FUZZY RELIABILITY OF SERIES SYSTEMS

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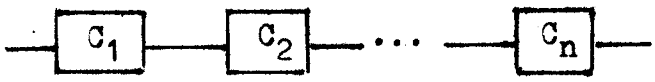
ABSTRACT: This paper is one of continuation for <Fuzzy Reliability>. It provides the calculating method which can be used in calculation of the fuzzy reliability for the series systems. In this paper authors derive the series formulas by means of the basic concepts and principles of fuzzy reliability. This paper only consider the indexes of FA mode.

KEY WORDS: fuzzy reliability of the series systems, fuzzy failure rate of the series systems, fuzzy mean life of the series systems.

I. INDEXES OF FUZZY RELIABILITY

We still employ the classificatory method of a system in general reliability theory. Suppose a system consists of several elements. If failure of any one element in the system would result in failure of the system, then the system is called the series system.

It is supposed that a series system consists of n distinct elements C_1, C_2, ..., C_n (as shown in figure). Further, it may be assumed that failure of any element would



occur independently of the operation of other components. If the sign C denotes the normal operation of the system, the sign C_j denotes the normal operation of the element C_j, $j = 1, 2, \ldots, n$, and the sign $\underset{\sim}{A}_i$ denotes discussing one of fuzzy performance subsets, then by means of the definition of fuzzy conditional probability we have

$$P(C \wedge \underset{\sim}{A}_i) = P(\underset{\sim}{A}_i | C)\, P(C) \tag{1}$$

$$P(C_j \wedge \underset{\sim}{A}_i) = P(\underset{\sim}{A}_i | C_j)\, P(C_j) \tag{2}$$

where the sign $\wedge$ denotes algebraic product.

When failure of any element would occur independently of the operation of other components, by means of general probability theory we obtain

$$P(C) = \prod_{j=1}^{n} P(C_j) \tag{3}$$

Substituting Eqs. (1) and (2) into Eq. (3), we obtain

$$\frac{P(C \wedge \underset{\sim}{A}_i)}{P(\underset{\sim}{A}_i | C)} = \prod_{j=1}^{n} \frac{P(C_j \wedge \underset{\sim}{A}_i)}{P(\underset{\sim}{A}_i \mid C_j)} = \frac{\prod_{j=1}^{n} P(C_j \wedge \underset{\sim}{A}_i)}{\prod_{j=1}^{n} P(\underset{\sim}{A}_i | C_j)} \tag{4}$$

In terms of the definition of series systems, we have

$$C = \bigcap_{j=1}^{n} C_j$$

then

$$P(\underset{\sim}{A}_i | C) = P(\underset{\sim}{A}_i | \bigcap_{j=1}^{n} C_j) = \prod_{j=1}^{n} P(\underset{\sim}{A}_i | C_j) \tag{5}$$

Substituting Eq. (5) into Eq. (4), we obtain

$$P(C \wedge \underset{\sim}{A}_i) = \prod_{j=1}^{n} P(C_j \wedge \underset{\sim}{A}_i) \tag{6}$$

Now we employ the denotations as follows:

R_s -- general reliability of series systems;

$\underset{\sim}{R}_s$ -- fuzzy reliability of series systems;

R_j — general reliability of element C_j;

$\underset{\sim}{R}_j$ -- fuzzy reliability of element C_j;

$\mu_{\underset{\sim}{A}_i}(R_s)$ -- degree of membership of R_s in $\underset{\sim}{A}_i$;

$\mu_{\underset{\sim}{A}_i}(R_j)$ -- degree of membership of R_j in $\underset{\sim}{A}_i$.

Then by means of general reliability theory and fuzzy reliability theory we have

$P(C) = R_s$	$P(C_j) = R_j$
$P(C \wedge \underset{\sim}{A}_i) = \underset{\sim}{R}_s$	$P(C_j \wedge \underset{\sim}{A}_i) = \underset{\sim}{R}_j$
$P(\underset{\sim}{A}_i \vert C) = \mu_{\underset{\sim}{A}_i}(R_s)$	$P(\underset{\sim}{A}_i \vert C_j) = \mu_{\underset{\sim}{A}_i}(R_j)$

Therefore Eqs. (1), (2), (5) and (6) may be respectively rewritten as follows:

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_i}(R_s)\, R_s \tag{7}$$

$$\underset{\sim}{R}_j = \mu_{\underset{\sim}{A}_i}(R_j)\ R_j \tag{8}$$

$$\mu_{\underset{\sim}{A}_i}(R_s) = \prod_{j=1}^{n} \mu_{\underset{\sim}{A}_i}(R_j) \tag{9}$$

$$\underset{\sim}{R}_s = \prod_{j=1}^{n} \underset{\sim}{R}_j \tag{10}$$

The Eqs. (7) and (10) are the general expressions of the fuzzy reliability of series systems.

By means of the relation between fuzzy reliability ($\underset{\sim}{R}_j$) and fuzzy failure rate ($\underset{\sim}{\lambda}_j$) for element C_j, we have

$$\underset{\sim}{R}_j = e^{-\int_0^t \underset{\sim}{\lambda}_j dt}$$

Substituting it into Eq. (10), we obtain

$$\underset{\sim}{R}_s = \prod_{j=1}^{n} e^{-\int_0^t \underset{\sim}{\lambda}_j dt} = e^{-\int_0^t \sum_{j=1}^{n} \underset{\sim}{\lambda}_j dt} \tag{11}$$

Let $\underset{\sim}{\lambda}_s$ be equal to $\sum_{j=1}^{n} \underset{\sim}{\lambda}_j$, in fact, $\underset{\sim}{\lambda}_s$ is fuzzy failure rate of the system, then

$$\underset{\sim}{R}_s = e^{-\int_0^t \underset{\sim}{\lambda}_s dt} \tag{12}$$

If $\underset{\sim}{\lambda}_s$ is equal to constant, then

$$R_s = e^{-\underset{\sim}{\lambda}_s t} \tag{13}$$

By means of the relation between fuzzy mean life and fuzzy reliability, the fuzzy mean life of the systems is expressed

$$\underset{\sim}{MTTF}_s = \int_0^\infty \underset{\sim}{R}_s \, dt$$

$$= \int_0^\infty e^{-\underset{\sim}{\lambda}_s t} \, dt$$

$$= \frac{1}{\underset{\sim}{\lambda}_s} = \frac{1}{\sum_{j=1}^{n} \frac{1}{\underset{\sim}{MTTF}_j}} \tag{14}$$

where $\underset{\sim}{MTTF}_j$ is the fuzzy mean life of element C_j.

If the fuzzy failure rate of every element is an equality, then we obtain

$$\underset{\sim}{R}_s = e^{-n \underset{\sim}{\lambda}_j t} \tag{15}$$

$$\underset{\sim}{\lambda}_s = n \underset{\sim}{\lambda}_j \tag{16}$$

$$\underset{\sim}{MTTF}_s = \frac{\underset{\sim}{MTTF}_j}{n} \tag{17}$$

From the Eq. (7) to Eq. (17), it is concluded that as follows:

1. Fuzzy reliability of the series systems is equal to product of fuzzy reliability of every component.

2. If fuzzy reliability of every component have a exponental distribution, then fuzzy reliability of the series systems also have a exponental distribution.

③ Fuzzy reliability of the series systems still is expressed by product of general reliability of the systems and degree of membership of the systems. The degree of membership of the systems is equal to product of the degree of membership of every component.

④ Fuzzy failure rate of the series systems is equal to sum of the fuzzy failure rate of every component.

⑤ Fuzzy mean life of the series systems is equal to a recipracal of the fuzzy failure rate of the systems.

II. EXAMPLE

A bicycle consists of thirteen major elements, namely, bicycle frame, front wheel, rear wheel, handlebar, front axle and bearing, bottom bracket bearing, rear axle and bearing, sprocket wheel, chain drive, chain, front fork, hand brake, and crank.

We have two lots Gold Star bicycles. First lot bicycles were employed on half a year. Second lot bicycles were employed on three point five years. Suppose the general reliability of every element of first lot bicycles is equal to 0.998, the general reliability of every element of second lot bicycles is equal to 0.986, and assume that:

$$\mu_{\text{extremely large performance}}(0.998) = 0.99$$

$$\mu_{\text{extremely large performance}}(0.986) = 0.6$$

$$\mu_{\text{more large performance}}(0.998) = 0.5$$

$$\mu_{\text{more large performance}}(0.986) = 0.986$$

Find indexes of the fuzzy reliability for the two lots bicycles in "extremely large performance" (i.e. "extremely reliable bicycle") and "more large performance" (i.e. "more reliable bicycle").

Suppose that the sign ELP denotes "extremely large performance", and the sign MLP denotes "more large performance". We shall respectively solve the problems above mentioned in the two fuzzy performance subsets.

1. In ELP

At first, consider first lot bicycles.

Since

$$\mu_{ELP}(R_s) = \prod_{j=1}^{13} \mu_{ELP}(R_j)$$

$$= (0.99)^{13} = 0.8775$$

$$R_s = \prod_{j=1}^{13} R_j = (0.998)^{13} = 0.9743$$

therefore

$$\underset{\sim}{R}_s = \mu_{ELP}(R_s)\, R_s = 0.8775 \times 0.9743 = 0.855$$

Assuming that $\underset{\sim}{\lambda}_s$ is equal to constant, then we obtain

$$\underset{\sim}{R}_s = e^{-\underset{\sim}{\lambda}_s t}$$

$$\underset{\sim}{\lambda}_s = \frac{\ln \underset{\sim}{R}_s}{-t} = \frac{-0.1567}{-0.5} = 0.313 \quad [1/\text{year}]$$

$$\underset{\sim}{MTBF}_s = \frac{1}{\lambda s} = \frac{1}{0.313} = 3.195 \quad \{year\}$$

The general mean life of first lot bicycles is

$$MTBF_s = \frac{1}{\lambda s} = \frac{-t}{\ln R_s} = \frac{-0.5}{-0.026} = 19.23 \quad [year]$$

From these results it is concluded that the general mean life of first lot bicycles is equal to 19.23 (year), at present the lot bicycles find oneself in a extremely reliable work period, and this period still continue for 2.695 (year).

Next, consider second lot bicycles.

Since

$$\mu_{ELP}(R_s) = \prod_{j=1}^{13} \mu_{ELP}(R_j)$$

$$= (0.6)^{13} = 0.0013$$

$$R_s = \prod_{j=1}^{13} R_j = (0.986)^{13} = 0.8325$$

therefore

$$\underset{\sim}{R}_s = \mu_{ELP}(R_s)\, R_s = 0.0013 \times 0.8325 = 0.001$$

From this result it is concluded that second lot bicycles broken away from extremely reliable work period.

2. In MLP

At first, consider second lot bicycles.

Since

$$\mu_{MLP}(R_s) = \prod_{j=1}^{13} \mu_{MLP}(R_j)$$

$$= (0.986)^{13} = 0.8325$$

$$R_s = \prod_{j=1}^{13} R_j = (0.986)^{13} = 0.8325$$

therefore

$$\underset{\sim}{R}_s = \mu_{MLP}(R_s)\ R_s = 0.8325 \times 0.8325 = 0.693$$

Assuming that $\underset{\sim}{\lambda}_s$ is equal to constant, then we obtain

$$\underset{\sim}{R}_s = e^{-\lambda_s t}$$

$$\lambda_s = \frac{\ln \underset{\sim}{R}_s}{-t} = \frac{-0.3667}{-3.5} = 0.105 \qquad [1/\text{year}]$$

$$\underset{\sim}{MTBF}_s = \frac{1}{\underset{\sim}{\lambda}_s} = \frac{1}{0.105} = 9.544 \qquad [\text{year}]$$

The general mean life of second lot bicycles is

$$MTBF_s = \frac{1}{\lambda_s} = \frac{-t}{\ln R_s} = \frac{-3.5}{-0.1833} = 19.06\ [\text{year}]$$

From these result it is concluded that the general mean life of second lot bicycles is equal to 19.06 (year), at present the lot bicycles find oneself in a more reliable work period,and this period still continue for 6.044(year).

Next, consider first lot bicycles.

Since

$$\mu_{MLP}(R_s) = \prod_{j=1}^{13} \mu_{MLP}(R_j) = (0.5)^{13} = 0.00012$$

$$R_s = \prod_{j=1}^{13} R_j = (0.998)^{13} = 0.9743$$

therefore

$$R_s = \mu_{MLP}(R_s) * R_s = 0.00012 * 0.9743 = 0.0001$$

From this result it is conclude that first lot bicycles at present find oneself in a extremely reliable work period,and do not get into a more reliable work period.

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