Some Other Types of Fuzzy Connectedness

Dewan M. Ali
Department of Mathematics
Rajshahi University
Rajshahi, Bangladesh.

In this paper, we propose some other types of fuzzy connectedness and discuss their several properties. We only give statements of our findings without their proofs. Also, we use standard fuzzy topological concepts and notations in our work.

__Definition 1.__ Let (X, t) be an fts and $\alpha \in I_0$.

(a) Let $\mu > 1-\alpha$ be a fuzzy set in X. Then (u,v), $u,v \in t$, is called an __α-separation__ of μ iff $u \neq \mu \neq v$, $u \vee v = \mu$ and $u \wedge v \leq 1-\alpha$.

(X,t) is called __α-C3__ iff no clopen $\mu > 1-\alpha$ can be α-separated.

(b) (X,t) is called __α-C4__ iff there do not exist $u,v \in t$ with $u \vee v > 1-\alpha$, $u \wedge v \leq 1-\alpha$ and $u^{-1}(1-\alpha,1] \neq \phi \neq v^{-1}(1-\alpha,1]$.

(c) (X,t) is called __strongly-C4__ (S-C4) iff it is α-C4 for all $\alpha \in I_0$.

(d) (X,t) is called __2_α-C__ iff there do not exist aproper non-empty subset $A \subseteq X$ such that $\alpha 1_A$, $\alpha 1_{X-A} \in t$. ([2]) .

(e) (X,t) is called __D-C__ iff it is 2_α-C for all $\alpha \in I_0$ ([2]).

(f) (X,t) is called __α-C__ iff there do not exis t $u,v \in t-\{0,1\}$ such that $u \vee v > 1-$ and $u \wedge v = 0$ ([4]) .

__Remark:__ In theorem 1, FC(i), FC(ii), FC(iii), FC(iv), FC(v) and FC(vi) will respectively denote the fuzzy connectedness concepts introduced in [1], [3], [5], [4] and [6] .

Note that 1-C = FC(v) .

__Theorem1:__ The following implications exist:

(a) S-C4 $\Rightarrow$ FC(v) $\Rightarrow$ FC(i) $\Rightarrow$ D-C
$\swarrow$
FC(ii) $\Rightarrow$ FC(iii) $\Leftarrow$ FC(vi)

Moreover, no other implications exist among these concepts.

(b) α-C3 $\Leftarrow$ α-C4 $\Rightarrow$ α-C

Furthermore, these implications are not reversible.

(c) (i) For $\alpha > \beta$, α-C3 $\Rightarrow$ $2_{1-\beta}$-C, $\alpha, \beta \in I_0$.

(ii) For $\alpha > 1/2$, α-C3 $\Rightarrow$ 2_α-C.

(iii) 1-C3 $\Leftrightarrow$ FC(i) and 1-C4 $\Leftrightarrow$ FC(v) .

The implications in (i) and (ii) being not reversible ,

Theorem2: The following are equivalent:

(i) (X,t) is α-C4

(ii) (X, $\iota_{1-\alpha}(t)$) is connected .

Theorem3: α-C3, α-C4 and S-C4 are preserved under continuous fuctions, but α-C3, α-C4, S-C4 and α-C are not preserved under almost and weakly continuous functions

Theorem 4: α-C3, α-C4, S-C4 and α-C properties are good extensions (in the sense of Lowen [1]) .

Theorem5: Let (X,t) be an fts and (A,t_A) be a subspace of (X,t), then for any B with $A \subseteq B \subseteq \bar{A}$, we have

(i) (A,t_A) is α-C3 $\Rightarrow$ (B,t_B) is α-C3 .

(ii) (A,t_A) is α-C4 $\Rightarrow$ (B,t_B) is α-C4 .

(iii) (A,t_A) is S-C4 $\Rightarrow$ (B,t_B) is S-C4 .

Theorem 6: If $\{ A_j, t_{A_j} : j \in J \}$ is a family of α-C3 (α-C4 or S-C) subspaces of (X,t) with $\bigcap_{j \in J} A_j \neq \phi$, then if $A = \bigcup_{j \in J} A_j$, (A,t_A) is α-C3 (α-C4 or S-C4).

Theorem 7 : A non-empty product space is α-C3 (α-C4 or S-C4) iff each factor space is α-C3 (α-C4 or S-C4).

References

1. R. Lowen, Connectedness in fuzzy topological spaces, Rocky Mount. J. Math. 11(1981) 427- 433 .

2. R. Lowen and A.K. Srivastava, On Preuss' connectedness concept in FTS (to app

3. P.M. Pu and Y.M. Liu, Fuzzy topology I : Neighbourhood structure of a fuzzy point and Moore- Smith convergence, J. Math: Anal:. Appl . 76 (1980) 571- 599.

4. S.E, Rodabaugh , Connectivity and the L-fuzzy unit interval, Rocky Mount. J. Math. 12(1982) 113- 121 .

5. C.Y. Zheng, Fuzzy path and fuzzy connectedness, Fuzzy Sets and Systems 14 (1984) .

6. R. Srivastava, Topics in Fuzzy Topology, Ph.D. Thesis, BHU, INDIA (1983).

....