COMPATIBILITY IN QUASI-ORTOKOMPLEMENTED POSETS

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1. INTRODUCTION

One of the actual problems of the mathematical description of quantum mechanics is the problem of simultaneous measurement of several observables. In the classical Kolmogorov model [1] measurement of non-quantum observables is performed within the framework of Boolean σ-algebra model. For quantum mechanics observables there exists a model of quantum logics [2]. There are also observables in the quantum logics which have the classical character, i. e., their ranges are embeded into a Boolean σ-algebra. The conditions showing when some observables have this property are called compatibility theorems. There are many results in this field using different notions of the compatibility [2],[3],[4] .

A different axiomatic model for measurements of quantum mechanical observables based on fuzzy sets ideas, called an F-quantum space, is presented in [5], where this problem has been stated, too.

In the present paper we show that the Boolean algebras in a quasi-orthocomplemented poset may be embedded into a Boolean algebra.

2. DEFINITIONS AND NOTIONS

Let P be a poset (σ-poset) with a quasi-orthocomplement $\perp$, i.e., with a mapping $\perp : P \to P$ such that :

/2.1/ $(a^{\perp})^{\perp} = a$ for any $a \in P$;

/2.2/ if $a \leqq b$, then $b^{\perp} \leqq a^{\perp}$;

/2.3/ $a \neq a^{\perp}$ for any $a \in P$;

/2.4/ if $a_n \leqq a_m^{\perp}$, for $n \neq m$, then there exists $\bigvee_{n=1}^{\infty} a_n := \sup_n a_n$ in P .

P is called a quasi-orthocomplemented poset /q o p/. Every q- -algebra, suggested by Suppes [6], and every F-quantum space are examples of quasi-orthocomplemented posets. A compatibility theorem for quasi-orthocomplemented lattice has been studied by Dvurečenskij [7].

Let $\mathcal{B}(R)$ be the Borel σ-algebra of the subsets of the real line R. By an observable of P we mean a mapping $x : \mathcal{B}(R) \to P$ such that :

/2.5/ $x(A^c) = x(A)^{\perp}$, $A \in \mathcal{B}(R)$, $A^c := R - A$;

/2.6/ $x(\bigcup_i A_i) = \bigvee_i x(A_i)$, if $A_i \cap A_j = \emptyset$ for $i \neq j$, $A_i \in \mathcal{B}(R), i \geqq 1$.

Let us denote by $\mathcal{R}(x)$ the range of an observable x , i.e., $\mathcal{R}(x) = \{ x(E) : E \in \mathcal{B}(R) \}$. Then $\mathcal{R}(x)$ is a Boolean σ-algebra of P with the minimal and maximal elements $x(\emptyset)$ and $x(R)$, respectively.

In accordance with the theory of quantum logics, we say that two elements $a, b \in P$ are :

/i/ compatible /$a \leftrightarrow b$/ if $a \wedge b$, $a \wedge b^{\perp}$, $a^{\perp} \wedge b \in P$ and $a = a \wedge b \vee a \wedge b^{\perp}$, $b = a \wedge b \vee a^{\perp} \wedge b$;

/ii/ strongly compatible /$a \overset{s}{\leftrightarrow} b$/ if $a \leftrightarrow b \leftrightarrow a^{\perp} \leftrightarrow b^{\perp} \leftrightarrow a$.

We say that two nonempty subsets A and B of P are compatible /strongly compatible/ if $a \leftrightarrow b$ /$a \overset{s}{\leftrightarrow} b$/ for all $a \in A, b \in B$.

A nonvoid subset A of P is said to be f-compatible /"f" for finiteness/ if for all $a_1,\dots,a_{n+1} \in A$ we have :

/i/ $u := a_1 \wedge \dots a_n \wedge a_{n+1} \in P$, $v := a_1 \wedge \dots \wedge a_n \wedge a_{n+1}^{\perp} \in P$;

/ii/ $u \vee v = a_1 \wedge \dots \wedge a_n$.

The subset $A \subset P$ is strongly f-compatible if $A \cup A^{\perp}$ is f-compatible, where $A^{\perp} = \{ a^{\perp} : a \in A \}$.

3. COMPATIBILITY THEOREM

We say that P has c-σ-distributive property if $a \leftrightarrow a_n$, $n \geq 1$, then $a \wedge (\bigvee_{n=1}^{\infty} a_n) = \bigvee_{n=1}^{\infty} (a \wedge a_n)$.

Any F-quantum space has the c-σ-distributive property.

THEOREM 1. Let P be q o p with the c-σ-distributive property and let $\{A_t : t \in T\}$ be a system of Boolean subalgebras of P. The following statements are equivalent :

/1/ $\bigcup_t A_t$ is strongly f-compatible .

/2/ There is a Boolean algebra of P containing all A_t, $t \in T$.

THEOREM 2. Let P be q o p with the c-σ-distributive property and let A be a nonempty set of P. Then, in order to exist a Boolean algebra of P containing A, it is necessary and sufficient for A to be strongly f-compatible.

A similar theorem for Boolean σ-algebras in fuzzy quantum spaces has been proved in [8].

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