ON THE g-FUZZY LINEAR SYSTEMS[1]

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1. Introduction

Let us consider the classical linear system of equalities and inequalities

$$\sum_{j=1}^{n} a_{ij}x_j = a_{i0}, \qquad i\in J_1, \tag{1}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leqq a_{i0}, \qquad i\in J_2. \tag{2}$$

In the problem (1)-(2) the parameters $a_{ij}\in\mathbf{R}$ are supposed to be well defined characteristics of the modelled problem. However, these parameters are generally known only approximatively. It is known, that the problem (1)-(2) in general belongs to the class of ill posed problems, so a small perturbation in the input data may cause a large deviation in the solution. Therefore it is a very important to know in what degree can be acceptable a solution as the solution of the original problem.

In this paper we will examine the possibilistic solution of (1)-(2) assuming that the same function g defines the side functions of the fuzzy numbers and the t-norm.

2. Preliminaries

Let X be a universe and I be the unit interval of the real line **R**. Denote $\mathcal{F}(X) = \{\mu|\mu:X \to I\}$ the set of all ***fuzzy subsets*** on X. The characteristic function of $A\subset X$ as a special fuzzy subset will be denoted by χ_A.

A binary operation T: $I^2 \to I$ is said to be a ***t-norm*** iff it is commutative, associative, non-decreasing and T(a,1) = a for all $a\in I$. A t-norm is ***Archimedean*** iff it is continuous and T(a,a)<a for all 0<a<1. A t-norm T is Archimedean iff it admits the representation

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$$T(a,b) = g^{(-1)}(g(a)+g(b))$$

where the ***generator function*** g: $I \to \mathbf{R}_+$ is strictly decreasing, continuous on $(0,1]$, $g(1)=0$, $\lim_{x\to 0} g(x)=g_0$ ($g_0=\infty$ is also allowed) and $g^{(-1)}(x)$ denotes the pseudoinverse of g, i.e. $g^{(-1)}(x)=g^{-1}(x)$ if $x\in[0,g_0]$ and $g^{(-1)}(x)=0$ for all $x\geq g_0$. Every t-norm induces an n-ary operation T^{n-1}: $I^n\to I$ with the following rule

$$T^{n-1}(a_1,a_2,\dots,a_n) = T(T^{n-2}(a_1,a_2,\dots,a_{n-1}),a_n).$$

If T is Archimedean then

$$T^{n-1}(a_1,a_2,\dots,a_n) = g^{(-1)}\Big(\sum_{j=1}^{n} g(a_j)\Big).$$

The ***T-fuzzy intersection*** and the ***T-fuzzy Cartesian product*** of a fuzzy sets will be defined as follows:

If $\mu,\nu\in\mathcal{F}(X)$ then $(\mu\wedge_T\nu)(x) = T(\mu(x),\nu(x))$, and if $\mu\in\mathcal{F}(X)$, $\nu\in\mathcal{F}(Y)$ then $(\mu\times_T\nu)(x,y) = T(\mu(x),\nu(y))$.

At every fixed $x\in X$ a ***T-fuzzification*** of the function value of the parametrical function $f(a,x) = f(a_1,a_2,\dots,a_n,x)$ by the fuzzy parameter vector $\mu_a=(\mu_1,\mu_2,\dots,\mu_n)$, $\mu_i\in\mathcal{F}(\mathbf{R})$, $i=1,2,\dots n$, is a fuzzy set on $\mathbf{R}$:

$$\tilde{f}_T(\mu_a,x)(y) = \begin{cases} \sup_{a\in A(x,y)\neq\emptyset} T^{n-1}(\mu_1(a_1),\mu_2(a_2),\dots,\mu_n(a_n)) \\ 0 \text{ if } A(x,y) = \emptyset \end{cases},$$

where $A(x,y) = \{a=(a_1,a_2,\dots,a_n)\,|\,a_i\in\mathbf{R}, y=f(a,x)\}$.

Let R be an unfuzzy relation on X. The T-fuzzification $\tilde{R}$ of R on $\mathcal{F}(X)$ is a fuzzy set on $\mathcal{F}(X)\times\mathcal{F}(X)$:

$$\tilde{R}(\mu,\nu) = \sup_{xRy} T(\mu(x),\nu(y)).$$

3. g-fuzzification of linear functions, linear equalities and inequalities

The following definitions and theorems are the generalizations of the analogous definitions and theorems in [1],[2].

Let g: $I \to \mathbf{R}_+$ be a fixed function with the properties of a generator function of an Archimedean T-norm.

Let be denoted by $\mathcal{F}_g$ the subset of $\mathcal{F}(\mathbf{R})$ containing the fuzzy sets with the membership function

$$\mu(x) = \begin{cases} g^{(-1)}(|x-\alpha|/d) & \text{if } d>0 \\ \chi_{\{\alpha\}}(x) & \text{if } d=0 \end{cases}$$

for all $\alpha\in\mathbf{R}$, $d\in\mathbf{R}_+\cup\{0\}$. An element of $\mathcal{F}_g$ will be called quasitriangular fuzzy number generated by g with the center α and the width d and we will recall for it by the pair (α,d).

Let T_g denote the Archimedean t-norm given also by the same generator function g.

Shortly the T_g-fuzzification of a parametrical function value or a relation by the fuzzy vector parameter $\mu_a \in \mathcal{F}_g^n$ will be called ***g-fuzzification***.

Let us introduce the following denotions:

$$D(x) = \max_{j=1,\dots,n} d_j |x_j|; \qquad D_0(x) = \max(d_0, D(x)).$$

Theorem 3.1. ***The g-fuzzification $\tilde{\ell}_g(\mu_a,x)$ of a linear function $\ell(a,x)=\sum_{j=1}^{n} a_j x_j$ by the fuzzy vector parameter $\mu=(\mu_1,\dots,\mu_n)$, $\mu_j=(\alpha_j,d_j)\in\mathcal{F}_g$, $j=1,\dots,n$, is $(\sum_{j=1}^{n}\alpha_j x_j, D(x))\in\mathcal{F}_g$, i.e.***

$$\tilde{\ell}_g(\mu_a,x)(y) = \begin{cases} g^{(-1)}(|y - \sum_{j=1}^{n}\alpha_j x_j|/D(x)) & \text{if } D(x)\neq 0 \\ \chi_{\{\ell(\alpha,x)\}}(y) & \text{if } D(x)=0 \end{cases}.$$

Proof. Let $K=\{i: 1\le i\le n, d_i\neq 0\}$, the cardinal number of which $k=|K|$ and divide $\ell(a,x)$ in two parts $\ell(a,x)=\ell_0(a,x)+\ell_1(a,x)$, where $\ell_0(a,x)=\sum_{i\in K} a_i x_i$ and $\ell_1(a,x)=\sum_{i\notin K} a_i x_i$. Let us introduce the following denotions $y_0=y-\ell_1(\alpha,x)$, $A(x,y)=\{a\in\mathbf{R}^n \mid y=\ell(a,x)\}$, $A_0(x,y)=\{a\in\mathbf{R}^k \mid y_0=\ell_0(a,x)\}$. By virtue of the extension principle and the g-fuzzy Cartesian product rule we have

$$\tilde{\ell}_g(\mu_a,x)(y) = \begin{cases} \sup_{a\in A(x,y)} g^{(-1)}(\sum_{j=1}^{n} g(\mu_j(a_j))) & \text{if } A(x,y)\neq\emptyset \\ 0 & \text{if } A(x,y)=\emptyset \end{cases} =$$

$$= \begin{cases} \sup_{a\in A(x,y)} g^{(-1)}(\sum_{i\in K}\min(g_0, |a_i-\alpha_i|/d_i) + \sum_{i\notin K} g(\chi_{\{\alpha_i\}}(a_i))) \\ 0 \end{cases}.$$

If $K\neq\emptyset$ then

$$\tilde{\ell}_g(\mu_a,x)(y) = \begin{cases} g^{(-1)}(\inf_{a\in A_0(x,y)} \sum_{i\in K} |a_i-\alpha_i|/d_i) & \text{if } A_0(x,y)\neq\emptyset \\ 0 & \text{if } A_0(x,y)=\emptyset \end{cases}.$$

If there exist $i\in K$ such that $x_i\neq 0$ then $A_0(x,y)\neq\emptyset$. In this case the minimization problem in the argument is equivalent to the linear programming problem

$$\sum_{j\in K}(z_j^+ + z_j^-) \longrightarrow \min$$

subject to

$$\sum_{j\in K}(z_j^+ - z_j^-)x_j = y_0-\ell_1(\alpha,x) = y-\ell(\alpha,x) \ ,$$

$$z_j^+ \geqq 0, \quad z_j^- \geqq 0, \ j\in K.$$

Indeed, introducing the nonnegative variables

$$z_j^+=\max(0,a_j-\alpha_j), \ z_j^+=\max(0,\alpha_j-a_j), \ j\in K,$$

we have

$$|a_j-\alpha_j|=z_j^+-z_j^- \quad \text{and } a_j-\alpha_j=z_j^+-z_j^- \text{ for } j\in K.$$

The optimal object value of this linear programming problem is equal to the optimal solution of its dual problem

$$(y-\ell(\alpha,x))u \longrightarrow \max$$

subject to

$$x_j u\leqq 1/d_j, \quad -x_j u\leqq 1/d_j, \quad j\in K.$$

From the constraints conditions follows that

$$|u|\leqq 1/\max_{j\in K} d_j|x_j|=1/\max_{j=1,\ldots,n} d_j|x_j|,$$

consequently the optimal object value is $|y-\ell(\alpha,x)|/D(x)$. So $\tilde{\ell}_g(\mu_a,x)(y)=g^{(-1)}(|y-\ell(\alpha,x)|/D(x))$ if $\exists i\in K: x_i\neq 0$, i.e. if $D(x)\neq 0$. If $K\neq\emptyset$, but $x_i=0$ for every $i\in K$, then $D(x)=0$. In this case $A_0(x,y)\neq\emptyset$ only if $y=\ell_1(\alpha,x)$ and $A_0(x,\ell_1(\alpha,x))=\mathbf{R}^k$, therefore

$$\tilde{\ell}_g(\mu_a,x)(y)=\begin{cases} g^{(-1)}(\inf_{a_i\in\mathbf{R}^k} \sum_{i\in K}|a_i-\alpha_i|/d_i) & \text{if } y=\ell_1(\alpha,x),\ x_i=0\ \forall i\in K \\ 0 & \text{if } y\neq\ell_1(\alpha,x),\ x_i=0\ \forall i\in K \end{cases} =$$

$$=\chi_{\{\ell_1(\alpha,x)\}}(y)=\chi_{\{\ell(\alpha,x)\}}(y).$$

If $K=\emptyset$ then $D(x)=0$ and

$$\tilde{\ell}_g(\mu_a,x)(y)=\begin{cases} \sup_{a\in A(x,y)} g^{(-1)}(\sum_{j=1}^{n} g(\chi_{\{\alpha_i\}}(a_i))) & \text{if } A(x,y)\neq\emptyset \\ 0 & \text{if } A(x,y)=\emptyset \end{cases} =$$

$$=\begin{cases} 1 & \text{if } y=\ell(\alpha,x) \\ 0 & \text{otherwise} \end{cases}$$

$$=\chi_{\{\ell(\alpha,x)\}}(y).$$

■

The Theorem 3.1 states that for every fixed $x\in\mathbf{R}^n$ the g-fuzzification is a mapping $\tilde{\mathcal{L}}_g:\mathcal{F}_g^n \to \mathcal{F}_g$.

Theorem 3.2. *The g-fuzzification $\sigma(x)$ of a linear equality $\mathcal{L}(a,x)-a_0=\sum_{j=1}^{n} a_j x_j - a_0 = 0$ by $\mu_a=(\mu_1,\ldots,\mu_n)$ and μ_0, $\mu_j=(\alpha_j,d_j)\in\mathcal{F}_g$, $j=0,\ldots n$, is the fuzzy set $\sigma\in\mathcal{F}(R^n)$:*

$$\sigma(x) = \begin{cases} g^{(-1)}(|\sum_{j=1}^{n}\alpha_j x_j - \alpha_0|/D_0(x)) & \text{if } D_0(x)\neq 0 \\ \chi_{\{z:\mathcal{L}(\alpha,z)=\alpha_0\}}(x) & \text{if } D_0(x)=0 \end{cases}.$$

Proof. Using the g-fuzzification of the equality relation between $\tilde{\mathcal{L}}_g(\mu_a,x)$ and μ_0 we have

$$\sigma(x)=\sup_{y=a_0} g^{(-1)}(g(\tilde{\mathcal{L}}_g(\mu_a,x)(y))+g(\mu_0(a_0)))=$$

$$= \begin{cases} g^{(-1)}(\inf_{y=a_0}(|y-\mathcal{L}(\alpha,x)|/D(x)+|a_0-\alpha_0|/d_0)) & \text{if } d_0\neq 0, D(x)\neq 0 \\ g^{(-1)}(|\alpha_0-\mathcal{L}(\alpha,x)|/D(x)) & \text{if } d_0=0, D(x)\neq 0 \\ g^{(-1)}(|\mathcal{L}(\alpha,x)-\alpha_0|/d_0) & \text{if } d_0\neq 0, D(x)=0 \\ 1 & \text{if } \mathcal{L}(\alpha,x)=\alpha_0 \text{ and } d_0=0, D(x)=0 \\ 0 & \text{if } \mathcal{L}(\alpha,x)\neq\alpha_0 \text{ and } d_0=0, D(x)=0 \end{cases}$$

The minimization problem in the argument in the case of $d_0\neq 0$, $D(x)\neq 0$ is equivalent to the linear programming problem

$$(z^+ + z^-)/D(x)+(v^+ + v^-)/d_0 \longrightarrow \min$$

subject to

$$z^+ - z^- + v^+ - v^- = \alpha_0 - \mathcal{L}(\alpha,x)$$
$$z^+\geqq 0, z^-\geqq 0, v^+\geqq 0, v^-\geqq 0,$$

where $z^+=\max(0,y-\mathcal{L}(\alpha,x))$, $z^-=\max(0,\mathcal{L}(\alpha,x)-y)$, $v^+=\max(0,a_0-\alpha_0)$, $v^-=\max(0,\alpha_0-a_0)$.

The corresponding dual problem

$$(\alpha_0-\mathcal{L}(\alpha,x)u \longrightarrow \max$$
$$u\leqq 1/D(x),\ -u\leqq 1/D(x),\ -u\leqq 1/d_0,\ u\leqq 1/d_0$$

can be immediatelly solved, since from the constraints inequalities follows that

$$|u|\leqq 1/\max(d_0,D(x)).$$

Therefore

$$\max_u(\alpha_0-\ell(\alpha,x))u=\min_{y=a_0}(|y-\ell(\alpha,x)|/D(x)+|a_0-\alpha_0|/d_0)=$$

$$=|\alpha_0-\ell(\alpha,x)|/\max(d_0,D(x))=|\alpha_0-\ell(\alpha,x)|/D_0(x).$$

Since

$$\max(d_0,D(x))=D_0(x)=\begin{cases}D(x) & \text{if } d_0=0\\ d_0 & \text{if } D(x)=0\end{cases}$$

we obtain the wanted statement for $\sigma(x)$. ■

Theorem 3.3. *The g-fuzzification $\theta(x)$ of a linear inequality $\ell(a,x)-a_0=\sum_{j=1}^{n}a_jx_j-a_0\leqq 0$ by $\mu_a=(\mu_1,\dots,\mu_n)$ and μ_0, $\mu_j=(\alpha_j,d_j)\in\mathcal{F}_g$, $j=0,\dots,n$, is the fuzzy set $\theta\in\mathcal{F}(R^n)$:*

$$\theta(x)=\begin{cases}\theta_+(x) & \text{if } \sum_{j=1}^{n}\alpha_jx_j\geqq\alpha_0\\ 1 & \text{if } \sum_{j=1}^{n}\alpha_jx_j\leqq\alpha_0\end{cases},$$

where

$$\theta_+(x)=\begin{cases}g^{(-1)}((\sum_{j=1}^{n}\alpha_jx_j-\alpha_0)/D_0(x)) & \text{if } D_0(x)\neq 0\\ \chi_{\{x:\ell(\alpha,x)\leq\alpha_0\}}(x) & \text{if } D_0(x)=0\end{cases}.$$

Proof. The g-fuzzified inequality relation gives

$$\theta(x)=\sup_{y\leqq a_0}g^{(-1)}(g(\tilde{\ell}_g(\mu_a,x)(y))+g(\mu_0(a_0)))=$$

$$=\begin{cases}g^{(-1)}(\inf_{y\leqq a_0}(|y-\ell(\alpha,x)|/D(x)+|a_0-\alpha_0|/d_0)) & \text{if } d_0\neq 0,D(x)\neq 0\\ g^{(-1)}(\inf_{y\leqq\alpha_0}(|y-\ell(\alpha,x)|/D(x))) & \text{if } d_0=0,D(x)\neq 0\\ g^{(-1)}(\inf_{\ell(\alpha,x)\leqq a_0}(|a_0-\alpha_0|/d_0)) & \text{if } d_0\neq 0,D(x)=0\\ 1 & \text{if } \ell(\alpha,x)\leqq\alpha_0 \text{ and } d_0=0,D(x)=0\\ 0 & \text{if } \ell(\alpha,x)>\alpha_0 \text{ and } d_0=0,D(x)=0\end{cases}$$

The minimization problems in the arguments are equivalent to the following linear programming problems:

$$\left.\begin{array}{c}(z^++z^-)/D(x)+(v^++v^-)/d_0\longrightarrow\min\\ -z^++z^-+v^+-v^-\geqq\ell(\alpha,x)-\alpha_0\\ z^+\geqq 0,\; z^-\geqq 0,\; v^+\geqq 0,\; v^-\geqq 0\end{array}\right\}\quad\text{if } d_0\neq 0,D(x)\neq 0$$

$$\left.\begin{array}{c}(z^++z^-)/D(x)\longrightarrow\min\\ -z^++z^-\geqq\ell(\alpha,x)-\alpha_0\\ z^+\geqq 0,\; z^-\geqq 0\end{array}\right\}\quad\text{if } d_0=0,D(x)\neq 0$$

$$\left.\begin{array}{c}(v^{+}+v^{-})/d_{0} \longrightarrow \min \\ v^{+}-v^{-}\geqq \ell(\alpha,x)-\alpha_{0} \\ v^{+}\geqq 0,\ v^{-}\geqq 0\end{array}\right\}\quad \text{if } d_{0}\neq 0, D(x)=0$$

where $z^{+}=\max(0,y-\ell(\alpha,x))$, $z^{-}=\max(0,\ell(\alpha,x)-y)$, $v^{+}=\max(0,a_{0}-\alpha_{0})$, $v^{-}=\max(0,\alpha_{0}-a_{0})$.

The dual of all these problems is

$$(\ell(\alpha,x)-\alpha_{0})u \longrightarrow \max$$
$$0\leqq u\leqq 1/\max(d_{0},D(x))=1/D_{0}(x),$$

the solution of which

$$\max_{u}(\ell(\alpha,x)-\alpha_{0})u=\begin{cases}(\ell(\alpha,x)-\alpha_{0})/D(x) & \text{if } \ell(\alpha,x)\geqq\alpha_{0} \\ 0 & \text{if } \ell(\alpha,x)\leqq\alpha_{0}\end{cases}.$$

Consequently

$$\theta(x)=\begin{cases} g^{(-1)}((\ell(\alpha,x)-\alpha_{0})/D_{0}(x)) & \text{if } D_{0}(x)\neq 0 \text{ and } \ell(\alpha,x)\geqq\alpha_{0} \\ 1 & \text{if } D_{0}(x)\neq 0 \text{ and } \ell(\alpha,x)\leqq\alpha_{0} \\ & \text{or } D_{0}(x)=0 \text{ and } \ell(\alpha,x)\leqq\alpha_{0} \\ 0 & \text{if } D_{0}(x)=0 \text{ and } \ell(\alpha,x)>\alpha_{0}\end{cases}$$

which is equivalent formulation to the desired statement. ■

σ and θ express the possibilities of the corresponding relations at the point $x\in\mathbf{R}^{n}$.

4. g-fuzzy solution of the g-fuzzy linear system

The g-fuzzy solution set ν of the g-fuzzified system of linear equalities and inequalities is obtained by the g-fuzzy intersection of the g-fuzzification of the corresponding equalities and inequalities.

Theorem 4.1. *Let the linear system (1)-(2) be fuzzified by $\mu_{ij}=(\alpha_{ij},d_{ij})\in\mathcal{F}_{g}$, $i\in J_{1}\cup J_{2}$, $j=0,\dots n$. Then the fuzzy solution $\nu\in\mathcal{F}(R^{n})$ is*

$$\nu(x)=\begin{cases} g^{(-1)}(G(x)) & \text{if } x\in S_{1}\cap S_{2} \\ 0 & \text{if } x\notin S_{1}\cap S_{2}\end{cases},$$

where

$$G(x)=\sum_{i\in P(x)}|\ell(\alpha_{i},x)-\alpha_{i0}|/D_{i0}(x)+\sum_{i\in Q(x)}\max(0,(\ell(\alpha,x)-\alpha_{i0})/D_{i0}(x)),$$

$$D_{i0}(x)=\max(d_{i0},D_{i}(x)),\ D_{i}(x)=\max_{j=1,\dots n}d_{ij}|x_{j}|,$$

$$S_{1}=\{x\in R^{n}\mid \ell(\alpha_{i},x)=\alpha_{i0},\forall i\in J_{1}\setminus P(x)\},$$
$$S_{2}=\{x\in R^{n}\mid \ell(\alpha_{i},x)\leqq\alpha_{i0},\forall i\in J_{2}\setminus Q(x)\},$$
$$P(x)=\{i\in J_{1}\mid D_{i0}(x)\neq 0\},\ Q(x)=\{i\in J_{2}\mid D_{i0}(x)\neq 0\}.$$

__Remark.__ It is possible that either $P(x)$ or $Q(x)$ or both of them are empty, in these cases the empty sums are equal to zero. Furthermore, $S_1=\mathbf{R}^n$ if $P(x)=J_1$ and $S_2=\mathbf{R}^n$ if $Q(x)=J_2$.

__Proof.__ Let $\sigma_i(x), i\in J_1$ and $\theta_i(x), i\in J_2$ be the possibilities of the corresponding g-fuzzified equalities and inequalities. Then the g-intersection of the g-fuzzifications of the equalities is

$$\nu_1(x)=g^{(-1)}\Big(\sum_{i\in J_1} g(\sigma_i(x))\Big)=$$

$$=g^{(-1)}\Big(\sum_{i\in P(x)} \min(g_0, |\ell(\alpha_i,x)-\alpha_{i0}|/D_{i0}(x))+$$

$$+\sum_{i\notin P(x)} g(\chi_{\{z:\ell(\alpha_i,x)=\alpha_{i0}\}}(x))\Big)=$$

$$=\begin{cases} g^{(-1)}\Big(\sum_{i\in P(x)} |\ell(\alpha_i,x)-\alpha_{i0}|/D_{i0}(x)\Big) & \\ & \text{if } P(x)=J_1 \text{ or } P(x)\subset J_1 \text{ and } x\in S_1, \text{ i.e. } \forall x\in S_1 \\ 0 & \text{otherwise} \end{cases}$$

For the g-intersection of the fuzzified inequalities we have

$$\nu_2(x)=g^{(-1)}\Big(\sum_{i\in J_2} g(\theta_i(x))\Big)=$$

$$=g^{(-1)}\Big(\sum_{i\in Q(x)} \chi_{\{z:\ell(\alpha_i,x)\geq\alpha_{i0}\}}(x)\min(g_0, (\ell(\alpha_i,x)-\alpha_{i0})/D_{i0}(x))+$$

$$+\sum_{i\notin Q(x)} g(\chi_{\{z:\ell(\alpha_i,x)\leq\alpha_{i0}\}}(x))\Big)=$$

$$=\begin{cases} g^{(-1)}\Big(\sum_{i\in Q(x)} \max((\ell(\alpha_i,x)-a_{i0})/D_{i0}(x))\Big) & \\ & \text{if } Q(x)=J_2 \text{ or } Q(x)\subset J_2 \text{ and } x\in S_2, \text{ i.e. } \forall x\in S_2 \\ 0 & \text{otherwise} \end{cases}$$

Consequently the solution of the joint system of fuzzified equalities and inequalities

$$\nu(x)=g^{(-1)}(g(\nu_1(x))+g(\nu_2(x)))=$$

$$=\begin{cases} g^{(-1)}\Big(\sum_{i\in P(x)} |\ell(\alpha_i,x)-\alpha_{i0}|/D_{i0}(x)+ & \\ \quad +\sum_{i\in Q(x)} \max(0,(\ell(\alpha_i,x)-\alpha_{i0})/D_{i0}(x))\Big) & \text{if } x\in S_1\cap S_2 \\ g^{(-1)}\Big(\sum_{i\in P(x)} |\ell(\alpha_i,x)-\alpha_{i0}|/D_{i0}(x)+g_0\Big) & \text{if } x\in S_1, x\notin S_2 \\ g^{(-1)}\Big(g_0+\sum_{i\in Q(x)} \max(0,(\ell(\alpha_i,x)-\alpha_{i0})/D_{i0}(x))\Big) & \text{if } x\notin S_1, x\in S_2 \\ g^{(-1)}(2g_0) & \text{if } x\notin S_1, x\notin S_2 \end{cases}$$

$$=\begin{cases} g^{(-1)}(G(x)) & \text{if } x\in S_1\cap S_2 \\ 0 & \text{otherwise} \end{cases}. \qquad \blacksquare$$

The last theorem shows, the connection between the penalization of a subset on $\mathbf{R}^n$ defined by equality and inequality relations and the fuzzification of this subset, since $G(x)$ may be considered as a penalty function. In the case when only the right hand side parameters are fuzzified, $G(x)$ is a classical penalty function.

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