FUZZY METHODS IN SIGNAL PROCESSING. I.

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Abstract: This paper introduces classes of fuzzy signals and methods of their processing. The accent is on the binary fuzzy reprezentation of signals.

Key words: fuzzy signals, fuzzy numbers, fuzzy binarry numbers, fuzzy processing of signals

1. Introduction

There are many ways to define fuzzy signals (fs). Interpreting them as mappings, one can derive the following types:

i) according to the variable: fuzzy numbers, or variable of a linguistic type; or real numbers;

ii) according to the values: fuzzy numbers-, or linguisticvvariable-valued, or real numbers-valued.

Because each linguistic variable (degrees) can be interpreted as a fuzzy number (fn), and real numbers are a special type of fns, we restrict the discussion to classes of fns. Furtheron, note that the signals can be of discrete type, or continuous type. For example, the linguistic degrees-valued signals are of discrete type.

Our opinion is that fuzzy signals are the'true' practical signals. They are not the'fuzzified' (horribile dictu!) version of the crisp signals. It is much more natural to consider the crisp signals as simplified versions of the true signals,as the signals are always measured with uncertainty, or include noise a.s.o.

2. Fuzzy binary numbers

Fuzzy digital technology is emerging by the contribution of Yamakawa's group. However, digital has in the presently developed hardware only the sense of 'sampled'. Because the advantages of digital computers in processing the crisp signals, one is motivated to try a counterpart for the fuzzy signal processing. The first question arising is if fns can be and how they must be represented under binary form, if a specific fuzzy binary form does exist.

Note first that the fuzzy numbers with triangular membership function (mf) can be represented by a set of three real (crisp) numbers, and denote $(a,n,b) = {}_a\hat{n}_b$ ($\hat{n}$, when no confusion is possible) the fn n with the limits of the mf a and b (i.e. $m(n,x) > 0$ for $a < x < b$, $m(n,x)$ the mf of $\hat{n}$).

<u>Def.1</u>. The basical fuzzy binary numbers (fbn) are ${}_o0_1$ and ${}_o1_1$. Below we denote $\hat{0} = {}_o0_1$, $\hat{1} = {}_o1_1$.

The summ and the product of the basical fbn are:

	${}_o0_1$	${}_o1_1$
${}_o0_1$	${}_o0_1$	${}_o1_1$
${}_o1_1$	${}_o0_1$	${}_o1_1$

(product)

${}_o0_1$	${}_o1_1$	${}_o0_2$	${}_o1_2$	.....
${}_o0_1$	${}_o0_2$	${}_o1_2$	${}_o0_3$	${}_o1_3$ .....
${}_o1_1$	${}_o1_2$	${}_o2_2$	${}_o1_3$	${}_o2_3$ .....
${}_o0_2$	${}_o0_3$	${}_o1_3$	${}_o0_4$	${}_o1_4$ .....
${}_o1_2$	${}_o1_3$	${}_o2_3$	${}_o1_4$	${}_o2_4$ .....
${}_o0_3$	${}_o0_4$	${}_o1_4$	${}_o0_5$	${}_o1_5$ .....
${}_o1_3$	${}_o1_4$	${}_o2_4$	${}_o1_5$	${}_o2_5$ .....
⋮				

(summ)

<u>Remarks:</u>

1.The basical fbns and the operation "." generate a monoid with unity ${}_o1_1$. The same is not true for the operation "+", the set of basical fbns being not closed to "+". However,

2. The set ${}_o0_o$, ${}_o0_1$, ${}_o1_1$ and all the derived fns(by +) with'+'is a monoid with unity ${}_o0_o$.

3. The set generated by the basical fbn and "+" does not include $\hat{N}$, the set of all fuzzy natural numbers. For example, ${}_5 6_7$ can not be expressed as a summ of $\hat{0}$ and $\hat{1}$.

4. The set generated by 0, $\hat{0}$, 1, $\hat{1}$ and "+" includes $\hat{N}$.

5. The set of fuzzy numbers of the form

$$ {}_a b_c = \sum_k {}_o^n \; 2^k \, \mathfrak{I}_k \, , \quad \mathfrak{I}_k = \hat{0} \text{ or } \hat{1} $$

does not include $\hat{N}$. Following, the above representation of the fuzzy numbers can not be used to quantize them and to represent them by strings of $\hat{0}$ and $\hat{1}$, such as $\hat{0}\hat{1}\hat{1}\hat{0}\hat{1}\hat{0}\hat{0}\hat{0}\hat{1}$.... It is natural to ask if the use of some kind of fuzzy binary weights allows a representation of fn's by strings of 0 and 1's.

<u>Def. 2.</u> A basical fuzzy binary weight is of the form:

$$\hat{2}^{k} = {}_{2^{k-1}}2^{k}{}_{2^{k+1}} \quad , \quad k \in N$$

<u>Remark:</u> The set of fn's of the form:

$$\hat{n} = \sum_{o}^{n} \hat{2}^{k}\cdot\hat{\imath}_{k} \quad , \quad \hat{\imath}_{k} = 0 \text{ or } 1$$

does not include $\hat{N}$. Take for example ${}_{4}5_{5}$.

One can ask if substituting $\hat{\jmath}_k$ instead of $\hat{\imath}_k$ in the above expression one can describe any $\hat{n}$ from $\hat{N}$. It is easy to see the answer is negative. The answer is the same for

$$\hat{n} = \sum_{o}^{n} 2^{h}\hat{\imath}_{h} + \sum_{o}^{m} 2^{k}\hat{\jmath}_{k}$$

However, there are the following possible representations of fn's using the basical fbn and fuzzy binary weights:

<u>Remark:</u> The following decompositions hold:

(i) $${}_{p}s_{q} = \sum_{o}^{n} 2^{m}\hat{\imath}_{m} + \sum_{k\in K} 2^{k}\cdot\hat{0} + \sum_{h\in H} 2^{h}\cdot\hat{1} \quad ; K, H \subset N$$

which is equivalente with the system:

$$p = \sum_{o}^{n} 2^{m}\hat{\imath}_{m}$$

$$s = \sum_{o}^{n} 2^{m}\hat{\imath}_{m} + \sum_{h\in H} 2^{h}$$

$$q = \sum_{o}^{n} 2^{m}\hat{\imath}_{m} + \sum_{h\in H} 2^{h} + \sum_{k\in K} 2^{k}$$

(ii) $${}_{p}s_{q} = \sum_{K}\hat{2}^{k}\cdot\hat{0} + \sum_{H}\hat{2}^{h}\cdot\hat{1} + \sum_{M} 2^{m}$$

which is equivalente with the system:

$$p = \sum 2^{m} \; ; \quad s = \sum 2^{h} + \sum 2^{m} \; ; \quad q = \sum 2^{k+1} + s$$

or :

$$p = \sum 2^{m}; \quad s - p = \sum 2^{h} \; ; \quad q/2 - s = \sum 2^{k}$$

(iii) $${}_{p}s_{q} = \sum_{K}\hat{2}^{k}\,\hat{0} + \sum_{H}\hat{2}^{h}\cdot\hat{1} + \sum_{M}\hat{2}^{m}\cdot 1$$

equivalente with:

$$p = \sum 2^{m-1}; \quad s = 2p + \sum 2^{k}; \quad q = 2s + \sum 2^{k+1}$$

3. The analogue-to-digital conversion of fuzzy signals

Each of the above decompositions of the fuzzy numbers can be used to convert the fs into fuzzy-quantized signals. Moreover, note that other, more complex decompositions are possible. One can choose one of the above forms in accor - dance with the meaning we like to attribute to the conversion: based on crisp weights, or on fuzzy binary weights, or on both classes of binary weights. The equivalente systems of equations provide the algorithms of conversion. The simplest way of AD conversion is probably (i), as it does not requires any product/division operations. According to (i), the basical ADC for fuzzy signals includes two difference blocks (s-p and q-s) and three (crisp) ADC: for p, s-p and q-s.

4. What filtering of fuzzy signals can mean?

Consider only one-dimensional (1-D) fuzzy signals, i.e. fs's of the form ${}_{p(t)}s(t)_{q(t)}$. (A two dimensional fs, e.g. a fs representing an image in a well defined, crisp plane, is of the form ${}_{p(x,y)}s(x,y)_{q(x,y)}$. We restrict here to fuzzy valued signals, the variable t, or (x,y) being crisp). Finally, a fuzzy-valued 1D signal consists in three crisp 1D signals p(t), s(t) and q(t) bounded by the condition:

$$\forall t: \quad p(t) \leqslant s(t) \leqslant q(t)$$

The assignement of membership function, which is a time-dependent membership function $m_s(x, t)$, equates finding p(t) and q(t) for a triangular mf. The assignement itself is dependent on the application in hand. If the uncertainty is due to stationary noise, or to constant measuring error, one can suppose that the spreading of s(t) is symetrical, i.e. $\forall t: s(t) - p(t) = q(t) - s(t) = w(t)$. If the error involved in the measurement is dependent on the actual value of the signal, one can suppose w(t) = = a· s(t). For stationary, independent errors, w(t) = ct.

In general, a filter for fs's is a system transforming the functions (p(t), s(t), q(t)) into (P(t), S(t), Q(t)) such as $\forall$ t: P(t) $\leq$ S(t) $\leq$ Q(t). This condition is not satisfied even if the same transform is applied to all the three initial functions, except particular cases.

We shall name uniform filtering of a fs the processing method involving the same transform for all the components, i.e., in the frequency domain,:

$$\underline{P}(w) = H(w).\underline{p}(w); \quad \underline{S}(w) = H(w).\underline{s}(w); \quad \underline{Q}(w) = H(w).\underline{q}(w)$$

where $\underline{P}$ denotes the Fourier transform of P a.s.o.

When the uniform filtering is not possible due to the constraint P(t) $\leq$ S(t) $\leq$ Q(t), one is motivated to introduce the "almost uniform" filter H(w) defined by the three transfer functions H'(w), H(w), H''(w), where H' and H'' are the nearest transfer functions to H satisfying the imposed condition. By the nearest function to H(w) we understand that function (or one of those functions) satisfying the minimum squared error with respect to H(w), under the given condition.

One can be motivated to search for non-uniform filters. For example, one can ask the output with a 'medium' membership function. This can be accomplished by filtering the signals s(t) - p(t) and q(t) - p(t) by the same type of low-pass filters, leaving s(t) unchanged. If the uncertainty is due to the noise, such a processing can be interpreted as noise reduction, and represents rather a filtering of the membership function than the filtering of the signal itself.

One must take care when interpreting the result of the filtering of membership functions. One can be tempted to believe that it is possible to reduce the uncertainty by filtering. This is generally false. Filtering rather puts into evidence the way uncertainty is changing (slowly or quikly). However, if the signal is stationary, it is reasonable to consider that the mean values of q(t) - s(t) and of s(t) - p(t) are the 'true' uncertainties. Thus, the low pass (average) filtering of the above difference-signals allow us to decrease the uncertainty of certain samples of the signals, if they have the corresponding values of s-p and q-s greater than the average values.

It is worth to note some properties of the fuzzy signals with linear dependent uncertainty, i.e.

$$q(t) - s(t) = a + b.s(t)$$

$$s(t) - p(t) = c + d.s(t)$$

For such a signal, one can assign to each of the frequency components of s(w) its ambiguity, given by the components of the same frequency of s - p and q - s. Consider now the linear model is assumed, and the actual p and q functions have frequency components which do not exist in the spectrum of S(w). Then, one is motivated to consider that there exists a zero-valued noise, ${}_{u}z_{v}$, such as z(t) = 0, u and v being given by those components in P(w) and Q(w) which do not exist in S(w). Following, one can reduce the uncertainty by deducing from p and q the values of u and respectively v.

5. Final remarks and conclusions

Only some particular cases of filtering were discussed above. Further interesting aspects arise when considering the signals having fuzzy variable, or those linguistic degrees-valued. Moreover, one can interpret the fuzzy signals as crisp signals belonging to fuzzy sets of (crisp) signals. Some of these aspects are discussed in /1/, /2/.

Digital signal processing of fuzzy signals can be performed after conversion into crisp binary form of p, s and q. However, a more specific way would be to use the fuzzy binary representations introduced in this paper (see also /2/). Such representations also require a fewer number of bits than the convertion in crisp binary form. The representation requiring only crisp binary weights and the basical fbn's (and 1) seems the most appropriate. The representation

$$_{p}s_{q} = A.\hat{0} + B.\hat{1} + C.1$$

yields the signals A(t), B(t) and C(t), which can be themselves processed. Finally, specific criteria could be derived for the filtering of these signals.

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/2/ H.N. Teodorescu - Pattern-oriented robust fuzzy filtering of images, submited, Iizuka Conf., Kyoto, 1988