The Application of the Criteria of the Maximum Membership Domination to the Recognition of logging lithology

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Abstract

In the process of the recognition of the logging lithology, the measured lithology is a blurring concept. Therefore, it is possible to employ the method of fuzzy pattern recognition to do so.

Suppose that there are m states of the lithology of strata in the logging area,expressed by the fuzzy subset $\underset{\sim}{A}i$(i=1,...,m).we assume again that there are n observed quantities relating to the lithology of strata, and we think of the result of each observation as a n dimensionality vector, indicated by X=(X1,X2,...,Xn). So It can be used to describe the states of the lithology of strata which have been penetrated by drilling.

$\mu_{\underset{\sim}{A}i}(X)$ denotes the degree of X subordinated to the lithology $\underset{\sim}{A}i$, while the aim of fuzzy pattern recognition is usually to work out each $\mu_{\underset{\sim}{A}i}(X)$,(i=1,...m). if $\mu_{\underset{\sim}{A}i}(X)$ =max$\{\mu_{\underset{\sim}{A}i}(X) ,1 \le i \le m\}$we can judge that X belongs to $\underset{\sim}{A}i$ and further predict the lithology of strata in correspondence to X. Its crucial question is to get $\mu_{\underset{\sim}{A}i}(X)$. However,the approaches applied to get $\mu_{\underset{\sim}{A}i}(x)$ have much to be desired because fuzzy sets have limited ways to operate and invisibly lose a lot of indispensable information. This article deliberately steers clear of the attempt to get $\mu_{\underset{\sim}{A}i}(X)$, but introduces the concept of the membership domination and the criteria of the maximum membership domination to the fuzzy pattern recognition for the logging lithology.

Upon the basis of the principle mentioned above,this article puts forward a new method to the fuzzy pattern recognition for the logging lithology . The crucial question of the method lies in how to work out $\mu_{\underset{\sim}{A}i}(X_j)$, (i=1,...m ; j=1,...n). Moreover,it also introduces two practical ways of getting $\mu_{\underset{\sim}{A}i}(X_j)$.

1 Introduction

Generally, the logging interpreter analyses subjectively and judges by his own experience the lithology of strata which have been penetrated by drilling in terms of some logging data. It is extremely difficult to artificially judge the lithology of strata because of the influence of the complexity of strata and various factors, somtimes leading to the partial or most partial overlap of the logging curve in the different lithologic strata. With the enlargement of the application of the logging in geology, the method of traditional, artificial, and subjective recognition of the lithology of strata has not been suitable to the requirement of the complex geology tasks.

In order to overcome the errors brought in by the artificial method, the triangulation method and the recognition method of statistical probability have come on recent years. But these methods are too complex in themselves, or too high in condition,to popularize in their practical uses. Recently,considering that the lithology of strata has not certain outlay,it is a blurring concept.We can apply the method of fuzzy sets to the fuzzy pattern recognition of the logging lithology. This article is further going to introduce a new method of the fuzzy pattern recognition, that is, applying the criteria of the membership domination and the maximum membership domination to the recognition of the logging lithology. The advantages of the method are:

(1) to synthetically judge the information to influence various kinds of factors of the lithology of strata.

(2) to have a strong entire recognition ability.

(3) to lose little information in its operation.

2. Method Description

We assume that there are m states of the lithology of strata in the logging area,such as gravel,gritstone, middle sandstone, fine sandstone, and mudstone, etc. Since each lithology state is a blurring concept,it can be described as a fuzzy subset . So its lithology of strata can respectively be named $\underset{\sim}{A1}$, $\underset{\sim}{A2}$,... $\underset{\sim}{Am}$. In general, the real state of lithology of strata in the logging area, for the logging interpreter,is an unknown number and it can not be directly observed, either. In order to make a correct decision, we need collect an amount of information and have a careful observation of several observed variables relating to the lithology of strata for the description of the state of the lithology of strata.If each variable can get k discrete values,the result of each observation is a n dimensionality vector, indicated by X= (X1, X2, ... Xn), and all the possible vectors have altogetherI=k ,whose sets can be expressed by U .

Now we assume that the section observed is U .$\underset{\sim}{A1}$, $\underset{\sim}{A2}$,...,$\underset{\sim}{Am}$ is a fuuzy subset in F(U),where F(U) is aggregate formed by the whole fuzzy subsets on U. The vector X=(X1,X2,...Xn) is called the sample on U. The degree of sample X belonging to lithology $\underset{\sim}{Ai}$ can be expressed by $\mu_{\underset{\sim}{Ai}}(X)$,(i=1,2,...,m),that is , $\mu_{\underset{\sim}{Ai}}(x)$ is the membership function of $\underset{\sim}{Ai}$, while the degree of the sample X belonging to the fuzzy subset $\underset{\sim}{Ai}$ only from the observation of the Xi-th index of sample X can be expressed by $\mu_{\underset{\sim}{Ai}}(X)$(i=1, 2,...,m;j=1,2,...,n).In general, the process of the pattern recognition is to work out $\mu_{\underset{\sim}{Ai}}(X)$ (i=1,2,...,m), and we obtain the $\mu_{\underset{\sim}{Ai}}(X)=\max\{\mu_{\underset{\sim}{Ai}}(X), 1\le i\le m\}$,while $i\in\{1, 2,...,m\}$,we can judge X correspondingly belongs to Ai so as to get the lithology of strata relating to X.This is the main essence of applying the criteria of the maximum membership to the recognition of the logging lithology, here the crucial question lies in how to work out $\mu_{\underset{\sim}{A_i}}(X)$. It is a difficult question and there is no good way out at present.

Now we deliberately steer clear of the direct attempt to get $\mu_{\underset{\sim}{A_i}}(X)$, but introduce the concept of the membership domination and the criteria of the maximum membership domination to the recognition of the logging lithology.

2--1 Membership Domination and Criteria of Maximum Membership Domination

Let $\max\limits_{1\le i\le m}\{\mu_{\underset{\sim}{A_i}}(Xj)\}$ is the maximum value of $\{\mu_{\underset{\sim}{A_i}}(Xj),\ 1\le i\le m\}$, $\mathrm{Smax}\limits_{1\le i\le m}\{\mu_{\underset{\sim}{A_i}}(X)\}$ is the submaximum value of $\{\mu_{\underset{\sim}{A_i}}(Xj),\ 1\le i\le m\}$. Then we define the partial membership domination as

$$C\mu_{\underset{\sim}{A_i}}(Xj)=\begin{cases}\max\limits_{1\le i\le m}\{\mu_{\underset{\sim}{A_i}}(Xj)\}-\mathrm{Smax}\limits_{1\le i\le m}\{\mu_{\underset{\sim}{A_i}}(Xj)\}=\mu_{\underset{\sim}{A_k}}(Xj)-\mu_{\underset{\sim}{A_l}}(Xj), & i=k\\ 0, & i\ne k\end{cases}$$

Where the partial domination $C\mu_{\underset{\sim}{A_i}}(Xj)$ denotes that the membership function of fuzzy subset Ak is maximum in terms of the index j-th of X . Therefore, only from the observation of index Xj-th,the degree of X belonging to $\underset{\sim}{A}k$ is superior to the other $\underset{\sim}{Ai}$ $(i \ne k)$,whose membership domination is $\mu_{\underset{\sim}{A_k}}(Xj)-\mu_{\underset{\sim}{A_l}}(Xj)$. It is clear that the domination belonging to $\underset{\sim}{Ai}$ $(i\ne k)$ is naught, where i=1, 2,...,k-1,K+1,...,m.In addition, we also define the entire membership domination as

$$C\mu_{\underset{\sim}{A_i}}(X)=\sum_{j=1}^{n} C\mu_{\underset{\sim}{A_i}}(Xj),\quad i=1,2,\dots,m.$$

where $C\mu_{\underset{\sim}{A_i}}(X)$ denotes the domination of hte sample belonging to the state $\underset{\sim}{Ai}$ of the lithology of strata in terms of every index Xj-th (j=1,2,...,m).

Then the criteria of the maximum membership domination is:

$$C\mu_{\underset{\sim}{A_i}}(X)=\max\{C\mu_{\underset{\sim}{A_1}}(X),C\mu_{\underset{\sim}{A_2}}(X),\dots,C\mu_{\underset{\sim}{A_m}}(X)\}$$

while i = 1,2,...,m, we judge that the sample X correspondingly belongs to fuzzy subset $\underset{\sim}{Ai}$.

It is known from the above that we marshal the measurement value of the lithology of strata into the vector X= (X1, X2,...,Xn) in terms of the data. Then we work out $C\mu_{\underset{\sim}{A_i}}(Xj)$ and $C\mu_{\underset{\sim}{A_i}}(X)$ respctively.If the entire membership domination of X in some stratum in correspondence to lithology $\underset{\sim}{A}i$ corresponds to:

$$C\mu_{\underset{\sim}{A_i}}(X)=\max\{C\mu_{\underset{\sim}{A_1}}(X),\ C\mu_{\underset{\sim}{A_2}}(X),\dots,C\mu_{\underset{\sim}{A_m}}(X)\}$$

we can judge the lithology of strata in correspondance to X as $\underset{\sim}{A}i$. A new method of the fuzzy pattern recognition of logging lithology can be formed in terms of the criteria of the maximum membership domination .

2--2 The Determination of Membership Function $\mu_{\underset{\sim}{A_i}}(Xj)$

It is known from the above that we must choose $C\mu_{A_i}(Xj)$ before we work out each $C\mu_{A_i}(X)$. So how to choose membership function $\mu_{A_i}(Xj)$ is the crucial question to employ the method of the fuzzy pattern recogniation to recognize the lithology of strata correctly. There are many common methods to determine the membership function,but each has its own limitation of its way. Here we use the following two methods.

(1) We assume that the same kind of logging quantity of the same lithology strata in the identical area obeys its normal distribution. Then we obtain

$$\mu_{A_i}(Xj) = e^{-\frac{1}{2}\left(\frac{Xj-aij}{\sigma ij}\right)^2}$$ where a_{ij} , σ_{ij} is the mean value and the variance of index j of X in the state of lithology i. From the mean value of the logging curve of various lithologic strata of the logging section statistics of the known lithology in the working area , we obtain

$$a_{ijp} = \frac{1}{p}\sum_{p=1}^{p}\bar{X}_{ijp} \qquad \sigma_{ijp} = \sqrt{\frac{\sum_{p=1}^{p}(\bar{X}_{ijp} - a_{ij})^2}{p-1}},$$

$i=1,2,...,m;\ j=1,...,n.$

where p is the number of strata accounted for lithology i.

(2) In terms of the observed samples in the logging section of the known lithology within the working area , we can get $P_{A_i}(Xj)$, where $P_{A_i}(Xj)$ denotes the sample of $\in$ Ai-fuzzy subset in correspondance to the probability density distribution of the changing index Xi, where symoble $\in$ stands for X correspodingly belonging to fuuzzy subset Ai which means $\mu_{A_i}(X) \geqslant \mu_{A_K}(X)$ (k=1,...,i-1,i+1,..., m).By using the following formula:

$$\mu_{A_i}(Xj) = \frac{P_{A_i}(Xj)}{\sum_{i=1}^{m} P_{A_i}(Xj)}$$ we can obtain $\mu_{A_i}(Xj)$.

for $P_{A_i}(Xj)$, we can work out in the following way.According to the result of geo-exploration, we can give m standard lithlogoy A1, A2,...,Am of a well section in the working area,which is m known subsets of U ,and the lithology of each stratum in this section has been already given . Union distribution $P_{A_i}(X)=P_{A_i}(X1,X2,...,Xn)$ is hardly estimated in its practice. With the condition of the statistical independance of each variable, we have:

$$p_{A_i}(X) = \prod_{j=1}^{n} p_{A_i}(Xj) \quad \text{where} \quad p_{A_i}(Xj) = \begin{cases} P_{ij1}, & \text{when } Xj=V_1 \\ P_{ij2}, & \text{when } Xj=V_2 \\ \vdots & \vdots \\ P_{ijK}, & \text{when } Xj=V_K \end{cases}$$

$$\sum_{s=1}^{k_i} P_{ijs} = 1 \quad \text{where} \quad P_{ijs} = P_{A_i}(Xj = V s), s=1,2,\ldots,k \ ,$$

Vs —— Xj can obtain s-th discrete values.
The subsample estimation of digit group$\{P_{ijs}\}$can usually be expressed as m matrices of(n,k)-type, indicated by $Ri=r_{ijs}$,where r_{ijs} is the frequency of j-th variable taking s-th values in i-th sub-sample, s=1,2,..., k; j=1,2,...n; i=1,2,..., m. As for i,j, they have the $\sum_{s=1}^{k} r_{ijs} = 1$.
When k=2 , P_{A_i} (Xj) can be worked out in the following way $P_{A_i}(Xj)=(p_{ij})^{x_j} \cdot (q_{ij})^{1-x_j}$ where $P_{ij} = \hat{P}_{A_i}(Xj=1)$, $q_{ij} = \hat{P}_{A_i}(Xj=0)$, $P_{ij} + q_{ij} = 1$. The sub-sample estimation of digit group$\{P_{ij}\}$ and$\{q_{ij}\}$ can be expressed as m matrices of (n,2)-type, indicated by Ri. rij , where rij is the same as the above,but s=0,1.Similarly,P_{A_i} (Xj) is $P_{\underset{\sim}{A_i}}$ (Xj) (P_{A_i} (X) ——$P_{\underset{\sim}{A_i}}$ (X).)

THe older the geological body, the more the overlap caused by the geological process.Therefore the geological phenomenon have not only random uncertainty,but also fuzzy uncertainty which makes the lithology of strata too sophisticated to recognize . The second method above correctly connect randomness with fuzziness,basically reflecting the features of the geological phenomenon with randomness and fuzziness . In addition , when using the methods above, we unnecessarily assume that the same logging area obeys the normal distribution , and again unnecessarily consider the individual interference and the influence of the environment condition during the work so as to make the method have strong adoptability.

3. Illustration

We introduce this method into the recognition of logging lithology in a coal field. There are five kinds of lithology,that is, gravel $\underset{\sim}{A1}$, gritstone $\underset{\sim}{A2}$,middle sandstone $\underset{\sim}{A3}$, fine sandstone $\underset{\sim}{A4}$, and mudstone $\underset{\sim}{A5}$.The measurement sample X in each stratum consists of the mean value Ra, GR, Jrr of the logging curve in the three logging methods. Then X= (X1, X2, X3) , where X1=Ra, X2=GR, X3=Jrr.In the lithology i (i=1,...,5),the mean value a and variance σ_{ij} of index j-th of sample X have already worked out (as in the following diagram 1).

The sample X=(X1,X2,X3) of 15 strata in the working area of the coal field is given out in diagram 3 . The lithology of strata is given by the logging interpreter. Now we have the fuzzy pattern recognition by computer , that is, applying the criteria of the maximum membership domination to the lithologic recognition of each stratum, Firstly,work out stratum p (p=1,...,15) $\mu_{\underset{\sim}{A_i}}(Xj)$ (i=1,..., 5; j=1,2,3) Then, $C\mu_{\underset{\sim}{A_i}}(Xj)$ of each stratum p.Lastly, synthetically work out $C\mu_{\underset{\sim}{A_i}}(X)$. Getting max $\{c\mu_{A_i}(X),\ 1 \le i \le 5\}$ from each stratum p,we can recognize the lithology of each stratum. The result is listed below (in diagram 4).

Diagram 1

aij / order		the order number of loggimg method		
		1 Ra (Ω,m)	2 GR (γ)	3 Jrr(x10[4] cpm)
i	1,gravel	114.83	9.417	6.315
	2,gritstone	67.25	15.25	7.513
	3,middle-sandstone	46.50	17.71	8.375
	4,fine sandstone	25.58	20.00	10.42
	5,mudstone	7.375	27.50	13.75

Diagram 2

σij		the order number of logging method j		
		1 Ra (Ω,m)	2 GR(γ)	3 Jrr(x10 cpm)
i	1,gravel	16.43	2.275	1.493
	2,grit-stone	7.557	2.261	2.569
	3,middle-sandstone	5.469	1.671	1.873
	4,fine-sandstone	4.379	2.089	3.282
	5,mudstone	1.811	3.030	3.468

Diagram 3

order	logging value			original
	Ra= X1	GR = X2	Jrr=X3	identified lithology
1	115.0	10.0	4.5	gravel
2	120.0	9.2	5.0	gravel
3	130.0	7.5	5.8	gravel
4	48.0	16.0	8.0	middle-sandstone
5	5.5	28.0	14.5	mudstone
6	50.0	15.0	6.0	middle-sandstone
7	24.0	20.5	9.0	fine-sandstone
8	4.0	30.0	12.0	mudstone
9	140.0	7.5	5.0	gravel
10	85.0	20.0	15.0	grit stone
11	25.0	21.0	10.0	fine-sandstone
12	12.0	30.0	20.0	mudstone
13	65.0	15.0	7.5	gritstone
14	30.0	13.0	4.0	middle-sandstone
15	10.0	12.0	2.0	mudstone

Diagram 4

ONS P	OIL	the membership domination of sample X to A					$\max\{\mu_{A_i}(X)\}$	$X \in A_i$	oberved lithology of logging	COR
		$\mu_{A_1}(X)$	$\mu_{A_2}(X)$	$\mu_{A_3}(X)$	$\mu_{A_4}(X)$	$\mu_{A_5}(X)$				
1	gravel	1.899	0.0482	0	0	0	1.899	A1	gravel	✓
2	gravel	1.9537	0	0	0	0	1.9537	A1	gravel	✓
3	gravel	1.4790	0	0	0	0	1.4790	A1	gravel	✓
4	middle sandstone	0	0.3561	0.9240	0	0	0.9240	A3	middle sandstone	✓
5	mudstone	0	0	0	0	2.086	2.086	A5	mudstone	✓
6	middle sandstone	0.1282	0.7255	0.7410	0	0	0.7410	A3	middle sandstone	✓
7	fine sandstone	0	0	0	1.9367	0	1.9367	A4	fine sandstone	✓
8	mudstone	0	0	0.0352	0.001	0.8876	0.8876	A5	mudsone	✓
9	gravel	1.0422	0	0	0	0	1.0422	A1	gravel	✓
10	gritstone	0.1291	0	0	0.609	0.5595	0.609	A4	gritstone	✗
11	fine sandstone	0	0	0	2.1736	0	2.1736	A4	fine sandstone	✓
12	mudstone	0	0	0	0	0.9245	0.9245	A5	mudstone	✓
13	gritstone	0	1.7753	0	0	0	1.7753	A2	gritstone	✓
14	middle sandstone	0	0.4305	0	0.5903	0	0.5903	A4	middle sandstone	✗
15	mudstone	0.169	[illegible]	0	0	0.3480	0.3480	A5	mudstone	✓

ONS ------order number of strata
OIL ------original identified lithology
COR ------correspondency

4. Conclusion

From the result in the diagram 4 above,we know that there are two strata not corresponding to the original identification.Coincident rate is 86%.From the diagram 4 the maximum membership domination of the lithology of sample X of the strata to be observed is all above 0.5 (except one).The two strata of the recognition erro are that the maximum membership domination of sample X to each lithology A_i is either naught or has a little difference between each other.We estimate the reason that it is for many errors in the data-collection or the bad logging condition .In order to raise the coincident rate of recognition,the following formula can be also employed in the determination of the membership function $\mu_{A_i}(Xj)$, in addition to overcoming the factors above:

$$\mu_{A_i}(Xj) = \frac{P_{A_i}(Xj)}{\sum_{i=1}^{m} P_{A_i}(Xj)}$$ From which we can get

$\mu_{A_i}(Xj)$.The recognition error caused by the error of data to result in inaccuracy and unstability of a_{ij} and σ_{ij} , can thus be avoided.

Above all,if the method of the recogntion of the logging lithology introduced by the article would be programmed into the computer, it has the charactors of rapidness, simpleness, effectiveness and high economic profits. It is also suitable to the popularization in practical use, and its applied region can be enlarged, such as applying to the seismic signal recognition in geophyasical exploration.

References

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