ON THE DISTANCE BETWEEN MODIFIED FUZZY NUMBERS

B l a h o s l a v   H a r m a n

Technical University, Liptovský Mikuláš - CSSR

In [1] is discused the modified approach to the fuzzy numbers calculus. The notion of the modified fuzzy number is the fundamental one. Let $\mathcal{L} = \{ f \in L_1(\mu) ; f \geqq 0 \}$ where μ is the Lebesgue measure on the real line and $L_1(\mu)$ be a set of the Lebesgue integrable functions. Let $\mathcal{D}$ be the set of the distributions on the fundamental function space $\mathcal{L}$ (for details see e.g. [2], [3]). Let us denote $\mathcal{F} = \mathcal{L} \cup \mathcal{D}$. Let us introduce the equivalence relation $\sim$ on the set $\mathcal{F}$ in the following way

$$u,v \in \mathcal{F} \quad , u \sim v \Longleftrightarrow \exists \alpha \in (0,+\infty) : \alpha u = v \ [\mu] \text{ a.e.}$$

Now we can recall the definition of the modified fuzzy number.

Definition 1. Let $\Phi = \mathcal{F}/\sim$ be the factor set which corresponds to the equivalence relation $\sim$. The elements of the set Φ will be called as the modified fuzzy numbers.

Remark: Let us denote $c(u) = \{ v \in \mathcal{F} ; u \sim v \}$. Let $\xi \in \Phi$ be such an element of Φ for which exists $u \in \mathcal{L}$ such that $\xi = c(u)$. The elements of such types represents the "proper" fuzzy numbers on the contrary to the elements of types $c(\delta_a)$ which can be looked at as a representative of the crisp number a . ($\delta_a(x) = \delta(x-a)$, $a > 0$ is the translation of the Dirac function $\delta(x)$.)

Let $\xi , \eta \in \Phi$. Let f,g be such that $\xi = c(f)$, $\eta = c(g)$,

$$\| f \| = \| g \| = \int_{-\infty}^{+\infty} f(x) \mu(dx) = \int_{-\infty}^{+\infty} g(x) \mu(dx) = 1 .$$

Without fear of being confused we can use, for the sake of simplicity, the representatives f resp. g insted of ξ , η respectively. The modified fuzzy numbers f,g will be called as orthogonal iff $supp(f) \cap supp(g) = \emptyset$.

Definition 2. Let $\varrho : \Phi \times \Phi \rightarrow \langle 0,+\infty)$ be the map such that the following conditions are satisfied:

i/ $\varrho(f,g) \geqq 0$, $\varrho(f,g) = 0 \Leftrightarrow f = g\ [\mu]$ a.e.

ii/ $\varrho(f,g) = \varrho(g,f)$

iii/ if f,g,h are pairwise orthogonal elements of Φ then $\varrho(f,h) \leqq \varrho(f,g) + \varrho(g,h)$.

The map ϱ will be called as the quasimetric on the space of the modified fuzzy numbers .

It is possible to prove the following theorems.

<u>Theorem 1.</u> The map ϱ defined by the relation

$$\varrho(f,g) = \frac{1}{2}\int_{-\infty}^{+\infty} |f(x)-g(x)| \left\{ \int_{-\infty}^{+\infty} |x-y|\,|f(y)-g(y)|\, \mu(dy) \right\} \mu(dx)$$

is the quasi metric on Φ .

Further fundamental properties of the quasimetric ϱ are formulated as follows:

<u>Theorem 2.</u> i/ Let $supp(f_n) \supset supp(f_{n+1})$, $\bigcap_{n=1}^{\infty} supp(f_n) = \{a\}$

$supp(g_n) \supset supp(g_{n+1})$, $\bigcap_{n=1}^{\infty} supp(g_n) = \{b\}$.

Then $\lim_{n\to\infty} \varrho(f_n,g_n) = |b-a|$.

ii/Let f_a, g_a be translations of the fuzzy numbers f,g. Let f,g are orthogonal such that $\sup(supp(f)) \leqq \inf(supp(g))$. Then

$\varrho(f,g_a) = \varrho(f,g) + a$

$\varrho(f_a,g_a) = \varrho(f,g)$

iii/Let $supp(f) \subset (a_1,b_1)$, $supp(g) \subset (a_2,b_2)$,

$a_1 < b_1 \leqq a_2 < b_2$. Then

$$a_2 - b_1 \leq \varrho(f,g) \leq b_2 - a_1 \quad .$$

iv/ Let $\omega(g) = \frac{1}{2}\iint_{-\infty}^{+\infty} |x-y|\, g(x)g(y)\mu(dx)\mu(dy)$. Then

$$\varrho(\delta_a, g) = \int_{-\infty}^{+\infty} |x-a|\, g(x)\mu(dx) + \omega(g) \quad .$$

References: [1] Harman,B.: The modified approach to the fuzzy numbers calculus. Submitted to Proceed. of the Conference on the interval and fuzzy mathematics, Poznań 1988

[2] Kolmogorov,A.N.,Fomin,S.V.: Elementy teorii funkcij i functionalnovo analiza. Moskva 1972

[3] Gelfand,I.M.,Šilov,G.E.: Obobščenyje funkciji i dejstvija nad nimi. Moskva 1959