Distribution of Fuzzy Subsets Projected from Random Areas

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Abstract

Fuzzy mathematics has been found its many applications, but any application of fuzzy set theory is based on the membership function. Hence the determination of membership function of fuzzy set is very important. In the paper [1] we given the formulae of membership function of one dimensional fuzzy set. The formulae of membership function of two dimensional fuzzy set will be given in this paper.

Keywords: Fuzzy set, Membership function, Random area, Distribution function.

1 Two dimensional fuzzy set

An element which belongs to a set A needs a two dimensional array to denote sometime. Such as target of shooting, the situation of polluting location etc. One situation is whether or not a polluting location, of course, is of fuzzyiness, so it should be discribed by the two dimensional fuzzy set.

Definition 1: Let U denote the set of all the array (x,y), define the mapping

$$\mu_{\underset{\sim}{A}}: U \longrightarrow [0,1]$$
$$(x,y) \longmapsto \mu_{\underset{\sim}{A}}(x,y)$$

the two dimensional subset $\underset{\sim}{A}$ is thus determined by $\mu_{\underset{\sim}{A}}$. Where $\mu_{\underset{\sim}{A}}$ is called the membership function of $\underset{\sim}{A}$. $\mu_{\underset{\sim}{A}}(x,y)$ is called the degree of membership to which (x,y) belongs to $\underset{\sim}{A}$.

2 The determination of the membership function of two dimensional fuzzy set

What is the degree of membership of fuzzy subset to a given point (x,y) to belong to? We can study it by fuzzy statistic test. If n persons do the test, each of them gives the two dimensional area A_i $(i=1,2,\dots,n)$ of a fuzzy set $\underset{\sim}{A}$. The number that area A_i includes the given point (x,y) is f_n, then the f_n is called frequency of including point (x,y). Of course, greater the number $\frac{f_n}{n}$ is, the more the membership degree of point (x,y) to belonging to fuzzy set $\underset{\sim}{A}$ is, and further, n is greater, then the number $\frac{f_n}{n}$ is more truely the membership degree of point (x,y) to belonging to fuzzy set $\underset{\sim}{A}$.

Definition 2: Given point (x,y), define

$$\mu_{\underset{\sim}{A}}(x,y)=\lim_{n\to\infty}\frac{f_n}{n}$$

Assuming that each investigated object gives circle area which has the centre (ξ,η) with radius ζ, then the circle areas including point (x,y) should satisfy

$$\sqrt{(\xi-x)^2+(\eta-y)^2}\leq\zeta \qquad (\zeta\geq 0)$$

That is to say that three dimensional point (ξ,η,ζ) should fall in to the interior of the cone, i.e

$$\sqrt{(X-x)^2+(Y-y)^2}\leq Z$$

Since the circle area given has the random, then (ξ,η,ζ) forms three dimensional random vector. The degree of membership of point (x,y) belonging to fuzzy set $\underset{\sim}{A}$ should be the probability of random vector (ξ,η,ζ) satisfying

$$\sqrt{(\xi-x)^2+(\eta^2-y)}\leq\zeta$$

Theorem 1: Let A be projected random area on U, $\underset{\sim}{A}$ be fuzzy subset projected from random areas, (ξ, η, ζ) be three dimensional random vector having distribution density p(x,y,z), then

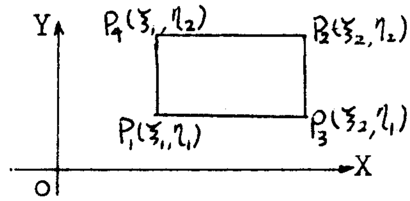
$$\mu_{\underset{\sim}{A}}(x,y)=\iiint_{v} p(x,y,z)\,dxdydz$$

where v satisfies

$$\sqrt{(X-x)^2+(Y-y)^2}\le Z$$

Assuming that each investigated object gives a rectangle area, that is to say , two points $P_1(\xi_1, \eta_1)$ and $P_2(\xi_2, \eta_2)$ are given in plane, where $\xi_2 \ge \xi_1$, $\eta_2 \ge \eta_1$ which form a rectangle area with vertices : P_1 ; P_2 ,and $P_3(\xi_2, \eta_1)$, $P_4(\xi_1, \eta_2)$, thus the rectangle includes point(x,y) if and only if

$$\xi_1 \le x \le \xi_2$$
$$\eta_1 \le y \le \eta_2$$



hold.

Since the two points given by the investigated person have random, (ξ_1, ξ_2) and (η_1, η_2) both are two dimensional random vectors . If $\xi=(\xi_1, \xi_2)$, $\eta=(\eta_1, \eta_2)$ are independent, then the membership degree of point (x,y) belonging to fuzzy set $\underset{\sim}{A}$ should be the probability of random variables ξ_1 , ξ_2 , η_1 , η_2 stisfying

$$\xi_1 \le x \le \xi_2 \quad \text{and} \quad \eta_1 \le y \le \eta_2$$

that is

$$\mu_{\underset{\sim}{A}}(x,y)=P\{\xi_1 \le x \le \xi_2\}\cdot P\{\eta_1 \le y \le \eta_2\}$$

Theorem 2 : Let A be projected random area on U, $\underset{\sim}{A}$ be fuzzy subset projected from random area A, $\xi=(\xi_1, \xi_2)$ $\eta=(\eta_1, \eta_2)$ be independent random vectors having distribution function $F_\xi(x_1, x_2)$, $F_\eta(y_1, y_2)$ respectivly, then

$$\mu_{\underset{\sim}{A}}(x,y)=[F_\xi(x+0,+\infty)-F_\xi(x+0,x)]\cdot[F_\eta(y+0,+\infty)-F_\eta(y+0,y)]$$

Proof: $\mu_{\underset{\sim}{A}}(x,y)=P\{\xi_1 \le x \le \xi_2,\ \eta_1 \le y \le \eta_2\}$

$$=P\{\xi_1 \le x \le \xi_2\}\cdot P\{\eta_1 \le y \le \eta_2\}$$
$$=P\{\xi_1 \in(-\infty, x],\ \xi_2 \in[x,+\infty)\}P\{\eta_1 \in(-\infty, y],\ \eta_2 \in[y,+\infty)\}$$

$$=\{P\{\xi_1\le x,\xi_2<+\infty\}-P\{\xi_1\le x,\xi_2<x\}\}\cdot\{P\{\eta_1\le y,\eta_2<+\infty\}-P\{\eta_1\le y,\eta_2<y\}\}$$
$$=\{F_\xi(x+0,+\infty)-F_\xi(x+0,x)\}\cdot\{F_\eta(y+0,+\infty)-F_\eta(y+0,y)\}$$

Remark: The definition of distribution function is as follows

$$F(x_1,x_2)=P\{\xi_1<x_1,\xi_2<x_2\}$$ (i.e left continuous)

Corollary 1: If random vectors $\xi=(\xi_1,\xi_2)$, $\eta=(\eta_1,\eta_2)$ are independent and continuous, and the distribution functions are $F_\xi(x_1,x_2)$, $F_\eta(y_1,y_2)$ respectivly, then

$$\mu_{\underset{\smile}{A}}(x,y)=\{F_\xi(x,+\infty)-F_\xi(x,x)\}\{F_\eta(y,+\infty)-F_\eta(y,y)\}$$

Corollary 2: If random vectors $\xi=(\xi_1,\xi_2)$, $\eta=(\eta_1,\eta_2)$ are independent and continuous. Random variables ξ_1,ξ_2 are independent and continuous having distribution functions $F_{\xi_1}(x)$, $F_{\xi_2}(x)$ respectivly, and random variables η_1, η_2 are also independent and continuous having distribution functions $F_{\eta_1}(y)$, $F_{\eta_2}(y)$, then

$$\mu_{\underset{\smile}{A}}(x,y)=F_{\xi_1}(x)F_{\eta_1}(y)\{1-F_{\xi_2}(x)\}\cdot\{1-F_{\eta_2}(y)\}$$

Corollary 3: Let random vectors $\xi=(\xi_1,\xi_2)$, $\eta=(\eta_1,\eta_2)$ be independent and discrete, and the distributions of ξ,η be $P\{\xi=(x_{1i},x_{2i})\}=p_i$, $P\{\eta=(y_{1j},y_{2j})\}=p_j$ respectivly, then

$$\mu_{\underset{\smile}{A}}(x,y)=\sum_{\substack{x_{1i}\le x\\ x_{2i}\ge x}}p_i\cdot\sum_{\substack{y_{1j}\le y\\ y_{2j}\ge y}}p_j$$

Corollary 4: Let random vectors $\xi=(\xi_1,\xi_2)$, $\eta=(\eta_1,\eta_2)$ be independent and discrete. If random variable ξ_1,ξ_2 are independent and discrete having distribution laws $P\{\xi_1=x_{1i}\}=p_{1i}$, $P\{\xi_2=x_{2i}\}=p_{2i}$ respectivly, and η_1, η_2 are also independent and discrete having distribution laws $P\{\eta_1=y_{1j}\}=p_{1j}$, $P\{\eta_2=y_{2j}\}=p_{2j}$ respectivly then

$$\mu_{\underset{\smile}{A}}(x,y)=\Big(\sum_{x_{1i}\le x}p_{1i}\Big)\Big(\sum_{x_{2i}\ge x}p_{2i}\Big)\Big(\sum_{y_{1j}\le y}p_{1j}\Big)\Big(\sum_{y_{2j}\ge y}p_{2j}\Big)$$

References

1 Jiang Pei rong , Distribution of Fuzzy Subset Projected from Random Intervals,BUSEFAL 15/1983.

2 Wang Pei zhuang, Sanchez,E., Treating a fuzzy subset as projectable random subset, 'Fuzzy Information and Decision Processes',Edited by M.M.Gupta,E.Sanchez (1982) 213-219.