Some Marginal Remarks on Fuzzy Manager

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The problem of a decision-maker who is to satisfy his manager's unknown and vague global preferences was investigated in [3]. The methods described there indicate a possibility to characterize also the rationality of manager's approach to the acceptability of the realized decisions. This marginal by-product of the mentioned model is worth to be remembered here at least for the discussion purposes, what is also the main goal of the contribution presented below.

The Considered Model

In the whole paper we denote by $\mathbb{R}$ the set of all real numbers.

Let us suppose that the *decision-maker* may choose his decisions from a non-empty set D, and that every decision is evaluated by $n \geq 2$ criteria, represented by utility functions

$$u_i : D \longrightarrow \mathbb{R} \ , \quad i=1,\ldots,n.$$

Besides the decision-maker there exists another subject, let us call him the *manager*, with his own global utility function $u : D \longrightarrow \mathbb{R}$ such that

$$u(d) = w_1 u_1(d) + \ldots + w_n u_n(d) \quad , \quad d \in D,$$
$$w_1 + \ldots + w_n = 1 \ , \ w_i \geq 0, \ i=1,\ldots,n.$$

The decision-maker aims to satisfy the manager's global preferences but he does not know the global utility function u, (i.e. he does not know the weights $w_1,\ldots,w_n$). He is only allowed to choose a few "experimental" decisions

$$d^{(1)}, \ \ldots \ , d^{(m)} \in D \ , \ m \geq 2,$$

and to estimate the global preferences using some information about the manager's evaluation of the chosen decisions. This problem was generally discussed in [2].

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Vague Comparative Information

The most interesting type of information about the manager's preferences is the global one. After choosing decisions $d^{(1)}, d^{(2)} \in D$, the decision-maker obtaines an information which one of them was globally more acceptable, i.e. which of the relations

$$u(d^{(1)}) \geq u(d^{(2)}) \quad \text{or} \quad u(d^{(1)}) \leq u(d^{(2)})$$

holds. Here we suppose, as well as in [3], that this information may be rather vague and that it is valid with certain degree of possibility. It can be described by a fuzzy relation with a membership function

$$f : D \times D \longrightarrow \langle 0,1 \rangle$$

where $f(d^{(1)},d^{(2)})$ is the possibility that $u(d^{(1)}) \geq u(d^{(2)})$. The complemantary relation $u(d^{(1)}) < u(d^{(2)})$ is valid with the possibility $1-f(d^{(1)},d^{(2)})$, and the possibility of $u(d^{(1)}) = u(d^{(2)})$ is equal to

$$\min\left[f(d^{(1)},d^{(2)}), f(d^{(2)},d^{(1)})\right]$$

(both inequalities are valid simultaneously).

Let us suppose that the decisions $d^{(1)}, d^{(2)} \in D$ were chosen and denote

$$\varphi(i,w,d^{(1)},d^{(2)}) = \frac{\sum_{j=n, j\neq i}^{n} w_j \left[u_j(d^{(2)}) - u_j(d^{(1)})\right]}{u_i(d^{(1)}) - u_i(d^{(2)})} .$$

Then it is not difficult to derive (c.f.[2]) that the inequality

$$u(d^{(1)}) \geq u(d^{(2)}) \tag{1}$$

implies

$$1 \geq w_i \geq \varphi(i,w,d^{(1)},d^{(2)}) , \tag{2}$$

and, on the other hand, the inequality

$$u(d^{(1)}) < u(d^{(2)})$$

implies

$$0 \leq w_i < \varphi(i,w,d^{(1)},d^{(2)}) . \tag{3}$$

Relations (2) or (3) divide the set

$$W = \left\{w=(w_1,\dots,w_n)\in\mathbb{R}^n:\ w_1+\dots+w_n=1,\ w_i\geq 0,\ i=1,\dots,n\right\}$$

into two disjoint subsets connected with certain values of possibility. It means that the set of possible wight-vectors $(w_1,\dots,w_n)\in W$ estimated by means of information (1) about the global fuzzy preferences over $d^{(1)}$ and $d^{(2)}$ is a fuzzy subset of W with membership function $g^{(1,2)}: W \longrightarrow \langle 0,1\rangle$ such that

$$\begin{aligned} g^{(1,2)}(w) &= f(d^{(1)},d^{(2)}) \quad \text{for } w \text{ fulfilling (2)}, \\ &= 1-f(d^{(1)},d^{(2)}) \quad \text{for } w \text{ fulfilling (3)}. \end{aligned}$$

If the "experimental" decisions $d^{(1)},\dots,d^{(m)}$, $m\geq 2$, were chosen then at most n^2 membership functions $g^{(i,j)}$, $i,j=1,\dots,m$, are given (without any loss of generality we may put $g^{(i,i)}\equiv 1$, $i=1,\dots,m$). Some of these functions need not be defined if no information about the possibility of

$$u(d^{(i)}) \geq u(d^{(j)})$$

was obtained from the manager. In such a case we may define $g^{(i,j)}(w) \equiv 1$, too.

The set of possible weight vectors $(w_1,\dots,w_n)$ estimated according to the information obtained about the manager's global preferences over the "experimental" decisions $d^{(1)},d^{(2)},\dots,d^{(m)}$ is also a fuzzy subset of W. Its membership function

$$g^{(1,\dots,m)}: W \longrightarrow \langle 0,1\rangle$$

is defined by

$$g^{(1,\dots,m)}(w) = \min\left[g^{(i,j)}(w):\ i,j = 1,\dots,m\right]$$

(with the definitorically completed values in cases mentioned above).

The values of the membership function $g^{(1,\dots,m)}(w)$ indicate the possibility that the weight-vector $w\in W$ is admissible according to the information obtained about the global ordering of $d^{(1)},\dots,d^{(m)}$. On the other hand, the properties of the membership function $g^{(1,\dots,m)}$ can also characterise some qualities of the manager.

Characteristics of Manager

The characteristics of the rationality of the manager's preferences (or more exactly of the qualities of the information obtained about it) based on the global evaluation of the "experimental" decisions $d^{(1)},\dots,d^{(m)}$ can be derived from the form of the membership function $g^{(1,\dots,m)}$. There are namely two features that can be easily observed.

The consistency of preferences: If for all $w \in W$

$$g^{(1,\dots,m)}(w) < \tfrac{1}{2}$$

then there necessarily exist contradictions in the manager's preferences such that (using transitivity)

$$u(d^{(i)}) > u(d^{(j)}) > u(d^{(i)})$$

with possibility at least

$$1 - \max\left[g^{(1,\dots,m)}(w): \ w \in W\right] .$$

This conclusion can be easily derived from the construction of $g^{(1,\dots,m)}$. The lower the maximal value of $g^{(1,\dots,m)}(w)$ over W is the higher the possibility of manager's contradictory approach to the preferences over D is.

The sharpness of preferences: The difference between the values of $f(d^{(i)},d^{(j)})$ and $f(d^{(i)},d^{(j)})$ for arbitrary $d^{(i)}$, $d^{(j)}$, $i,j = =1,\dots,m$, $i \neq j$, indicates the sharpness with which the manager distinguishes the global acceptabilities of $d^{(i)}$ and $d^{(j)}$. Analogously, the difference between

$$\max\left[g^{(1,\dots,m)}(w): \ w \in W\right] \quad \text{and} \quad \min\left[g^{(1,\dots,m)}(w): \ w \in W\right]$$

characterises in certain degree the sharpness applied during the manager's evaluation of the chosen decisions $d^{(1)},d^{(2)},\dots,d^{(m)}$.

Questions

The considerations made above imply a few questions and comments.

- The first one of them is, if and how it is possible to introduce an effective quantitative measure of the manager's consistency or inconsistency based on his evaluation of $d^{(1)},\dots,d^{(m)}$, e.g. the value

(5) $$\tfrac{1}{2} - \max\left\{g^{(1,\dots,m)}(w):\ w \in W\right\}$$

  as a measure of inconsistency.
- Is it possible to suggest an analogous quantitative measure of the manager's sharpness manifested by his evaluation of $d^{(1)},\dots$ $\dots,d^{(m)}$, e.g. the difference between the maximal and minimal values of $g^{(1,\dots,w)}(w)$ over W, or the difference between the maximal and the second maximal value of that function? Or is it more convenient to consider the whole function $g^{(1,\dots,m)}$ for the best characteristic of the manager's sharpness?
- What will be the analytical properties of such quantitative measures of inconsistency and sharpness, especially, how they develop with increasing number of the "experimental" decisions, i.e. with increasing m ? It is evident that the value (5) is non-increasing with m, and as it is limited from below by 0 there exists it limit. An analogous property cannot be expected in case of measures of the sharpness mentioned above, as both parts of the proposed differences are non-increasing with m.

References

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