FUZZY RELIABILITY OF PARALLEL SYSTEMS

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ABSTRACT: This paper is one of continuation for <FUZZY RELIABILITY>. It provides the calculating method which can be used in calculation of the fuzzy reliability for the parallel systems. In this paper authors derive the series formulas by means of the basic concepts and principles of fuzzy reliability. This paper only considers the indexes of FA mode.

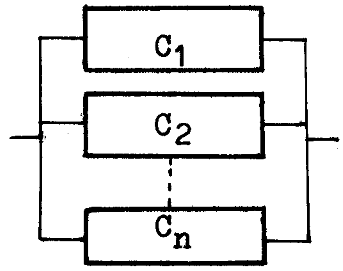
KEY WORDS: fuzzy reliability of the parallel Systems, fuzzy failure rate of the parallel Systems, fuzzy mean life of the parallel Systems.

I. INDEXES OF FUZZY RELIABILITY

We still employ the classificatory method of a system for general reliability theory. A parallel system is defined as a system consisting of several elements such that system failure occurs when and only when all elements fail. Equivalently, the system is successful when at least one of its elements is successful.

It is supposed that a parallel system consists of n element C_1, C_2, ..., C_n (as shown in figure), and it may be assumed that failure of any element would occur independently of the operation of other components.

Let C denote the event that the system is successful, $\bar{C}$ denote the event that the system fails, C_j denote the event that the element C_j is successful, $\bar{C}_j$ denote the event that the element C_j fails, $\underset{\sim}{A}i$ denote discussing one of fuzzy performance substs, then by meams of the definition of fuzzy conditional probability, we have



Figure

$$P(C \wedge \underset{\sim}{A}i) = P(\underset{\sim}{A}i | C)\ P(C) \qquad (1)$$

$$P(C_j \wedge \underset{\sim}{A}_i) = P(\underset{\sim}{A}_i | C_j)\ P(C_j) \qquad (2)$$

where the sign $\wedge$ denotes algebraic product.

In terms of the definition of a parallel system, the event

$$\bar{C} = \bigcap_{j=1}^{n} \bar{C}_j$$

and therefore

$$P(\bar{C}) = P\left(\bigcap_{j=1}^{n} \bar{C}_j\right) \qquad (3)$$

Equation (3) can also be written (using the conditional probability relations) as

$$P(\bar{C}) = P(\bar{C}_1 | \bar{C}_2 \cap \ldots \cap \bar{C}_n)\, P(\bar{C}_2 \cap \ldots \cap \bar{C}_n)$$

$$= P(\bar{C}_1 | \bar{C}_2 \cap \ldots \cap \bar{C}_n)\, P(\bar{C}_2 | \bar{C}_3 \cap \ldots \cap \bar{C}_n)\, P(\bar{C}_3 \cap \ldots \cap \bar{C}_n)$$

and in general

$$P(\bar{C}) = \left[\prod_{j=1}^{n-1} P(\bar{C}_j | \bar{C}_{j+1} \cap \ldots \cap \bar{C}_n)\right] P(\bar{C}_n) \qquad (4)$$

Because $\bar{C}_j$ are mutually independent, i.e. probability of success of the jth element is the same irrespective of occurrence of any combination of events of the other elements, which implies that

$$P(\bar{C}_j | \bar{C}_{j+1} \cap \ldots \cap \bar{C}_n) = P(\bar{C}_j) \;,\; j=1,2,\ldots,\; n-1$$

then E_q. (4) takes the form

$$P(\bar{C}) = \prod_{j=1}^{n} P(\bar{C}_j) \qquad (5)$$

In terms of general probability theory, use the probability relations of complementary events

$$\left.\begin{aligned} P(\bar{C}) &= 1-P(C) \\ P(\bar{C}_j) &= 1-P(C_j) \end{aligned}\right\} \qquad (6)$$

Substituting E_q. (6) into E_q.(5) , we obtain

$$P(C) = 1-\prod_{j=1}^{n} 1-P(C_j) \qquad (7)$$

Suppose that the denotations are defined as follows:

R_s -- general reliability of the systems;

R_j -- general reliability of jth element;

$\underset{\sim}{R}_s$ -- fuzzy reliability of the systems;

$\underset{\sim}{R}_j$ -- fuzzy reliability of jth element;

$\mu_{\underset{\sim}{A}i}(R_s)$ --- membership function of R_s in $\underset{\sim}{A}_i$;

$\mu_{\underset{\sim}{A}i}(R_j)$ --- membership function of R_j in $\underset{\sim}{A}i$;

then by means of general reliability theory and fuzzy reliability theory, we have

$$P(C) = R_s \qquad P(C_j) = R_j$$

$$P(C \wedge \underset{\sim}{A}i) = \underset{\sim}{R}_s \qquad P(C_j \wedge \underset{\sim}{A}_i) = \underset{\sim}{R}_j$$

$$P(\underset{\sim}{A}_i \mid C) = \mu_{\underset{\sim}{A}i}(R_s) \qquad P(\underset{\sim}{A}_i \mid C_j) = \mu_{\underset{\sim}{A}i}(R_j)$$

Therefore Eqs. (1). (2) and (7) are respectively rewritten as:

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}i}(R_s)\, R_s \tag{8}$$

$$\underset{\sim}{R}_j = \mu_{\underset{\sim}{A}i}(R_j)\, R_j \tag{9}$$

$$R_s = 1 - \prod_{j=1}^{n} (1 - R_j) \tag{10}$$

Substitutimg Eqs. (9) and (10) into Eq. (8) we obtain

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}i}(R_s) \left[1 - \prod_{j=1}^{n} (1 - R_j) \right] \tag{11}$$

or

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}i}(R_s) \left[1 - \prod_{j=1}^{n} \left(1 - \frac{\underset{\sim}{R}_j}{\mu_{\underset{\sim}{A}i}(R_j)}\right) \right] \tag{12}$$

E_{qs}. (8). (11) and (12) are the general expressions of the fuzzy reliability of the system.

By means of the relation between the fuzzy reliability ($\underset{\sim}{R}_j$) and the fuzzy failure rate ($\underset{\sim}{\lambda}_j$) for jth element, we have

$$\underset{\sim}{R}_j = \exp\left\{-\int_0^t \underset{\sim}{\lambda}_j \, dt\right\}$$

Substituting it into E_q.(12), we obtain

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}i}(R_s)\left\{1-\prod_{j=1}^{n}\left[1-\frac{\exp\left\{-\int_0^t \underset{\sim}{\lambda}_j \, dt\right\}}{\mu_{\underset{\sim}{A}i}(R_j)}\right]\right\} \quad (13)$$

If $\underset{\sim}{\lambda}_j$ is equal to constant, then

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}i}(R_s)\left\{1-\prod_{j=1}^{n}\left[1-\frac{e^{-\underset{\sim}{\lambda}_j t}}{\mu_{\underset{\sim}{A}i}(R_j)}\right]\right\} \quad (14)$$

If $\underset{\sim}{\lambda}_j$ is equal to $\underset{\sim}{\lambda}$, j=1, 2, ..., n, E_q. (13) may be also written as

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}i}(R_s)\left\{1-\left[1-\frac{e^{-\underset{\sim}{\lambda} t}}{\mu_{\underset{\sim}{A}i}(R_j)}\right]^n\right\} \quad (15)$$

Now derive a formula of fuzzy mean life $\underset{\sim}{MTTF}_s$ of the system. By means of the relation between fuzzy mean life and fuzzy reliability, $\underset{\sim}{MTTF}_s$ of the system is expressed

$$\underset{\sim}{MTTF}_s = \int_0^\infty \underset{\sim}{R}_s \, dt$$

$$= \int_0^\infty \mu_{\underset{\sim}{A}i}(R_s) \left[1 - \prod_{j=1}^{n} \left(1 - \frac{e^{-\int_0^t \underset{\sim}{\lambda}_j dt}}{\mu_{\underset{\sim}{A}i}(R_j)}\right)\right] dt \quad (16)$$

If $\underset{\sim}{\lambda}_j$ is equal to constant, and Variations of $\mu_{\underset{\sim}{A}i}(R_s)$ and $\mu_{\underset{\sim}{A}i}(R_j)$ with time t are more small, may substitute their mean values $\mu_{\underset{\sim}{A}i}(R_s)_m$ and $\mu_{\underset{\sim}{A}i}(R_j)_m$ for $\mu_{\underset{\sim}{A}i}(R_s)$ and $\mu_{\underset{\sim}{A}i}(R_j)$, then

$$\underset{\sim}{MTTF}_s = \mu_{\underset{\sim}{A}i}(R_s)_m \int_0^\infty \left[1 - \prod_{j=1}^{n} \left(1 - \frac{e^{-\underset{\sim}{\lambda}_j t}}{\mu_{\underset{\sim}{A}i}(R_j)_m}\right)\right] dt$$

$$= \mu_{\underset{\sim}{A}i}(R_s)_m \int_0^\infty \left[\sum_{j=1}^{n} \frac{e^{-\underset{\sim}{\lambda}_j t}}{\mu_{\underset{\sim}{A}i}(R_j)_m} - \sum_{\substack{j,k=1 \\ j\neq k}}^{n} \frac{e^{-(\underset{\sim}{\lambda}_j + \underset{\sim}{\lambda}_k)t}}{\mu_{\underset{\sim}{A}i}(R_j)_m \mu_{\underset{\sim}{A}i}(R_k)_m}\right.$$

$$\left. + \dots + (-1)^{n+1} \frac{e^{-\sum_{j=1}^{n} \underset{\sim}{\lambda}_j t}}{\prod_{j=1}^{n} \mu_{\underset{\sim}{A}i}(R_j)_m} \right] dt$$

$$= \mu_{\underset{\sim}{A}i}(R_s)_m \left[\sum_{j=1}^{n} \frac{1}{\mu_{\underset{\sim}{A}i}(R_j)_m \underset{\sim}{\lambda}_j} - \sum_{\substack{j,k=1 \\ j\neq k}}^{n} \frac{1}{\mu_{\underset{\sim}{A}i}(R_j)_m \mu_{\underset{\sim}{A}i}(R_k)_m (\underset{\sim}{\lambda}_j + \underset{\sim}{\lambda}_k)}\right.$$

$$\left. + \dots + (1-)^{n+1} \frac{1}{\prod_{j=1}^{n} \mu_{\underset{\sim}{A}i}(R_j)_m \sum_{j=1}^{n} \underset{\sim}{\lambda}_j} \right] \quad (17)$$

Usually, n is equal to 2 or 3 in practice. If n is equal to 2, then

$$\underset{\sim}{\text{MTTF}}_s = \mu_{\underset{\sim}{Ai}}(R_s)_m\left[\frac{1}{\mu_{\underset{\sim}{Ai}}(R_1)_m\underset{\sim}{\lambda}_1} + \frac{1}{\mu_{\underset{\sim}{Ai}}(R_2)_m\underset{\sim}{\lambda}_2} - \frac{1}{\mu_{\underset{\sim}{Ai}}(R_1)_m\mu_{\underset{\sim}{Ai}}(R_2)_m(\underset{\sim}{\lambda}_1+\underset{\sim}{\lambda}_2)}\right] \quad (18)$$

If n is equal to 3, then

$$\underset{\sim}{\text{MTTF}}_s = \mu_{\underset{\sim}{Ai}}(R_s)_m\left[\frac{1}{\mu_{\underset{\sim}{Ai}}(R_1)_m\underset{\sim}{\lambda}_1} + \frac{1}{\mu_{\underset{\sim}{Ai}}(R_2)_m\underset{\sim}{\lambda}_2} + \frac{1}{\mu_{\underset{\sim}{Ai}}(R_3)_m\underset{\sim}{\lambda}_3}\right.$$

$$- \frac{1}{\mu_{\underset{\sim}{Ai}}(R_1)_m\mu_{\underset{\sim}{Ai}}(R_2)_m(\underset{\sim}{\lambda}_1+\underset{\sim}{\lambda}_2)} - \frac{1}{\mu_{\underset{\sim}{Ai}}(R_2)_m\mu_{\underset{\sim}{Ai}}(R_3)_m(\underset{\sim}{\lambda}_2+\underset{\sim}{\lambda}_3)}$$

$$- \frac{1}{\mu_{\underset{\sim}{Ai}}(R_1)_m\mu_{\underset{\sim}{Ai}}(R_3)_m(\underset{\sim}{\lambda}_1+\underset{\sim}{\lambda}_3)}$$

$$\left. + \frac{1}{\mu_{\underset{\sim}{Ai}}(R_1)_m\mu_{\underset{\sim}{Ai}}(R_2)_m\mu_{\underset{\sim}{Ai}}(R_3)_m(\underset{\sim}{\lambda}_1+\underset{\sim}{\lambda}_2+\underset{\sim}{\lambda}_3)}\right] \quad (19)$$

If $\underset{\sim}{\lambda}_j$ is equal to constant $\underset{\sim}{\lambda}$, then

$$R_j = R$$

$$\mu_{\underset{\sim}{Ai}}(R_j)_m = \mu_{\underset{\sim}{Ai}}(R)_m \quad , j = 1,2, \ldots , n,$$

and

$$\underset{\sim}{\text{MTTF}}_s = \mu_{\underset{\sim}{Ai}}(R_s)_m\int_0^{\infty}\left[1 - \left(1 - \frac{e^{-\underset{\sim}{\lambda} t}}{\mu_{\underset{\sim}{Ai}}(R)_m}\right)^n\right] dt \quad (20)$$

Let $1-\dfrac{e^{-\lambda t}}{\mu_{\tilde{A}i}(R)_m}=X$, then Eq. (20) takes

the form

$$MTTF_s=\frac{\mu_{\tilde{A}i}(R_s)_m}{\lambda}\int_{1-\frac{1}{\mu_{\tilde{A}i}(R)_m}}^{1}\frac{1-X^n}{1-X}\,dX$$

$$=\frac{\mu_{\tilde{A}i}(R_s)_m}{\lambda}\int_{1-\frac{1}{\mu_{\tilde{A}i}(R)_m}}^{1}(1+X+X^2+\dots+X^{n-1})\,dX$$

$$=\frac{\mu_{\tilde{A}i}(R_s)_m}{\lambda}\sum_{j=1}^{n}\frac{1}{j}\left[1-\left(1-\frac{1}{\mu_{\tilde{A}i}(R)_m}\right)^j\right] \quad (21)$$

II. EXAMPLE

A control system of a aeroplane consists of two subsystems which have same function in order to increasing reliability. Suppose that the general reliability of every subsystem is equal to o.986, and fuzzy failure rate of every subsystem is equal to 0.38 (1/year), and assume that

$\mu_{\text{extremely reliable}}(0.986) = 0.6$

$\mu_{\text{very reliable}}(0.986) = 0.96$

$\mu_{\text{more reliable}}(0.986) = 0.6$

$\mu_{\text{extremely reliable}}(0.9998) = 0.99$

$\mu_{\text{very reliable}}(0.9998) = 0.6$

$\mu_{\text{more reliable}}(0.9998) = 0.4$

Find indexes of the fuzzy reliability for the control system.

This control system is a parallel system,its general reliability R_s is

$$R_s = 1-(1-R)^2$$
$$= 1-(1-0.986)^2 = 0.9998$$

Evidently,

$$\mu_{\text{extremely reliable}}(0.9998) > \mu_{\text{very reliable}}(0.9998) > \mu_{\text{more reliable}}(0.9998)$$

it follows that at present the system find oneself in a extremely reliable work period. Fuzzy reliability of the system is

$$\underset{\sim}{R}_s = \mu_{\text{extremely reliable}}(0.9998) \times 0.9998 = 0.9898$$

and fuzzy mean life of the system is

$$\underset{\sim}{MTBF}_s = \frac{\mu_{\text{extremely reliable}}(0.9998)}{\lambda}\left\{\frac{1}{\mu_{\text{extremely reliable}}(0.986)} + \frac{1}{2}\left[1-\left(1-\frac{1}{\mu_{\text{extremely reliable}}(0.986)}\right)^2\right]\right\}$$

$$= \frac{0.99}{0.38}\left\{\frac{1}{0.6} + \frac{1}{2}\left[1-\left(1-\frac{1}{0.6}\right)^2\right]\right\}$$

$$= 5.0658 \text{ (year)}$$

From these results it is concluded that a parallel control system consisting of two very reliable subsystems is extremely reliable, and the extremely reliable work period still continue for 5.0658 year.

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