FUZZY SWITCHING QUASI-BOOLEAN ALGEBRA AND VISION SIMULATION COLOR DISCERNING

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ABSTRACT

If the sub-lines on the line $I=[{}^{1}_{0}]$ are regarded as the elements in set L, then the Boolean algebra on L may be called line Boolean algebra or line-algebra. About this, there are several useful theorems. By these theorems and the laws of Boolean operation, the measurement of any line-function can be calculated by the maximun, minimun, addition and subtraction of the ends of the sub-lines. While we regard the ends of the sub-lines as fuzzy switching variables, hence we have fuzzy switching quasi-Boolean algebra, on which we can set up a terse color vision system mathematical model and obtain a way of vision simulation color discerning.

1. FUZZY SWITCHING QUASI-BOOLEAN ALGEBRA

Let the line I represented by a longitudinal interral $[{}^{1}_{0}]$, i.e. $I=[{}^{1}_{0}]$. Then sub-lines on I (Their lower ends are all 0) $A=[{}^{a}_{0}]$, $B=[{}^{b}_{0}]$, $C=[{}^{c}_{0}]$...form a set L. The function of the Boolean algebra on L or line-function may be signed as

$$Y = (A,B,C...).$$

Y may be some sub-lines that do not overlap each other. How to get the upper and lower ends of these sub-lines and $|Y|$ which is the measurement of Y ? This is to be done as following.

Suppose F(A,B,C...) contains three kinds of operation $\cdot$, $\vee$ and $^{-}$, and the $\cdot$ can be omitted. Then let $|\ |$, A, B, C... in $|F(A,B,C...)|$ be substituted by [ ], a, b, c ... respectively. The substituded result is [f(a,b,c...)], i.e.

$$|F(A,B,C...)| = [f(a,b,c...)].$$

According to the definition, f(a,b,c...) follows the laws of Boolean operation as well as F(A,B,C...). About [ ], there are several useful theorems.

Theorem 1. If there are two variables in f(...), then

$$[ab] = \min(a,b) \,,$$

$$[a\vee b] = \max(a,b),$$

$$[a\bar{b}] = \max(0,a-b) \,.$$

The above formulae are obvious. From theorem 1, we can deduce

$$[a] = a \,,$$

$$[\bar{a}] = 1-a \,,$$

$$[a\bar{a}] = 0 \,.$$

Theorem 2. If $f(a,b,c...)=y_1\vee y_2$, y_1 and y_2 are sub-functions, and $[y_1y_2]=0$, then

$$[f(a,b,c...)] = [y_1] + [y_2] \,.$$

For exsample

$$[a\vee \bar{a}] = [a] + [\bar{a}] = 1 \,.$$

Proof: Since $[y_1y_2]=0$, $Y_1Y_2 =\emptyset$.

Therefore

$$[f(a,b,c...)] = |F(A,b,c...)|$$

$$= |Y_1| + |Y_2|$$

$$= [y_1] + [y_2] \,.$$

Theorem 3. If w in f(...,w,...) is a sub-function that contains no negative or complementary operation $^-$, then

$$[f(...,w,...)] = [f(...,[w],...)] \,.$$

For example

$$[a\overline{bc}] = [a\overline{b\vee c}] = [a\overline{[b\vee c]}]$$

$$= \max(0,a-\max(b,c)) \,.$$

To prove this theorem needs only to prove [w]=w.

Proof: Since w contains no negative operation, W must be a line of which the lower end is 0, hence |W| must equal w that is the upper end of W. And account of [w]=|W| , therefore [w]=w.

Depending upon the above theorems and the laws of Boolean operat-

ion, it is not difficult to change any complex [f(a,b,c...)] into the operations maximun, minimun, addition and sutraction of a, b, c .:. . First, we may change [f(a,b,c...)] into the sum of several parts any one of which contains negative operation ‾ no more then one (This is possible after all because if there are n variabies in f(a,b,c...), we are certainly able to change f(a,b,c...) into the sum of 2^n miniterms, and then use Demogon's law to make every mini-term contain ‾ no more then one). Then we may use theorem 2, 3 and 1 in proper order to obtain the final result. At the same time, the upper and lower ends of the sub-lines represented by Y that do not overlap each other are given naturally.

Example. Calculat the value of line-function $Y=A\overline{B}\overline{\overline{C}\overline{D}}$.

Solution:

$$\begin{aligned}|Y| &= [a\overline{b}\overline{\overline{c}\overline{d}}] = [a\overline{b}(\overline{c}\vee d)] \\ &= [a\overline{b}\overline{c}\overline{d}] + [a\overline{b}d] \\ &= [a\overline{[b\vee c\vee d]}] + [[ad]\overline{b}] \\ &= \max(0,a-\max(b,c,d)) + \max(0,\min(a,d)-b)\ .\end{aligned}$$

From above result we can see Y represent the two sub-lines as following

$$\left[\begin{matrix} a \\ \max(b,c,d)\end{matrix}\right] \quad \text{and} \quad \left[\begin{matrix}\min(a,d) \\ b\end{matrix}\right]\ .$$

So far, there is no question for the resolution of the line-function in which the lower ends of the sub-lines are all 0.

If the lower end of a sub-line in F(A,B,C...) is not 0, then this sub-line can be regarded as the difference or relative complement of two sub-lines the lower ends of which are both 0. For example

$$A = \left[\begin{matrix} a_1 \\ a_2\end{matrix}\right] \quad (\ a_2 > a_1 \text{ is admitted, If } a_2>a_1, \text{ then } |A| = 0\ ).$$

We may suppose

$$A_1 = \left[\begin{matrix} a_1 \\ 0\end{matrix}\right]\ , \quad A_2 = \left[\begin{matrix} a_2 \\ 0\end{matrix}\right]\ .$$

Then

$$A = A_1\overline{A}_2\ ,$$

and

$$\begin{aligned}|Y| &= |F(A_1\overline{A}_2,B,C...)| \\ &= [f(a_1\overline{a}_2,b,c...)]\ .\end{aligned}$$

Up to now, there is no question, too, for the resolution of line- function in which the lower ends of the sub-lines are 0 or not.

If a, b, c... $\in R=[0,1]$ are fuzzy switching variables, we denifite:

The relation between fuzzy switching function $f^*(a,b,c...)$ and a, b,c... is the relation between the measuremen of line-function and the upper ends of the sub-lines the lower ends of which are all 0. That is

$$f^*(a,b,c...) = [f(a,b,c...)] = |F(A,B,C...)|.$$

The algebra on R definited by above formula may be called fuzzy switching quasi-Boolean algebra. It may be signed as FSQBA = < R,(·, ,$^-$)X[ ],0,1 >. It is worthy to be noted that [ ] is a special operation but a binary operation, and we can not regard [ ] as ( ) to discuss whether combinative law and distributive law in FSQBA are tenable.

In FSQBA, the negation is distinquished into relative negation and absolute negation. The relative and the absolute of a in [f(a,b,c...)] are represented by $\bar{a}$ and $[\bar{a}]$ respectively. For the relative negation, the complementary laws are tenable, i.e.

$$[a\bar{a}] = 0 ,$$

$$[a\vee \bar{a}] = 1 .$$

However, for the absolute negation, the complementary laws are usually untenable, i.e.

$$[a[\bar{a}]] = \min(a,1-a) \neq 0 ,$$

$$[a\vee [\bar{a}]] = \max(a,1-a) \neq 1 .$$

It is obvious that the above two formulae are really the different presenting of the untenability of complementary laws in Zadeh algebra.

It is not difficult to prove that, in FSQBA, if let R only contain 0 and 1, then FSQBA will shrink to switching Boolean algebra or switching algebra; and if the relative negation is substituded by the absolute negation, the FSQBA will shrink to Zadeh algebra. It can be seen that FSQBA is an algebra of more universal signification.

2. HUMAN COLOR VISION SYSTEM MATHEMATICAL MODEL

For a series of equal-energy monochromatic lights, if the output signals from three kinds of optic cones, but spectral tristimulus values, are b, g and r which are the functions of wavelenth and are shown in Fig. 1, then $[\bar{b}\bar{g}r]$, $[\bar{b}gr]$, $[\bar{b}g\bar{r}]$, $[bg\bar{r}]$, $[b\bar{g}\bar{r}]$, $[b\bar{g}r]$ and $[bgr]$ will reflect the quantities of the stimulus to seven kinds of cell in brain that produce red, yellow, green, cyan, blue, magenta and white color sensation respectively.

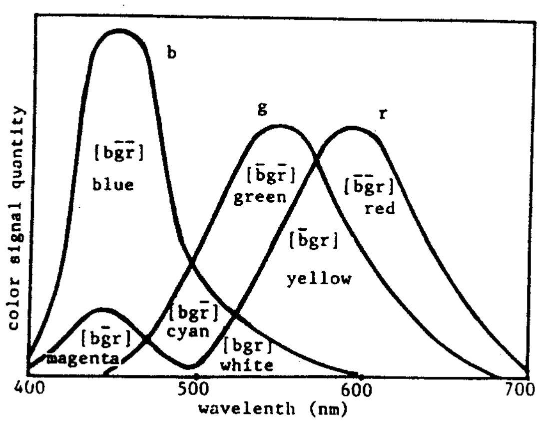

Fig. 1

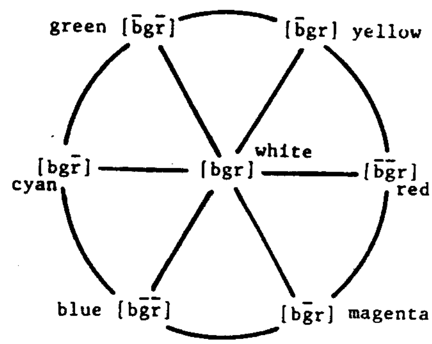

Fig. 2

For a certain color, among the seven quantities or function values, no more than three are probably not 0, and the three quantities must be at the three vertices of a sector in Fig. 2.

Suppose the three quantities are m_1, m_2 and m_3 , $m_3=[bgr]$, the angles of seat on the round of m_1 and m_2 are θ_1 and θ_2 respectively. We have

$$\theta = \frac{m_1\theta_1 + m_2\theta_2}{m_1 + m_2} ,$$

$$R = m_1 + m_2 ,$$

$$Z = m_1 + m_2 + m_3 .$$

The data verification showes that if CIE tristimulus values X, Y, Z are properly changed into b, g, r, then θ, R, Z from b, g, r will reflect hue, saturation and lightness better (The details will be shown in other paper).

The six quantities at the circle in Fig. 2 may be calculated by the way as following. First calculate

$$m = [bg \vee br \vee gr] \ .$$

Then do

$$[b\bar{m}] = [b\bar{g}\bar{r}], \qquad [\bar{b}m] = [\bar{b}gr] \ ;$$

$$[g\bar{m}] = [\bar{b}g\bar{r}], \qquad [\bar{g}m] = [b\bar{g}r] \ ;$$

$$[r\bar{m}] = [\bar{b}\bar{g}r], \qquad [\bar{r}m] = [bg\bar{r}] \ .$$

Clearly this way conforms to the conclusion of physiology optics that color signals exist in the form of tri-pigments at the stage of optic cones, and exist in the form of the opponent-colors at the stage of optic nerve.

3. VISION SIMULATION COLOR DISCERNING

Why do the human color vision systems transform the tristimulus signals into hue, saturation and lightness? The reason is that hue reflects the essential feature of objects or natural lights best, and next does saturation. It is obvious that the way by which spectral features are abstracted is worthy to be emulated for multispectral imagine recognition.

In order to verify the way, we made a device for color discerning. The device includes an APPLE II system with A/D card and a photosensor which can produce blue, green and red three color signals. The procedure of discerning is as following:

1. Let the machine look at a black sample and get b_0, g_0, r_0.
2. Let the machine look at a white sample and get b_1, g_1, r_1.
3. Let the machine look at a sample to be discerned and get b_2, g_2, r_2.

4. Let

$$b_3 = \frac{b_2-b_0}{b_1-b_0}, \quad g_3 = \frac{g_2-g_0}{g_1-g_0}, \quad r_3 = \frac{r_2-r_0}{r_1-r_0} .$$

and

$$b = 0.25b_3^{1/3} - 0.17 ,$$

$$g = 0.25g_3^{1/3} - 0.17 ,$$

$$r = 0.25r_3^{1/3} - 0.17 .$$

Selecting above nonlinear function, we refered to the relation between Munsell value V and luminosity Y. The experiment makedit clear that using above function can make R reflect Munsell chroma C well and decrease the errorsof discerning hue.

5. Calculate θ, R, Z from b, g, r and judge the hue, saturation and lightness of the sample by θ, R, Z respectively. At the same time, print the name such as "deep red", "light yellow", "dark blue" and all that.

6. If the other sample need be discerned, then go to 3 else end.

The experiment showes the printed color name and the name said by a man are uniform about 80 per cent, and simillar about 15 per cent. If the three sensitive curves of the photosensor are more alike to that of the human eye, uniformity of discerning will increase yet. It may be seen that the way presented by this paper is essentially effective.

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