value set.

1.1. Single value set

DEFINITION 1 If $\forall$ $u \in U$, $|\mu(u)|=1$, then $A(\mu)$ is called a single value set.
If $A(\mu)$ is a single value set and $\forall$ $u \in U$, $\mu(u) \subseteq [0,1]$ then is called a Cantor set.
If $A(\mu)$ is a single value set and $\forall$ $u \in U$, $\mu(u) \subseteq [0,1]$, then $A(\mu)$ is called a fuzzy set.

1.2. Complex value set

DEFINITION 2 Let $A(\mu)$ be a generalized set, if $\exists$ $u \in U$, and $|\mu(u)| > 2$, then $A(\mu)$ is called a complex value set.

Let $A(\mu)$ be a complex value set, if $\exists$ $u \in U$, $\sup\mu(u)$ or $\inf\mu(u) \in [0,1]$, then $A(\mu)$ is called a set of awaiting use.

If $\forall$ $u \in U$, $\mu(u) \subseteq [0,1]$, then $A(\mu)$ is called a generalized grey set of U.

If $A(\mu)$ is a generalized complex value grey set, then $A(\mu)$ is called a proper grey set.

If $\mu: U \to T$, $\forall$ $u \in U$ and $\mu(u)$ is called a closed sub-interval of [0,1], then $A(\mu)$ is called a fuzzy grey set, and therefore, $\forall$ $u \in U$, $\mu(u)=[\inf\mu(u), \sup\mu(u)]$.

2. THE OPERATION OF GENERALIZED SET

DEFINITION 1 Let $\mu(u)$ R

(I) if $\sup\mu(u) \in \mu(u)$ and $\inf\mu(u) \in \mu(u)$, then $[\inf\mu(u), \sup\mu(u)]$ is called a compact brackets set of $\mu(u)$;

(II) if $\sup\mu(u) \notin \mu(u)$, but $\inf\mu(u) \in \mu(u)$, then $[\inf\mu(u), \sup\mu(u))$ is called a compact brackets set of $\mu(u)$;

(III) if $\sup\mu(u) \in \mu(u)$, but $\inf\mu(u) \notin \mu(u)$, then $(\inf\mu(u), \sup\mu(u)]$ is called a compact brackets set of $\mu(u)$;

(IV) if $\sup\mu(u) \notin \mu(u)$, and $\inf\mu(u) \notin \mu(u)$, then $(\inf\mu(u), \sup\mu(u))$ is called a compact brckets set of $\mu(u)$.

The compact breakets set of $\mu(u)$ is written as $Q(\mu)$.

DEFINITION 2 Let $\mu(u) \subseteq R$

(I) if $\sup\mu(u) \in \mu(u)$, then $(-\infty, \sup\mu(u)]$ is called an upper brackets set of $\mu(u)$;

(II) if $\sup\mu(u) \notin \mu(u)$, then $(-\infty, \sup\mu(u))$ is called an upper brackets set of $\mu(u)$, which is written as $\overline{Q}(\mu)$.

DEFINITION 3 Let $\mu(u) \subseteq R$

(I) if $\inf\mu(u) \in \mu(u)$, then $[\inf\mu(u), +\infty)$ is called a lower brackets set of $\mu(u)$;

(II) if $\inf\mu(u) \notin \mu(u)$, then $(\inf\mu(u), +\infty)$ is called a lower brackets set of u(u), which is written as $\underline{Q}(\mu)$.

DEFINITION 4 The combining set of generalized set $A(\mu 1)$ and $A(\mu 2)$ of (U).

$$A(\mu 1) \cup A(\mu 2) = A((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2))$$

DEFINITION 5 Generalized sets $A(\mu 1)$ and $A(\mu 2)$ and communitive set of ($\cap$).

$$A(\mu 1) \cap A(\mu 2) = A((\mu 1 \cup \mu 2) \cap \overline{Q}(\mu 1) \cap \overline{Q}(\mu 2))$$

DEFINITION 6 Complete set(-) of generalized set $A(\mu)$

$$A(\mu) = A(\lambda), \lambda = \{1-\alpha \mid \alpha \in \mu(u)\}$$

DEFINITION 7 When $\mu 1(u) = \mu 2(u)$, $A(\mu 1)$ is said to be equal to $A(\mu 2)$, written as $A(\mu 1) = A(\mu 2)$.

DEFINITION 8 When $\inf \mu 1(u) > \sup\mu 2(u)$, $A(\mu 1)$ is said to conclude $A(\mu 2)$. and $A(\mu 2)$ is too called the subset of $A(\mu 1)$, written as $A(\mu 1) \supseteq A(\mu 2)$ or $A(\mu 2) \subseteq A(\mu 1)$.

It is obvious that the including relation is of the following properties:

(I) if $A(\mu 1) \subseteq A(\mu 2)$ and $A(\mu 2) \subseteq A(\mu 1)$ then $\inf\mu 1(u) = \inf\mu 2(u) = \sup\mu 1(u) = \sup\mu 2(u)$ therefore $A(\mu 1) = A(\mu 2)$ is the single value set.

(II) if $A(\mu 1) \supseteq A(\mu 2)$, $A(\mu 2) \supseteq A(\mu 3)$ then $A(\mu 1) \supseteq A(\mu 3)$.

3. THE OPERATIONAL PROPERTIES OF GENERALIZED SET

In order to discuss the properties of generalized set, we'll first study the following knowledge.

(I) $\underline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)) = \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)$

PROOF:

when $\inf \mu 1 < \sup \mu 1 < \inf \mu 2 < \sup \mu 2$
or $\inf \mu 1 < \inf \mu 2 < \sup \mu 1 < \sup \mu 2$
or $\inf \mu 1 < \inf \mu 2 < \sup \mu 2 < \sup \mu 1$

$$\underline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)) = \underline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 2))$$
$$= \underline{Q}(\mu 2) = \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)$$

when $\inf \mu 2 < \sup \mu 2 < \inf \mu 1 < \sup \mu 1$
or $\inf \mu 2 < \inf \mu 1 < \sup \mu 2 < \sup \mu 1$
or $\inf \mu 2 < \inf \mu 1 < \sup \mu 1 < \sup \mu 2$

$$\underline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)) = \underline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1))$$
$$= \underline{Q}(\mu 1) = \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)$$

hence $\underline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)) = \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)$

Same method can proof :

(II) $\underline{Q}((\mu 1 \cup \mu 2) \cap \overline{Q}(\mu 1) \cap \overline{Q}(\mu 2)) = \underline{Q}(\mu 1) \cup \underline{Q}(\mu 2)$

(III) $\overline{Q}((\mu 1 \cup \mu 2) \cap \overline{Q}(\mu 1) \cap \overline{Q}(\mu 2)) = \overline{Q}(\mu 1) \cap \overline{Q}(\mu 2)$

(IV) $\overline{Q}((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2)) = \overline{Q}(\mu 1) \cup \overline{Q}(\mu 2)$

Here is the discussion of its operational properties.

3.1. Commutative law

$$A(\mu 1) \cup A(\mu 2) = A(\mu 2) \cup A(\mu 1); A(\mu 1) \cap A(\mu 2) = A(\mu 2) \cap A(\mu 1)$$

PROOF :

$$A(\mu 1) \cup A(\mu 2) = A((\mu 1 \cup \mu 2) \cap \underline{Q}(\mu 1) \cap \underline{Q}(\mu 2))$$
$$= A((\mu 2 \cup \mu 1) \cap \underline{Q}(\mu 2) \cap \underline{Q}(\mu 1))$$
$$= A(\mu 2) \cup A(\mu 1)$$

Same method can proof :

$$A(\mu 1) \cap A(\mu 2) = A(\mu 2) \cap A(\mu 1)$$

3.2. Associative law

$$A(\mu 1) \cup [A(\mu 2) \cup A(\mu 3)] = [A(\mu 1) \cup A(\mu 2)] \cup A(\mu 3)$$
$$A(\mu 1) \cap [A(\mu 2) \cap A(\mu 3)] = [A(\mu 1) \cap A(\mu 2)] \cap A(\mu 3)$$

PROOF (omitted)

3.3. Distributive law

if $\mu 1(u) = \{\alpha \mid \alpha \in Q(\mu 1)\}$ then

$$A(\mu 1) \cup [A(\mu 2) \cap A(\mu 3)] = [A(\mu 1) \cup A(\mu 2)] \cap [A(\mu 1) \cup A(\mu 3)]$$
$$A(\mu 1) \cap [A(\mu 2) \cup A(\mu 3)] = [A(\mu 1) \cap A(\mu 2)] \cup [A(\mu 1) \cap A(\mu 3)]$$

PROOF :

$A(\mu 1)\cup(A(\mu 2)\cap A(\mu 3))$
$=A(\mu 1)\cup A((\mu 2\cup\mu 3)\cap\overline{Q}(\mu 2)\cap\overline{Q}(\mu 3))$
$=A((\mu 1\cup(\mu 2\cup\mu 3)\cap\overline{Q}(\mu 2)\cap\overline{Q}(\mu 3))\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 2\cup\mu 3)\cap\overline{Q}(\mu 2)\cap\overline{Q}(\mu 3))$
$=A((\mu 1\cup\mu 2)\cap(\mu 1\cup\overline{Q}(\mu 3))\cap(\mu 1\cup\overline{Q}(\mu 2))\cap(\underline{Q}(\mu 2)\cup\underline{Q}(\mu 3))\cap\underline{Q}(\mu 1))$
$=A(\mu)$

in it

$$\mu=(\mu 1\cap\underline{Q}(\mu 2)\cup\mu 1\cap\underline{Q}(\mu 3)\cup\mu 2\cap\overline{Q}(\mu 3)\cup 3\mu\cap\overline{Q}(\mu 2))\cap\underline{Q}(\mu 1).$$

$(A(\mu 1)\cup A(\mu 2))\cap(A(\mu 1)\cup A(\mu 3))$
$=A((\mu 1\cup\mu 2)\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 2))\cap A((\mu 1\cup\mu 3)\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 3)$
$=A(((\mu 1\cup\mu 2)\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 2)\cup(\mu 1\cup\mu 3)\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 3)($
$\cap\overline{Q}((\mu 1\cup\mu 3)\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 2))\cap\overline{Q}((\mu 1\cup\mu 3)\cap\underline{Q}(\mu 1)\cap\underline{Q}(\mu 3)))$
$=A(\mu')$

in it

$$\mu'=(\mu 1\cap\underline{Q}(\mu 2)\cup\mu 1\cap\underline{Q}(\mu 3)\cup\mu 2\cap\overline{Q}(\mu 3)\cup\mu 3\cap\overline{Q}(\mu 2)\cup(\mu 2\cup\mu 3)\cap\overline{Q}(\mu 1))\cap\underline{Q}(\mu 1)$$

If $\mu 1(u)=\{\alpha\mid\alpha\in Q(\mu 1)\}$
hence $A(\mu)=A(\mu')$ namely
$A(\mu 1)\cup(A(\mu 2)\cap A(\mu 3))=(A(\mu 1)\cup A(\mu 2))\cap(A(\mu 1)\cup A(\mu 3))$

Same method can proof, if $\mu 1(u)=\{\alpha\mid\alpha\in Q(\mu 1)\}$
then

$$A(\mu 1)\cap(A(\mu 2)\cup A(\mu 3))=(A(\mu 1)\cap A(\mu 2))\cup(A(\mu 1)\cap A(\mu 3))$$

3.4. DeMergan Law

(I) $\overline{(A(\mu 1)\cup A(\mu 2))}=\overline{A(\mu 1)}\cup\overline{A(\mu 2)}$

(II) $\overline{(A(\mu 1)\cap A(\mu 2))}=\overline{A(\mu 1)}\cap\overline{A(\mu 2)}$

3.5. Idempotent Law

$$A(\mu)\cup A(\mu)=A(\mu)\ ;\ A(\mu)\cap A(\mu)=A(\mu)$$

3.6. Absorption Law

If $\mu(u)=\{\alpha\mid\alpha\in Q(\mu)\}$, then $A(\mu)\cup(A(\mu)\cap A(\mu 1))=A(\mu)$;
$A(\mu)\cap(A(\mu)\cup A(\mu 1))=A(\mu)$.

3.7. Involution Law

$$\overline{\overline{A(\mu)}}=A(\mu)$$

PROOF (omitted)

REFERENCES

(1) Wu Heqin, Wang Qingyin, Preliminary Study of the Theory of Generalized Sets, Proceedings of NAFIPS'88.

(2) Wu Heqin, Wang Qingyin, the Theory of Compound Fuzzy Sets, Proceedings of NAFIPS'88.