Some Notes On The " A Value-Raising Method For Finding Fuzzy Transitive Closure "

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In paper (1), Xiao Xian has given the value-raising method for finding the transitive closure of a fuzzy similar matrix R. In this article ,we shall show that the result given by the value-raising algorithm doesn't agree with the results given by the usual algorithm . The value-raising method for finding transitive closure is called " the value-raising method" in following contents for short. For the matrix R, using the value-raising method and the usual method, we can obtain the transitive closures $\hat{R}'$ and $\hat{R}$ respectively. Here, the usual method means the fuzzy compound--computation.

§ 1 Finding $\hat{R}'$

Let R be a fuzzy similar matrix, r_{i-j-k} be a small rectangle with its one vertex on the main diagonal of the matrix R, where i,j,k denote the columns and rows of vertexes of r_{i-j-k} in the matrix R. From (1) , we can see that, in R, there exist i,j,k, $i<j<k$, such that a element ($\neq 1$) in r_{i-j-k} is smaller than the rest two elements. Then we replace the smallest element with the secondary big one in r_{i-j-k}. Symmetrically, we change the symmetric number in R. Such a changed-matrix is called the value-raising matrix of R. If all r_{i-j-k} of R satisfy the condition as the above small rectangel does, we can obtain the transitive closure $\hat{R}'$ of R, and the example 5 in (1) gave it a algorithm.

§ 2 Some Notes

1. The clustering result from the value-raising method doesn't agree with the result from the usual method.

Reverse example 1:

let R be a fuzzy similar matrix,

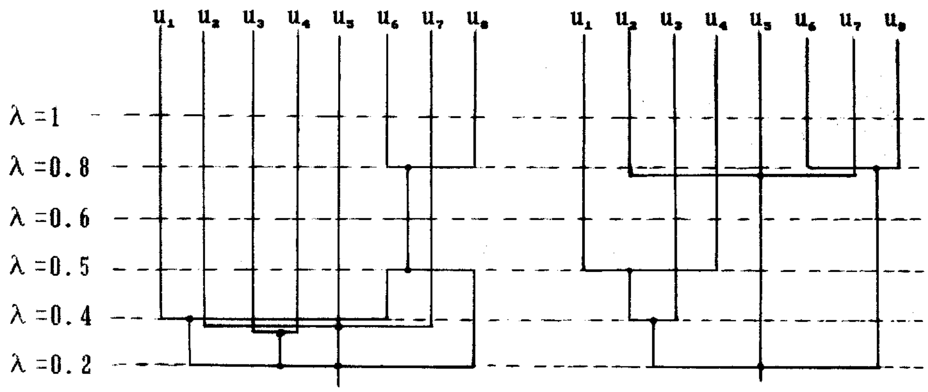
$$R=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.4 \\ 0 & 1 & 0 & 0 & 0.8 & 0 & 0.8 & 0.2 \\ 0 & 0 & 1 & 0.4 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0.4 & 1 & 0 & 0.2 & 0 & 0.5 \\ 0 & 0.8 & 0 & 0 & 1 & 0 & 0.4 & 0 \\ 0.5 & 0 & 0.2 & 0.2 & 0 & 1 & 0 & 0.8 \\ 0 & 0.8 & 0 & 0 & 0.4 & 0 & 1 & 0 \\ 0.4 & 0.2 & 0.2 & 0.5 & 0 & 0.8 & 0 & 1 \end{bmatrix} \qquad (1)$$

Using the value-raising method and the usual respectively, we obtain the transitive cloures $\hat{R}'$ and $\hat{R}$ as follows:

$$\hat{R}'=\begin{bmatrix} 1 & 0.2 & 0.2 & 0.2 & 0.2 & 0.5 & 0.2 & 0.4 \\ 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.2 & 0.2 & 1 & 0.4 & 0.2 & 0.2 & 0.2 & 0.4 \\ 0.2 & 0.2 & 0.4 & 1 & 0.2 & 0.2 & 0.2 & 0.5 \\ 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 & 0.4 & 0.2 \\ 0.5 & 0.2 & 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 & 0.2 & 0.4 & 0.2 & 1 & 0.2 \\ 0.4 & 0.2 & 0.4 & 0.5 & 0.2 & 0.8 & 0.2 & 1 \end{bmatrix} \qquad (2)$$

$$\hat{R}=\begin{bmatrix} 1 & 0.2 & 0.4 & 0.5 & 0.2 & 0.5 & 0.2 & 0.5 \\ 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.4 & 0.2 & 1 & 0.4 & 0.2 & 0.4 & 0.2 & 0.4 \\ 0.5 & 0.2 & 0.4 & 1 & 0.2 & 0.5 & 0.2 & 0.5 \\ 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 & 0.8 & 0.2 \\ 0.5 & 0.2 & 0.4 & 0.5 & 0.2 & 1 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 & 0.2 & 0.8 & 0.2 & 1 & 0.2 \\ 0.5 & 0.2 & 0.4 & 0.5 & 0.2 & 0.8 & 0.2 & 1 \end{bmatrix} \qquad (3)$$

Using the matrix (2) and (3), we have the culster figures when $\lambda=1$, 0.8, 0.6, 0.5, 0.4 and 0.2



(Fig.1 for $\hat{R}'$) (Fig.2 for $\hat{R}$)

It is well-known that when λ decreases, the classfication becomes thickness. When $\lambda=0.8$, Fig.1 shows that u_6 and u_8 belong to one class $\{u_6, u_8\}$. With λ decreasing (for $\lambda=0.5$), u_6 and u_8 split into two classes: $\{u_6\}$, $\{u_8\}$ and so on. Obviously, the above results don't agree with the results from $\hat{R}$.

We point out that:

1° The example 3 in [1] showed that, using the value-raising method, the results of $\hat{R}'$ and its clustering Fig. agree with those using the usual method. It is a concidence. If the result of example 5 in [1] is tested by using the value-raising method, the conclusion is not correct.

2° For the number i and j (i, j=1, 2, ⋯, n), if s_i and s_j are two classifications of U, then $\bigcup_{i=1}^{n} s_i = U$, and $s_i \cap s_j = \phi$ $(i \neq j)$. When λ is determined $(0 \leqslant \lambda \leqslant 1)$, the corresponding fuzzy clustering is unique for the concrete fuzzy matrix. This clustering doesn't depend on the algorithm.

3° In the clustering analysis, we often use the compound computation to find the $\hat{R}$ for a fuzzy similar matrix R. Indeed, there is a value-raising problem for $\mu_R(x_i, x_j)$. Here, the "value-raising" means that: suppose $\mu_R(x_i, x_j) = \min\{\mu_R(x_i, x_p), \mu_R(x_p, x_i)\}$, then we take the biggest one from all those $\mu_R(x_i, x_j)$, i.e.

$$\bigvee_{p=1}^{n} [\mu_R(x_i, x_p) \wedge \mu_R(x_p, x_j)] \quad \text{for } i, j = 1, 2, \cdots, n.$$

The value-raising in [1] is the value-raising between two elements $\mu_R(x_i, x_k)$, $\mu_R(x_i, x_k)$ or between $\mu_R(x_j, x_i)$ or $\mu_R(x_j, x_k)$ in R, this raised-value replace the raised-value of $\mu_R(x_i, x_j)$ on U. As for some clustering analysis, the result of value-raising method is not correct. Then, the value-rasing method in [1] is not practical.

2. The condition $x_1 < x_2 < x_3$ of Therom 1 in [1] is meaningless.

Reverse example 2:

Let U be a western-style clothes set of the same size. According to the similar degree of their colour, let $u_i, u_j \in U$, we have the similar matrix R $(i, j = 1, 2, \cdots, 8)$

$$R = \begin{pmatrix} 1 & 0 & 0.3 & 0.4 & 0.2 & 0 & 0.5 & 0.8 \\ 0 & 1 & 0.2 & 0.2 & 0.5 & 0.6 & 0 & 0.2 \\ 0.3 & 0.2 & 1 & 0.9 & 0 & 0.2 & 0.4 & 0 \\ 0.4 & 0.2 & 0.9 & 1 & 0.4 & 0.6 & 0.2 & 0.6 \\ 0.2 & 0.5 & 0 & 0.4 & 1 & 0 & 0 & 0.4 \\ 0 & 0.6 & 0.2 & 0.6 & 0 & 1 & 0.2 & 0.7 \\ 0.5 & 0 & 0.4 & 0.2 & 0 & 0.2 & 1 & 0.3 \\ 0.8 & 0.2 & 0 & 0.6 & 0.4 & 0.7 & 0.3 & 1 \end{pmatrix} \quad (4)$$

Obviously, we obtain $\hat{R}$ from R (the concrete form of $\hat{R}$ is omited). For the arbitrary $u_i, u_j, u_k \in U$ in $\hat{R}$, their subordinates degrees are $\mu_R(u_i, u_j)$, $\mu_R(u_j, u_k)$ and $\mu_R(u_i, u_k)$, their must have two of them being equal for a arbitrary r_{i-j-k}, and the third one is not smaller than the former two. Among u_i, u_j and u_k, there have not the relation: $u_i < u_j < u_k$. Even if we pick up the elements x_1, x_2, x_3 from $[0, 1]$ to construct a Fuzzy similar matrix R, without the condition $x_1 < x_2 < x_3$, we still can obtain the $\hat{R}$.

We should point out that:

1° In practice, the domain U is not taked from real set when we are doing clustering analysis.

2° For the example 5 in (1), $\hat{R}'$ is obtained by applying the value--raising method to the example 3, the condition $x_1<x_2<x_3$ in theorm 1 is useless.

§ 3 Revising Theorm 1

Theorm: Let R be a n-order, two-face fuzzy similar matrix on U×U. R_{i-j-k} be a set of small retangles with their one vertex on the main diagonal of R. For u_i, u_j, $u_k \in U$, let their subordinate degrees be $\mu_r(x_i, x_j)$, $\mu_r(x_j, x_k)$ and $\mu_r(x_i, x_k)$. For arbitrary $r_{i-j-k} \in R_{i-j-k}$, if and only if two of those subordinate degrees are equal and the third one is not smaller than either of former two, the R be a two-place fuzzy transitive matrix $\hat{R}$ on U×U, that is $R=\hat{R}$.

Collary: Let $\hat{R}$ be a two-place fuzzy transitive matrix on U×U. For u_i, u_j, $u_k \in U$, let their subordinate degrees be $\mu_r(u_i, u_j)$, $\mu_r(u_j, u_k)$, $\mu_r(u_i, u_k)$ respectively. For $r_{i-j-k} \in R_{i-j-k}$, There exist two subordinate degrees being equal, another is not smaller than the former two, where i, j, k $\in$ { 1, 2, …, n}.

References:

1. Xiao Xian, A Value-raising Method for Finding Fuzzy Transitive Closure. FUZZY MATHEMATICS, Vol. 5, No. 4, 1985, Wuhan, China.

2. Wang Peizhuang, Theories and Applications of Fuzzy Sets, Shanghai, China, 1983.

3. Dubois H. And Prade H., Fuzzy Sets and Systems, Theory and Applications, Academic Press, 1980.