

## A MATHEMATICAL MODEL OF EVALUATION ON QUALITY OF BOOK

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Abstract. In this paper, a mathematical model of evaluation approximate to objective on quality of book be made by the aid of fuzzy integral and convergence theorem of fuzzy integral sequence, make an attempt to rationally evaluate the quality of book.

Keywords. Factor set, Evaluator set, Fuzzy integral, Importance measure, Set-valued statistics, Comprehensive evaluation.

## 1. FACTOR SET AND EVALUATION SET

It depends on many factors whether the quality of book is good or not, for example, ideological content, scientific level, style of writing, binding and layout, error rate, printing etc. Generally, let  $X = \{x_1, x_2, \dots, x_n\}$  be a factor set on quality of book.

The evaluators are usually made of many persons of different strata, for example, expert and professor, intellectual, teacher, student etc. then we can let  $E_1 = \{e_{11}, e_{12}, \dots, e_{1k_1}\}$ ,  $E_2 = \{e_{21}, e_{22}, \dots, e_{2k_2}\}$ ,  $\dots$ ,  $E_m = \{e_{m1}, e_{m2}, \dots, e_{mk_m}\}$  be evaluator sets of  $m$  strata.

## 2. IMPORTANCE MEASURE AND ITS DETERMINED METHOD

The actions of numerous factors on quality of book are different, they may be large or little. In this paper, we described the important degree of every factor on quality of book by aid of importance measure, in Wang Zhenyuan (1985) it is shown that the concept of importance measure, it plays an important part in the evaluation on quality of book. In many papers (see, e.g., Wang Peizhuang and Liu Xihui, 1984) it is shown that the method of set-valued statistics, now we determined the importance measure  $\mu$  on factor set  $X$  and the importance measure  $\nu$  on evaluator set  $E$  by this method.

We show the method by determining importance measure  $\mu$  on factor set  $X$ .

$S$  experts of book publishing and management make statistical tests to important degree of every factor on quality of book. We

select initial value  $q$  ( $1 \leq q \leq n$ ) (When we determine the important degree  $\mu(x_1)$  on single factor  $x_1$ , select  $q=1$ ; When we determine the important degree  $\mu(x_{i_1}, x_{i_2})$  on double factors  $(x_{i_1}, x_{i_2})$ , select  $q=2$ ; ...; When we determine the important degree  $\mu(x_1, x_2, \dots, x_n)$  on  $n$  factors  $(x_1, x_2, \dots, x_n)$ , select  $q=n$ ), person  $j$

make the statistical tests according to following step:

First step, person  $j$  select  $q$  factors which he thought the most important in factor set  $X$ , we obtain subset

$$X_1^{(j)} = \{x_{i_1}^{(j)}, x_{i_2}^{(j)}, \dots, x_{i_q}^{(j)}\} = X$$

Second step, person  $j$  select  $2q$  factors which he thought the most important in factor set  $X$ , we obtain subset

$$X_2^{(j)} = \{x_{i_1}^{(j)}, \dots, x_{i_q}^{(j)}, x_{i_{q+1}}^{(j)}, \dots, x_{i_{2q}}^{(j)}\} = X_1^{(j)}$$

.....

Step  $u$ , person  $j$  select  $uq$  factors which he thought the most important in factor set  $X$ , we obtain subset

$$X_u^{(j)} = \{x_{i_1}^{(j)}, x_{i_2}^{(j)}, \dots, x_{i_{uq}}^{(j)}\} = X_{u-1}^{(j)}$$

.....

If natural number  $t$  satisfies  $n=tq+r$  and  $1 \leq r < q$ , then this iterative process ended to

Step  $t+1$ , assume  $X_{t+1}^{(j)} = X$

If selected initial value  $q$  satisfies  $n=tq$ , then the important degree of  $q$  factors  $x_{i_{k_1}}, x_{i_{k_2}}, \dots, x_{i_{k_q}}$  may be determined by

$$\mu(x_{i_{k_1}}, x_{i_{k_2}}, \dots, x_{i_{k_q}}) = \frac{2}{\text{sqrt}(t+1)} \sum_{j=1}^s \sum_{u=1}^t \sum_{l=1}^q \chi_{X_u^{(j)}}(x_{i_{k_l}})$$

If selected initial value  $q$  satisfies  $n=tq+r$  ( $1 \leq r < q$ ), then the important degree of  $q$  factors  $x_{i_{k_1}}, x_{i_{k_2}}, \dots, x_{i_{k_q}}$  may be determined by

$$\mu(x_{i_{k_1}}, x_{i_{k_2}}, \dots, x_{i_{k_q}}) = \frac{2}{s[qt(t+1)+2n]} \sum_{j=1}^s \sum_{u=1}^{t+1} \sum_{l=1}^q \chi_{X_u^{(j)}}(x_{i_{k_l}})$$

### 3. FUZZY INTEGRAL AND ITS CALCULUS

In many papers (see, e.g., M. Sugeno, 1974) it is shown that fuzzy integral of a measurable function  $f$  over  $X = \{x_1, x_2, \dots, x_n\}$  with

where  $\mu$  is a fuzzy measure

$$\mu(A) = \sup_{A \in \mathcal{P}(X)} [\mu(A) \wedge \inf_{x_i \in A} f(x_i)]$$

Because of easy to operate, the Sugeno's fuzzy integral may be calculated by the following method:

Order the value of larger to small, the  $f(x_i)$  ( $i=1, 2, \dots, n$ ) can be arranged in  $f^{(1)}, f^{(2)}, \dots, f^{(n)}$ ; Corresponding,  $\{x_1, x_2, \dots, x_n\}$  be arranged in  $x^{(1)}, x^{(2)}, \dots, x^{(n)}$ , write

$$A^{(i)} = \{x^{(1)}, x^{(2)}, \dots, x^{(i)}\}, i=1, 2, \dots, n$$

and calculate  $f^{(i)} \wedge \mu(A^{(i)})$  for every  $i$  ( $i=1, 2, \dots, n$ ), then find the minimum value

$$\min_{1 \leq i \leq n} \{f^{(i)} \wedge \mu(A^{(i)})\}$$

is the value of fuzzy integral  $\int f d\mu$ .

#### 4.2. EVALUATION ON QUALITY OF SINGLE FACTOR

Let  $k_p$  evaluator independently make a mark on every factor  $x_i$  ( $1 \leq i \leq n$ )

of book  $X$  which will be evaluated. Now assume  $f_{pq}(x_i)$  (limit to  $[0, 1]$ )

is the mark which is given by evaluator  $q$  ( $1 \leq q \leq k_p$ ) of stratum  $p$

of factor  $x_i$ . If  $i$  be fixed, it is a parent population  $F_{ip}$

with mathematical expectation  $g_{ip}$  that the marks be given by nu-

merous evaluators of  $p$  strata to factor  $x_i$  of book  $X$  time after time,

the marks  $\{f_{p1}(x_i), f_{p2}(x_i), \dots, f_{pk_p}(x_i)\}$  which be made by  $k_p$

evaluators of stratum  $p$  for factor  $x_i$  at a time, be a capacity  $k_p$

random subsample of the parent population  $F_{ip}$ , namely  $f_{p1}(x_i)$ ,

$f_{p2}(x_i), \dots, f_{pk_p}(x_i)$  are random variables of same distribution of

mutual independence and have same mathematical expentation  $g_{ip}$ . By

the Kolmogorov strong law of large numbers, in probability 1 have

$$\lim_{k_p \rightarrow \infty} \frac{1}{k_p} \sum_{q=1}^{k_p} f_{pq}(x_i) = g_{ip}$$

When  $k_p$  is finite, the above formula is all tenable for every  $p$ .

Using convergence theorem of fuzzy integral sequence, have

$$\lim_{k_p \rightarrow \infty} \int \frac{1}{k_p} \sum_{q=1}^{k_p} f_{pq}(x_i) d\nu = \int g_i d\nu = g(x_i)$$

$g(x_i)$  is called evaluation on single factor  $x_i$ , where  $\nu$  be a importance measure on  $E$ , it can be determined by the method of set-valued statistics.

##### 5. COMPREHENSIVE EVALUATION ON QUALITY OF BOOK

We have already obtained all evaluations on every single factor  $x_i$  ( $i=1, 2, \dots, n$ ), if the importance measure  $\mu$  on  $X$  be determined, then fuzzy integral

$$\int g d\mu = \sup_{A \in \mathcal{P}(X)} [\mu(A) \wedge \min_{x_i \in A} g(x_i)]$$

is evaluatable. Now its value is a result of comprehensive evaluation on quality of book  $X$ .

##### 6. CONCLUDING REMARKS

In this paper, a method of evaluating quality of book approximate to objective has been given, it be suitable for choose highly rated book through public appraisal, and evaluation result is more rational. But this method relate to more date and need organized lots of persons, so the concrete practice of evaluation is more difficult.

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