

THE PROGRAM SOLVING THE FUZZY RELATION N-D EQUATION OF A
FUZZY MATRIX

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ABSTRACT

In paper [1] we gave the definition of the fuzzy relational N-D equation of a fuzzy matrix.

Using the fuzzy relational N-D equation can solve Schein rank of a fuzzy matrix [2,3] .

Using the fuzzy relational N-D equation can deeply discuss the eigen fuzzy sets of a fuzzy matrix [4] .

Using the fuzzy relational N-D equation can study the nonsingular and singular fuzzy matrices [5] .

In the paper we give general flowchart of solving the fuzzy relational N-D equation of a fuzzy matrix. And we give the program solving the fuzzy relational N-D equation of a fuzzy matrix, too.

Keywords: The general flowchart of solving the fuzzy relational N-D equation of a fuzzy matrix. The program solving the fuzzy relation N-D equation of a fuzzy matrix.

Definition 1. Let $A = (a_{ij})_{m \times n}$. And A be a non-zero fuzzy matrix. A equation in the form of

$$A_{m \times n} = Y_{m \times t} X_{t \times n} \quad (1)$$

is called a fuzzy relational non-deterministic equation of the fuzzy matrix A or a N-D equation of A . (Where A is known, and both Y and X are unknown). t is called the index. Both Y and X such that (1) holds are called the solution matrices of the N-D equation of A while

The index is t .

[1] proved that:

Theorem 1. The N-D equation of a $m \times n$ fuzzy matrix A has the solutions if the index $t = n$ or $t = m$.

Theorem 2. For an index $t = t_0$, if the N-D equation of a $m \times n$ fuzzy matrix A has the solutions, then the N-D equation of A has the solutions when $t = t_0 + k$, too. Where k is a positive indeger.

Theorem 3. The N-D equation of a $m \times n$ fuzzy matrix $A = (a_{ij})_{m \times n}$ always the solutions if the index $t \geq \min\{m, n\}$.

[6]gived the definition of Schein rank of a fuzzy matrix.

Definition 2. The Schin rank $\rho_s(A)$ of a fuzzy matrix A is the least number of rank 1 matrices shose the sum is A .

[3] proved that:

Theorem 4. For a $m \times n$ fuzzy matrix A , $\rho_s(A) = s$ if and only if the N-D equation of A haves not solution while $1 \leq t \leq s-1$, but has a solution while $t = s$.

The $t_0 = \max\{m, n\}$ is called the maximum index of the N-D equation of a fuzzy matrix $A_{m \times n}$.

We give that the general flowohart of solving the fuzzy relation N-D equation of a fuzzy matrix. (see 3 pages)

And we give the program solving the fuzzy relation N-D equation of a fuzzy matrix. (see 4 pages).

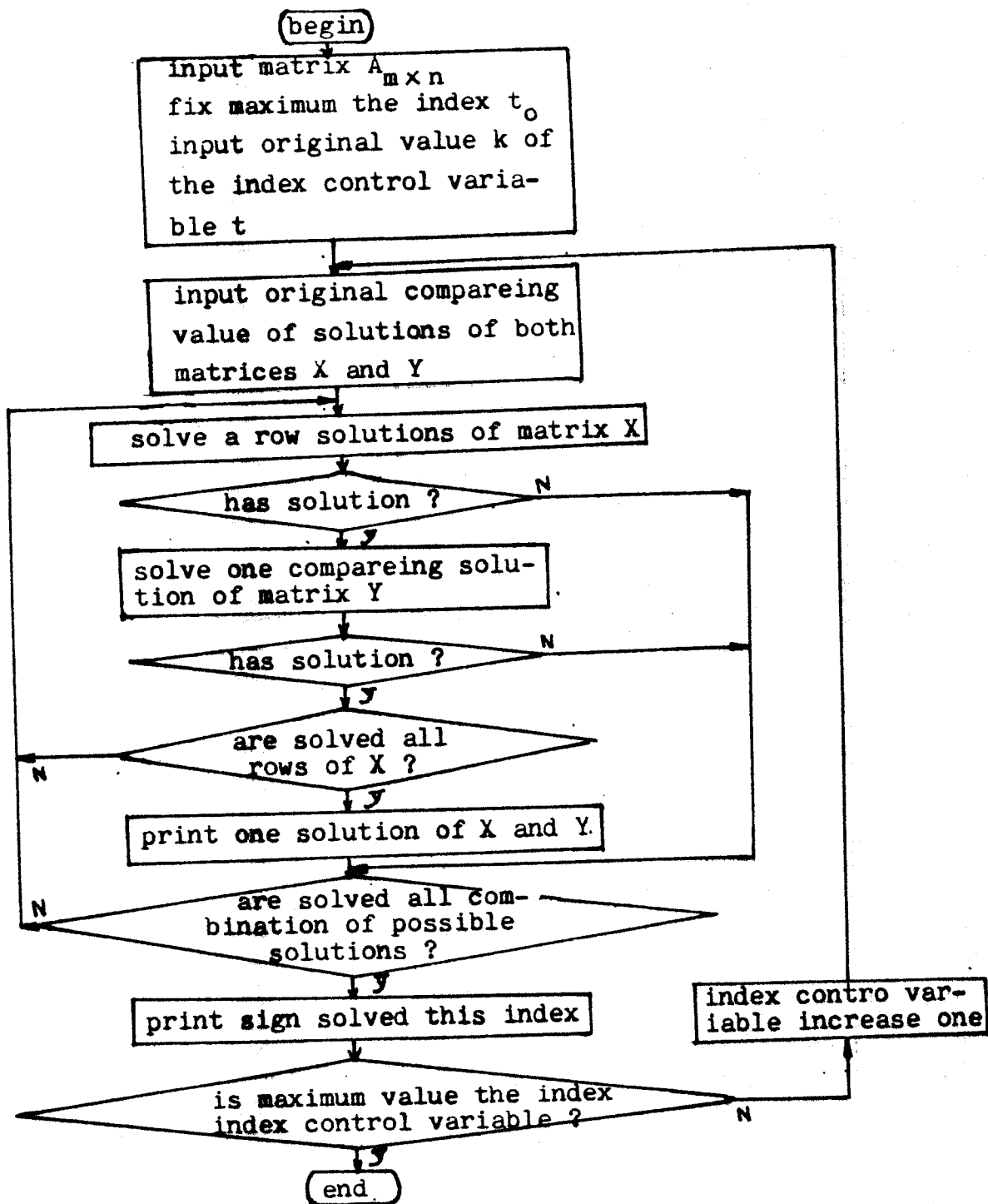
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Some Notes On The " A Value-Raising Method For Finding
Fuzzy Transitive Closure "

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In paper (1), Xiao Xian has given the value-raising method for finding the transitive closure of a fuzzy similar matrix R . In this article ,we shall show that the result given by the value-raising algorithm doesn't agree with the results given by the usual algorithm . The value-raising method for finding transitive closure is called " the value-raising method" in following contents for short. For the matrix R , using the value-raising method and the usual method, we can obtain the transitive closures \widehat{R}' and \widehat{R} respectively. Here, the usual method means the fuzzy compound-computation.

§ 1 Finding \widehat{R}'

Let R be a fuzzy similar matrix, r_{i-j-k} be a small rectangle with its one vertex on the main diagonal of the matrix R , where i, j, k denote the columns and rows of vertexes of r_{i-j-k} in the matrix R . From (1) , we can see that, in R , there exist i, j, k , $i < j < k$, such that a element ($\neq 1$) in r_{i-j-k} is smaller than the rest two elements. Then we replace the smallest element with the secondary big one in r_{i-j-k} . Symmetrically, we change the symmetric number in R . Such a changed-matrix is called the value-raising matrix of R . If all r_{i-j-k} of R satisfy the condition as the above small rectangel does, we can obtain the transitive closure \widehat{R}' of R , and the example 5 in (1) gave it a algorithm.

§ 2 Some Notes

1. The clustering result from the value-raising method doesn't agree with the result from the usual method.

Reverse example 1,

let R be a fuzzy similar matrix,

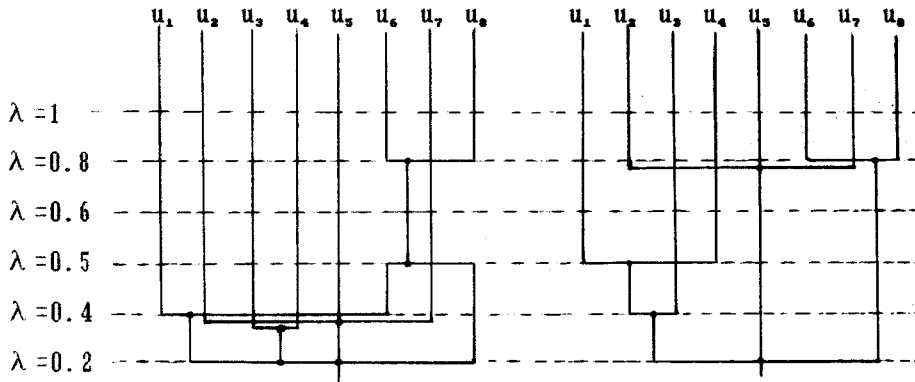
$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.4 \\ 0 & 1 & 0 & 0 & 0.8 & 0 & 0.8 & 0.2 \\ 0 & 0 & 1 & 0.4 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0.4 & 1 & 0 & 0.2 & 0 & 0.5 \\ 0 & 0.8 & 0 & 0 & 1 & 0 & 0.4 & 0 \\ 0.5 & 0 & 0.2 & 0.2 & 0 & 1 & 0 & 0.8 \\ 0 & 0.8 & 0 & 0 & 0.4 & 0 & 1 & 0 \\ 0.4 & 0.2 & 0.2 & 0.5 & 0 & 0.8 & 0 & 1 \end{pmatrix} \quad (1)$$

Using the value-raising method and the usual respectively, we obtain the transitive cloures \widehat{R}' and \widehat{R} as follows,

$$\widehat{R}' = \begin{pmatrix} 1 & 0.2 & 0.2 & 0.2 & 0.2 & 0.5 & 0.2 & 0.4 \\ 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.2 & 0.2 & 1 & 0.4 & 0.2 & 0.2 & 0.2 & 0.4 \\ 0.2 & 0.2 & 0.4 & 1 & 0.2 & 0.2 & 0.2 & 0.5 \\ 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 & 0.4 & 0.2 \\ 0.5 & 0.2 & 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 & 0.2 & 0.4 & 0.2 & 1 & 0.2 \\ 0.4 & 0.2 & 0.4 & 0.5 & 0.2 & 0.8 & 0.2 & 1 \end{pmatrix} \quad (2)$$

$$\widehat{R} = \begin{pmatrix} 1 & 0.2 & 0.4 & 0.5 & 0.2 & 0.5 & 0.2 & 0.5 \\ 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.4 & 0.2 & 1 & 0.4 & 0.2 & 0.4 & 0.2 & 0.4 \\ 0.5 & 0.2 & 0.4 & 1 & 0.2 & 0.5 & 0.2 & 0.5 \\ 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 & 0.8 & 0.2 \\ 0.5 & 0.2 & 0.4 & 0.5 & 0.2 & 1 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 & 0.2 & 0.8 & 0.2 & 1 & 0.2 \\ 0.5 & 0.2 & 0.4 & 0.5 & 0.2 & 0.8 & 0.2 & 1 \end{pmatrix} \quad (3)$$

Using the matrix (2) and (3), we have the cluster figures when $\lambda=1, 0.8, 0.6, 0.5, 0.4$ and 0.2



(Fig.1 for \hat{R}')

(Fig.2 for \hat{R})

It is well-known that when λ decreases, the classification becomes thickness. When $\lambda=0.8$, Fig.1 shows that u_6 and u_7 belong to one class $\{u_6, u_7\}$. With λ decreasing (for $\lambda=0.5$), u_6 and u_7 split into two classes, $\{u_6\}$, $\{u_7\}$ and so on. Obviously, the above results don't agree with the results from \hat{R} .

We point out that,

1° The example 3 in (1) showed that, using the value-raising method, the results of \hat{R}' and its clustering Fig. agree with those using the usual method. It is a coincidence. If the result of example 5 in (1) is tested by using the value-raising method, the conclusion is not correct.

2° For the number i and j ($i, j=1, 2, \dots, n$), if s_i and s_j are two classifications of U , then $\bigcup_{i=1}^n s_i = U$, and $s_i \cap s_j = \phi$ ($i \neq j$). When λ is determined ($0 < \lambda < 1$), the corresponding fuzzy clustering is unique for the concrete fuzzy matrix. This clustering doesn't depend on the algorithm.

3° In the clustering analysis, we often use the compound computation to find the \widehat{R} for a fuzzy similar matrix R . Indeed, there is a value-raising problem for $\mu_r(x_i, x_j)$. Here, the "value-raising" means that, suppose $\mu_r(x_i, x_j) = \min\{\mu_r(x_i, x_p), \mu_r(x_p, x_j)\}$, then we take the biggest one from all those $\mu_r(x_i, x_j)$, i.e.

$$\bigvee_{p=1}^n [\mu_r(x_i, x_p) \wedge \mu_r(x_p, x_j)] \quad \text{for } i, j=1, 2, \dots, n.$$

The value-raising in (1) is the value-raising between two elements $\mu_r(x_i, x_k)$, $\mu_r(x_i, x_n)$ or between $\mu_r(x_j, x_i)$ or $\mu_r(x_j, x_n)$ in R , this raised-value replace the raised-value of $\mu_r(x_i, x_j)$ on U . As for some clustering analysis, the result of value-raising method is not correct. Then, the value-raising method in (1) is not practical.

2. The condition $x_1 < x_2 < x_3$ of Theorem 1 in (1) is meaningless.

Reverse example 2:

Let U be a western-style clothes set of the same size. According to the similar degree of their colour, let $u_1, u_2 \in U$, we have the similar matrix R ($i, j=1, 2, \dots, 8$)

$$R = \begin{pmatrix} 1 & 0 & 0.3 & 0.4 & 0.2 & 0 & 0.5 & 0.8 \\ 0 & 1 & 0.2 & 0.2 & 0.5 & 0.6 & 0 & 0.2 \\ 0.3 & 0.2 & 1 & 0.9 & 0 & 0.2 & 0.4 & 0 \\ 0.4 & 0.2 & 0.9 & 1 & 0.4 & 0.6 & 0.2 & 0.6 \\ 0.2 & 0.5 & 0 & 0.4 & 1 & 0 & 0 & 0.4 \\ 0 & 0.6 & 0.2 & 0.6 & 0 & 1 & 0.2 & 0.7 \\ 0.5 & 0 & 0.4 & 0.2 & 0 & 0.2 & 1 & 0.3 \\ 0.8 & 0.2 & 0 & 0.6 & 0.4 & 0.7 & 0.3 & 1 \end{pmatrix} \quad (4)$$

Obviously, we obtain \widehat{R} from R (the concrete form of \widehat{R} is omitted). For the arbitrary $u_1, u_2, u_3 \in U$ in \widehat{R} , their subordinates degrees are $\mu_r(u_1, u_2)$, $\mu_r(u_2, u_3)$ and $\mu_r(u_1, u_3)$, their must have two of them being equal for a arbitrary $r_{1,2,3}$, and the third one is not smaller than the former two. Among u_1, u_2 and u_3 , there have not the relation, $u_1 < u_2 < u_3$. Even if we pick up the elements x_1, x_2, x_3 from $[0, 1]$ to construct a Fuzzy similar matrix R , without the condition $x_1 < x_2 < x_3$, we still can obtain the \widehat{R} .

We should point out that,

1° In practice, the domain U is not taken from real set when we are doing clustering analysis.

2° For the example 5 in (1), \hat{R}' is obtained by applying the value-raising method to the example 3, the condition $x_1 < x_2 < x_3$ in theorem 1 is useless.

§ 3 Revising Theorem 1

Theorem. Let R be a n -order, two-face fuzzy similar matrix on $U \times U$. $R_{1..j..k}$ be a set of small rectangles with their one vertex on the main diagonal of R . For $u_1, u_j, u_k \in U$, let their subordinate degrees be $\mu_r(x_1, x_j)$, $\mu_r(x_j, x_k)$ and $\mu_r(x_1, x_k)$. For arbitrary $r_{1..j..k} \in R_{1..j..k}$, if and only if two of those subordinate degrees are equal and the third one is not smaller than either of former two, the R be a two-place fuzzy transitive matrix \hat{R} on $U \times U$, that is $R = \hat{R}$.

Collary. Let \hat{R} be a two-place fuzzy transitive matrix on $U \times U$. For $u_1, u_j, u_k \in U$, let their subordinate degrees be $\mu_r(u_1, u_j)$, $\mu_r(u_j, u_k)$, $\mu_r(u_1, u_k)$ respectively. For $r_{1..j..k} \in R_{1..j..k}$, There exist two subordinate degrees being equal, another is not smaller than the former two, where $i, j, k \in \{1, 2, \dots, n\}$.

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