THE FUZZY LINEAR NORMED SPACE $V_{m \times n}$

Wang Hongxu

Dept. of Basis. Liaoyang Institute of Chemical Fiber Industry. Liaoyang, Liaoning, China

ABSTRACT

In this paper a fuzzy inner product space $V_{m \times n}$ and a fuzzy linear normed space are definited and their properties are discussed. A definition of fuzzy cubic matrix is given. Applying it, we prove: there exists exactly one standard orthogonal basis in each fuzzy linear normed space $V_{m \times n}$.

Key Words: Fuzzy inner product space of $V_{m \times n}$. Fuzzy Linear normed space of $V_{m \times n}$. Orthogonal basis of $V_{m \times n}$. Standard orthogonal basis and simple standard orthogonal basis of $V_{m \times n}$. Fuzzy cubi matrix.

I. FUZZY INNER PRODUCT SPACE $V_{m \times n}$

For definition and signs used in this paper, see [1], and [5].

Definition 1.1 Let $V_{m \times n}$ be a fuzzy semilinear space composed of fuzzy matrices. If for an arbitrary pair of elements A, B of $V_{m \times n}$, there is a number (A,B) of [0,1] such that satisfies

1) (A,B) = (B,A)

2) $(kA,B)=k(A,B)$, $k\in[0,1]$

3) $(A+B,C)=(A,C)+(B,C)$, $C\in V_{m\times n}$

4) $(A,A)=0$ iff $A=\theta$

then $V_{m\times n}$ is calleda a fuzzy inner product space $V_{m\times n}$.(A,B) is called the fuzzy inner product of A and B.

Proposition 1.1 In fuzzy inner product space $V_{m\times n}$, the following formulas hold:

1) $(kA,hB)=kh(A,B)$, $k,h\in[0,1]$

2) $(A,B+C)=(A,B)+(A,C)$. $C\in V_{m\times n}$

3) $(A,kB+hC)=k(A,B)+h(A,C)$, $k,h\in[0,1]$

4) If A or B is θ, then $(A,B)=0$

5) $(\sum_{i=1}^{m} k_i A_i , \sum_{j=1}^{n} h_j B_j)=\sum_{i=1}^{m}\sum_{j=1}^{n} k_i h_j (A_i ,B_j)$, $k_i ,h_j \in[0,1]$.

Theorem 1.1 In fuzzy semilinear space $V_{m\times n}$ for any fuzzy matrices $A=(a_{ij})_{m\times n}$ and $B=(b_{ij})_{m\times n}$, we define $(A,B)=\bigvee_{i,j}(a_{ij}\wedge b_{ij})$ as the inner product of A and B, then $V_{m\times n}$ is a inner product space.

The meanings of the following inner products are the same as one of the inner product of theorem 1.1.

II. THE DEFINITION OF A FUZZY LINEAR NORMED SPACE $V_{m\times n}$

Definition 2.1 Let $V_{m\times n}$ be a fuzzy semilinear space, if for every element A of $V_{m\times n}$, there is a number $\|A\|$ corresponding to it that satisfies the following condition:

1) $1\geqslant\|A\|\geqslant 0$, $\|A\|=0$ iff $A=\theta$

2) $\|kA\|=k\|A\|$, $k\in[0,1]$

3) $\|A+B\|\leq\|A\|+\|B\|$

then $V_{m\times n}$ is called fuzzy linear normed space, and $\|A\|$ is called the norm of A .

Theorem 2.1 In $V_{m\times n}$, let $\|A\|=(A,A)$, then $V_{m\times n}$ is a fuzzy linear normed space.

The following fuzzy linear normed space, and its norm are all same as in theorem 2.1.

Theorem 2.2 $\forall A,B\in V_{m\times n}$, Cauchy-Буняковский inequality stands:

$$(A,B)\leq\|A\|\cdot\|B\| \quad .$$

Proposition 2.1 In a fuzzy linear normed space $V_{m\times n}$ the followings hold:

1) $\|A+B\|\leq\|A\|+\|B\|$

2) $\|A\|^k=\|A\|$, $k\in N$

3) $\|A+B\|^2\leq\|A\|^2+\|B\|^2$

4) $\|A+B+\ldots\ldots+C\|^k\leq\|A\|^k+\|B\|^k+\ldots\ldots+\|C\|^k$, $k\in N$

III. A ORTHOGONAL BASIS AND A ORTHOGONAL SUBSPACE OF $V_{m\times n}$

Definition 3.1 Let A,B $V_{m\times n}$, if $(A,B)=0$, then A and B are called orthogonal.

A matrix group of consistiny of non-zero matrices is called an orthogonal group if every two matrices of it are orthogonal.

Proposition 3.1 In $V_{m\times n}$, the following hold:

1) $\|A+B\|=\|A\|+\|B\|$ iff $(A,B)=0$

2) $\|A+B\|^2=\|A\|^2+\|B\|^2$ iff $(A,B)=0$

3) $\|A+B+\ldots\ldots+C\|^k=\|A\|^k+\|B\|^k+\ldots\ldots+\|C\|^k$, $k\in N$ iff A,B,

...... , C are orthogonal each other.

Proposition 3.2 Let A be an element of $V_{m\times n}$ $S=\{B\mid(B,A)=0, B\in V_{m\times n}\}$ is said to be the maximum orthogonal subspace over A.

Definition 3.2 Let W_1 and W_2 be two subspaces of $V_{m\times n}$ if $(A_1, A_2)=0$ for arbitrary $A_1\in W_1$, $A_2\in W_2$ then the subspaces W_1 and W_2 are called orthogonal.

Proposition 3.3 Let S be a subspace of $V_{m\times n}$, then the set of all matrices to each of which S is orthogonal is a subspace, which is called the orthogonal subspace of S.

Definition 3.3 In $V_{m\times n}$, a matrix is identity norm matrix if its norm is 1. If matrices of a identity norm matrix group of $V_{m\times n}$ are orthogonal mutually. Then it is called a identity normed orthogonal group. For the sake of convenience, a identity norm matrix is also called orthogonal.

Definition 3.4 A matrix group $\{A_1,\ldots,A_t\}$ of $V_{m\times n}$ is independent if and only if there is no $A_i\in\{A_1,\ldots,A_t\}$ such that is represented as a linear combination of elements of $\{A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_t\}$. If there is a $A_i\in\{A_1, \ldots, A_t\}$ such that it is a linear combination of elements of $\{A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_t\}$, it is said to be dependent.

Definition 3.5 Let $A_1, \ldots, A_t\in S\subseteq V_{m\times n}$, if $A_1, \ldots, A_t$ are independent and $\forall A\in S$ can be denoted by a linear combination of $A_1, \ldots, A_t$, then $A_1, \ldots, A_t$ is called the maximal independent group of S.

Proposition 3.4 Let $A_1, \ldots, A_t\in V_{m\times n}$. If $A_1, \ldots, A_t$ are independent , then $A_1, \ldots, A_t$ is a greatest independent group of $L(A_1, \ldots, A_t)$.

Theorem 3.1 (1) A non-zero orthogonal matrix spanning group of $V_{m\times n}$ is a maximal independent group of $V_{m\times n}$.

(2) An identity normed orthogonal matrix spanning group of $V_{m\times n}$ is a maximal independent group of $V_{m\times n}$.

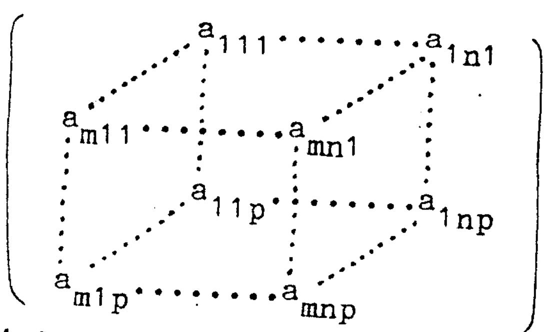
(3) The numbers of matrices of orthogonal matrix spanning groups of $V_{m\times n}$ are equal.The number of matrices of identity normed orthogonal matrix spanning group of $V_{m\times n}$ are equal.

Theorem 3.2 Let a set $\{A_1, \dots, A_t\}$ which $A_i \in V_{m\times n}, (i=1,\dots, t)$ be an orthogonal matrix group of $V_{m\times n}$, then $S=L(A_1,\dots A_t)$ is a subspace of $V_{m\times n}$ and is called a orthogonal subspace of $V_{m\times n}$. $\{A_1, \dots, A_t\}$ is called a orthogonal basis of S.

Definition 3.6 For two bases $\{A_1, \dots, A_t\}$ and $\{B_1, \dots, B_t\}$ of S that are two subsets of S,if $L(A_1, \dots, A_t)=L(B_1, \dots, B_t)$, then the two bases are called identical.

IV. FUZZY CUBIC MATRIX

Definition 4.1 An arrangement of m×n×p elements of [0,1] of m rows, n columns and p storeys (see (1)) is called a m×n×p cubic matrix of [0,1], denoted by $A=(a_{ijk})_{m\times n\times p}$.



(1)

Definition 4.2 Let $A=(a_{ijk})_{m\times n\times p}$. The elements at the intersections of the rows $i_1, \dots, i_r$, the columns $j_1, \dots, j_s$ and the storeys $k_1, \dots, k_t$ in A forms a r×s×t cubic matrix , which is called a subcubic matrix denoted by $L_A\left\{\begin{matrix} i_1, & \cdots & , i_r \\ j_1, & \cdots & , j_s \\ k_1, & \cdots & , k_t \end{matrix}\right\}$.

$L_A\left\{\begin{matrix} 1, & \cdots & , m \\ 1, & \cdots & , n \\ k_1, & \cdots & , k_t \end{matrix}\right\}$ is called the subcubic matrix of storeys k_1, $\cdots$, k_t. Specially, storey k of A, that is $L_A\left\{\begin{matrix} 1, & \cdots & , m \\ 1, & \cdots & , n \\ & k & \end{matrix}\right\}$,

is denoted by A_k. We denote A by $A=\left\{\begin{matrix} A_1 \\ \vdots \\ A_p \end{matrix}\right\}$ where $A_1, \cdots, A_p$

denote respectirely storey 1, ... , p of A.

$L_A\left\{\begin{matrix} & i & \\ & j & \\ 1, & \cdots & , p \end{matrix}\right\}$ is symboled of i, j r-c.

$L_A\left\{\begin{matrix} & i & \\ 1, & \cdots & , n \\ & k & \end{matrix}\right\}$ is symboled of i, k r-s.

$L_A\left\{\begin{matrix} 1, & \cdots, & m \\ & j & \\ & k & \end{matrix}\right\}$ is symboled j, k c-s.

$L_A\left\{\begin{matrix} i \\ j \\ k \end{matrix}\right\}$ is an element of row i, column j storey k of A and is symboled of i, j, k r-c-s.

Itself of a cubic matrix A can be denoted by $L_A\left\{\begin{matrix} 1, \ldots, m \\ 1, \ldots, n \\ 1, \ldots, p \end{matrix}\right\}$

To prove theorem 5.1 we shall need to use contents of this section.

V. THE STANDARD ORTHOGONAL BASIS OF $V_{m \times n}$

Definition 5.1 If $A_1, \cdots, A_t$ is an identity normed matrix group of $V_{m\times n}$ then $S=L(A_1, \cdots, A_t)$ is called a standard orthogonal subspace of $V_{m\times n}$ and $A_1, \cdots, A_t$ is called a standard orthogonal basis of S .

Proposition 5.1 If $A_1, \ldots, A_t \in V_{m\times n}$ is standard orthogonal basis of $L(A_1,\ldots,A_t)$ then $A_2, \ldots, A_t$ is standard orthogonal basis of $L(A_2, \ldots, A_t)$.

Theorem 5.1 There exists only a standard orthogonal basis in fuzzy linear normed space $V_{m\times n}$.

Theorem 5.2 The subspace spaned by some matrices of standard orthogonal basis of $V_{m\times n}$ forms a standard orthogonal subspace of $V_{m\times n}$.

VI. THE SIMPLE STANDARD ORTHOGONAL BASIS OF $V_{m\times n}$

Definition 6.1 let W be a finite spanning subspace . For $A\in W$, if there is non-ordered relation " $\leq$ " $B, C\in W$, so that $A = B + C$, then A is called a compound matrix of W, otherwise A is called a simple matrix.

Proposition 6.1 A is a simple matrix of $V_{m\times n}$, if and only if A is like following matrix

$$A = \begin{pmatrix} 0 \cdots 0 \cdots 0 \\ \vdots \quad \vdots \quad \vdots \\ 0 \cdots a_{ij} \cdots 0 \\ \vdots \quad \vdots \quad \vdots \\ 0 \cdots 0 \cdots 0 \end{pmatrix} i \qquad a_{ij}\in[0,1]$$
$$\quad j$$

where A is a $m\times n$ matrix that i,j r-c element is a_{ij} , other elements are all zero.

Definition 6.2 If a finite spanning subspace S of $V_{m\times n}$ possesses a standard orthogonal basis and every matrix of the basis is also a simple matrices of $V_{m\times n}$, then the basis is called a simple standard orthogonal basis of S and S is referred to as a simple standard orthogonal subspace of $V_{m\times n}$.

Proposition 6.2 A finite spanning subspace of $V_{m\times n}$. If there is a simple standard basis , then this basis possesses

1) In this basis every matrix is simple matrix of $V_{m\times n}$.
2) In this basis every matrix is identity normed matrix.
3) The matrices of this basis are orthogonal mutually.

Proposition 6.3 A is a indentity normed matrix if and only if A is just like

$$\begin{pmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 1 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & \underset{j}{0} & \cdots & 0 \end{pmatrix} i, \begin{pmatrix} i=1,\ldots,m \\ j=1,\ldots,n \end{pmatrix} \quad (2)$$

where i,j r-c is 1, other elements are 0.

Theorem 6.1 In $V_{m\times n}$ there exist only a simple standard orthogonal basis.

Theorem 6.2 Let S_1 and S_2 are the orthogonal subspaces of $V_{m\times n}$. If S_1 and S_2 are orthogonal, then $S_1 \cap S_2 = \{\theta\}$.

Proposition 6.4 Let S be a simple standard orthogonal subspace of $V_{m\times n}$, then the basis of S is spaned by some matrix like (2).

Proposition 6.5 Let S be a simple standard orthogonal subspace of $V_{m\times n}$, then $T = V_{m\times n} - S + \{\theta\}$ is also a simple standard orthogonal subspace and $S \cap T = \{\theta\}$.

Definition 6.3 Let S_1 and S_2 are two simple standard orthogonal subspaces of $V_{m\times n}$. If $S=S_1+S_2$ and $S_1 \cap S_2=\{\theta\}$, then S is called direct sum S_1 and S_2, and is denoted by $S = S_1 \dot{+} S_2$.

Theorem 6.3 Let S_1 and S_2 be two simple standard orthogonal subspaces of $V_{m\times n}$, $S = S_1+S_2$. Then $S=S_1 \dot{+} S_2 \Longleftrightarrow S_1$ and S_2 are orthogonal.

Theorem 6.4 Let S is a simple standard orthogonal subspace of $V_{m\times n}$ and let $T = V_{m\times n} - S + \{\theta\}$, then $V_{m\times n} = S \dot{+} T$, and we call T a direct complementary space of S. We denote by $S^{\perp}= T$, that is $S^{\perp}= V_{m\times n} - S + \{\theta\}$.

Proposition 6.6 The sum space of simple standard orthogonal subspaces of $V_{m\times n}$ is also a simple standard orthogonal subspace of $V_{m\times n}$.

VII. n-ary LINEAR NORMED SPACE V_n

Let $V_n=\{(\alpha_1, \ldots, \alpha_n); \alpha_i \in [0,1], i=1,\ldots,n\}$ then obviously V_n is special case of $V_{m\times n}$. All condusions of $V_{m\times n}$ are tenable for V_n thus V_n is aiso a fuzzy linear normed space its inner product is definited according inner product of $V_{m\times n}$.

Theorem 5.1' There exist only a standard orthogonal basis in fuzzy linear normed space V_n.

Similarly we can definite V^n and carry on some discusses. The conerete content is not given detaile.

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