FUZZY RELIABILITY

Li Tingjie Gao He
(Beijing Institute of Aeronautics and Astronautics)

ABSTRACT: In this paper, the problem of reliability with a fuzziness is discussed by means of fuzzy mathematics. This paper describes the basic concepts (definition of fuzzy reliability, fuzzy subsets of reliability, fuzzy failure) and the major indexes of fuzzy reliability. The purpose of this paper is to provide the new concepts and method which can be used in calculation of the fuzzy reliability for different systems.

KEY WORDS: fuzzy reliability, fuzzy failure, fuzzy performance subsets, fuzzy maintainability, indexes of fuzzy reliability.

I. BASIC CONCEPTS OF FUZZY RELIABILITY

1. Definition of general fuzzy reliability

The definition of general reliability is expressed as follows: "Fuzzy reliability is the ability of a device performing its purpose in varying degrees of success for the period of time intended under the operating conditions encountered".

It should be observed that the definition stresses five elements, namely, ability, device, performance in varying degrees of success, time, and operating conditions. This performance is called fuzzy performance.

From the point of view of fuzzy probability theory, the problems of fuzzy reliability are classified as three modes, namely, fuzzy event-accurate probability (i.e. FA mode), clear event-fuzzy probability (i.e. CF mode), and fuzzy event-fuzzy probability (i.e. FF mode). The definition above mentioned completely contains these three modes.

In view of the definition of fuzzy reliability, at first, there is no mistaking what the fuzzy reliability is aimed at; next, the fuzzy reliability is development of the general reliability.

2. Fuzzy subsets of reliability

The fuzzy performances may be expressed by fuzzy subsets by means of constructing rule of fuzzy language in fuzzy mathematics. At first, the fuzzy performances are expressed by two fuzzy subsets, namely, "large performance" and "small performance". Next, we make use of language operators in order to differentiate varying degrees of "large performance" and "small performance". The language operators may be classified as mood operator, fuzzy operator and judgement operator.

By means of mood operators — "more", "very" and "extremely", fuzzy performance subsets are classified as eight types, namely, "extremely large performance" (i.e. "extremely reliable device"), "very large performance" (i.e. "very reliable device"), "Large performance" (i.e. "reliable device"), "more large performance" (i.e. "more reliable device"), "more small performance" (i.e. "more unreliable device"), "small performance" (i.e. "unreliable device"), "very small performance" (i.e. "very unreliable device") and "extremely small performance" (i.e. "extremely unreliable device").

In the same way, by means of the fuzzy operator ("probably") and the judgement operator ("be tend to"), the fuzzy performance subsets may be classified as sixteen types.

If consider different compositions of the mood operator, the fuzzy operator and the judgement operator, then the quantity of fuzzy performance subsets still increase.

Except for the language operators are applied, we also make use of the concept of complement set and the operation rules of the language value in order to further differ the fuzzy performance subsets.

In short, by means of the methods above mentioned, the various fuzzy performances may be expressed by the various fuzzy subsets. These fuzzy performance subsets may fully satisfy the requirement for researches on fuzzy reliability.

3. Fuzzy failure

The definition of fuzzy failure is expressed as follows: "Fuzzy failure is a phenomenon, that appears when the true values of performance indexes of a device break away from the limits permitted by the performance specification in varying degrees.

In fuzzy reliability theory, must differentiate the degrees of the large failures and the small failures, the many and little indexes in excess of the limits permitted by performance specification of a device, the easy maintaining works and the difficult maintaining works.

Evidently, the fuzzy failure subsets correspond one by one to the fuzzy performance subsets.

II. MAJOR INDEXES OF FUZZY RELIABILITY

All indexes of the fuzzy reliability are classified as three modes, namely, FA mode, CF mode and FF mode. Now we discuss the indexes of FA mode. Indexes of FA mode may satisfy various operations of classical mathematics and general probability theory, due to they all are numerical values.

1. Fuzzy reliability

The definition of fuzzy reliability is expressed as follows: "Fuzzy reliability is the probability of a device performing its purpose in varying degrees of success for the period of time intended under operating conditions encountered", and that is denoted by the sign $\underset{\sim}{R}$. Evidently $\underset{\sim}{R}$ is a function of $\underset{\sim}{A}_i$. The sign $\underset{\sim}{A}_i$ denotes discussing one of fuzzy performance subsets. Certainly, this definition completely contains the three modes above mentioned.

Now we shall derive the expression of FA mode for the fuzzy reliability. If the sign A denotes that a device performs its purpose adequately, then in terms of the definition of fuzzy conditional probability, we have

$$P(A \wedge \underset{\sim}{A}_i) = P(\underset{\sim}{A}_i | A)\ P(A) \tag{1}$$

According to the definition of general reliability (R) and the definition of fuzzy reliability ($\underset{\sim}{R}$), we know

$$P(A) = R \qquad P(A \wedge \underset{\sim}{A}_i) = \underset{\sim}{R} \tag{2}$$

where the sign $\wedge$ denotes algebraic product.

Substituting Eq. (2) into Eq. (1), we obtain

$$\underset{\sim}{R} = P(\underset{\sim}{A}_i | A)\ R \tag{3}$$

Suppose that $\mu_{\underset{\sim}{A}_i}(R)$ is the degree of membership of R in $\underset{\sim}{A}_i$, and substitute $\mu_{\underset{\sim}{A}_i}(R)$ for $P(\underset{\sim}{A}_i | A)$, then

$$\underset{\sim}{R} = \mu_{\underset{\sim}{A}_i}(R) R \tag{4}$$

If the sign $\underset{\sim}{A}_i$ denotes one of fuzzy performance subsets from "more large performance" to "extremely large performance", then $\underset{\sim}{R}$ and R have a same tendency when they vary with time, i.e. $\underset{\sim}{R}$ is a monotonic decreasing function (as shown in Fig. 1).

If the sign $\underset{\sim}{A}_i$ denotes one of fuzzy performance subsets from "extremely small performance" to "more small performance", $\underset{\sim}{R}$ have a tendency as shown in Fig. 2 when it vary with time.

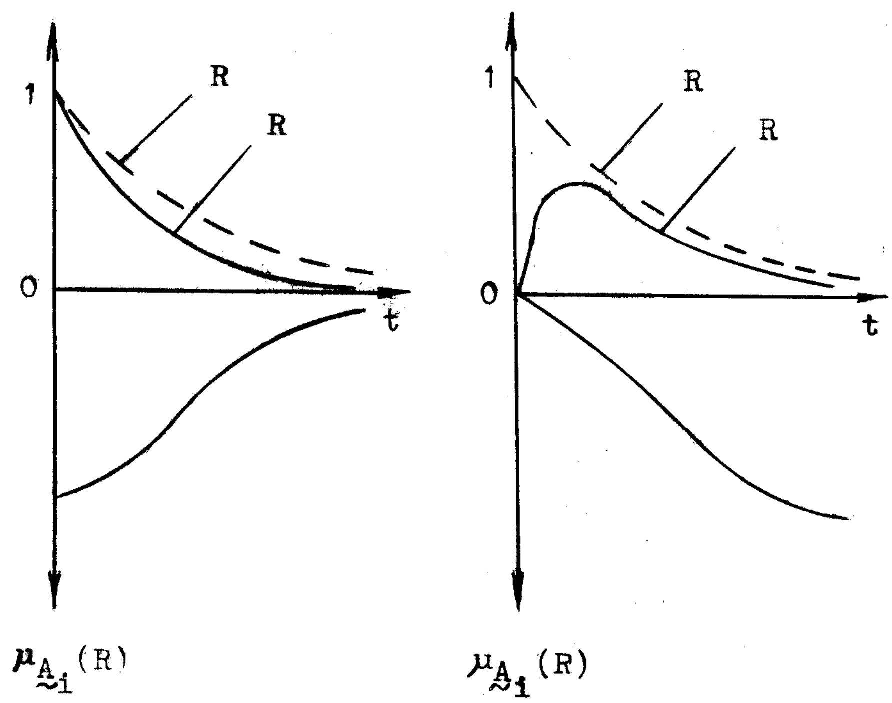

Fig. 1 Fig. 2

The fuzzy reliability ($\underset{\sim}{R}$) also may be expressed in terms of a probability density function [p(t)] for general reliability theory. According to general reliability theory, we have

$$R = \int_t^{\infty} p(t)dt$$

Substituting it into Eq. (4), we obtain

$$\underset{\sim}{R} = \mu_{\underset{\sim}{A}_i}(R) \int_t^{\infty} p(t)dt \qquad (5)$$

2. Fuzzy failure rate

The fuzzy failure rate is expressed in terms of a probability happened to one fuzzy failure per unit of time, and that is denoted by the sign $\underset{\sim}{\lambda}$.

Now we shall derive the expression of FA mode for the fuzzy failure rate. Suppose that the sign $\underset{\sim}{B}_1$ denotes that a device not heppen to one fuzzy failure over interval [o, t], and the sign $\underset{\sim}{B}_2$ denotes that a device happen to one fuzzy failure over interval(t, t + dt], then in terms of the definition of fuzzy conditional probability we have

$$P(\underset{\sim}{B}_2 | \underset{\sim}{B}_1) = \frac{P(\underset{\sim}{B}_1 \wedge \underset{\sim}{B}_2)}{P(\underset{\sim}{B}_1)} \qquad (6)$$

If the sign T denotes time not happened to one fuzzy failure, then we have

$$\underset{\sim}{B}_1 \wedge \underset{\sim}{B}_2 = \text{"} t < T \leqslant t + dt \text{"}$$

$$\underset{\sim}{B}_1 = \text{"} T > t \text{"}$$

Substituting they into Eq. (6), we obtain

$$P(\underset{\sim}{B}_2 | \underset{\sim}{B}_1) = \frac{P(t < T \leqslant t + dt)}{P(T > t)}$$

$$= \frac{P(T \leqslant t + dt) - P(T \leqslant t)}{P(T > t)}$$

$$= \frac{\underset{\sim}{F}(t + dt) - \underset{\sim}{F}(t)}{P(T > t)} = \frac{d\underset{\sim}{F}(t)}{P(T > t)}$$

where $\underset{\sim}{F}(t)$ is the probability distribution function of T, and $P(T > t) = \underset{\sim}{R}$.

According to the definition of probability distribution function, we have

$$\underset{\sim}{F}(t) \triangleq P(T \leq t) = 1 - P(T > t) = 1 - \underset{\sim}{R}$$

Thus

$$P(\underset{\sim}{B}_2 | \underset{\sim}{B}_1) = \frac{d(1 - \underset{\sim}{R})}{\underset{\sim}{R}} = - \frac{d\underset{\sim}{R}}{\underset{\sim}{R}dt} dt$$

Therefore

$$\underset{\sim}{\lambda} = - \frac{d\underset{\sim}{R}}{\underset{\sim}{R}\, dt} \tag{7}$$

Substituting Eq. (4) into Eq. (7), we obtain

$$\underset{\sim}{\lambda} = \frac{d[\mu_{\underset{\sim}{A}_i}(R)\, R]}{\mu_{\underset{\sim}{A}_i}(R)\, R\, dt} = \lambda - \frac{d\mu_{\underset{\sim}{A}_i}(R)}{\mu_{\underset{\sim}{A}_i}(R)\, dt} \tag{8}$$

where λ is general failure rate.

Now let us consider the problem expressed $\underset{\sim}{R}$ by $\underset{\sim}{\lambda}$. In terms of Eq. (7), we have

$$\underset{\sim}{\lambda} dt = \frac{1}{\underset{\sim}{R}} d\underset{\sim}{R}$$

Integrating this equation over interval $[o, t]$, we obtain

$$\int_o^t \underset{\sim}{\lambda} dt = - \ln \underset{\sim}{R}$$

Therefore

$$\underset{\sim}{R} = e^{-\int_0^t \underset{\sim}{\lambda} dt} \tag{9}$$

if $\underset{\sim}{\lambda}$ is equal to constant, then

$$\underset{\sim}{R} = e^{-\underset{\sim}{\lambda} t} \tag{10}$$

3. Fuzzy mean life

The fuzzy mean life is expressed in terms of a expectation of time not happened to one fuzzy failure, and that is denoted by the sign $\underset{\sim}{MTTF}$.

According to definition of a mathematical expectation, we have

$$\begin{aligned}\underset{\sim}{MTTF} &= \int_0^\infty t\left(\frac{d\underset{\sim}{F}(t)}{dt}\right)dt \\ &= \int_0^\infty t\,\frac{d(1-\underset{\sim}{R})}{dt}\,dt \\ &= -\int_0^\infty t\,d\underset{\sim}{R}\end{aligned}$$

By means of a integration by parts and L'Hospital's rule, we obtain in conclusion

$$\underset{\sim}{MTTF} = \int_0^\infty \underset{\sim}{R}\,dt \tag{11}$$

Now let us consider the problem expressed $\underset{\sim}{MTTF}$ by $\underset{\sim}{\lambda}$. Substituting Eq. (9) into Eq. (11), we obtain

$$\underset{\sim}{MTTF} = \int_0^\infty e^{-\int_0^t \underset{\sim}{\lambda}\,dt}\,dt \tag{12}$$

If $\underset{\sim}{\lambda}$ is equal to constant, then

$$\underset{\sim}{MTTF} = \int_0^\infty e^{-\underset{\sim}{\lambda} t}\,dt = \frac{1}{\underset{\sim}{\lambda}} \tag{13}$$

Thus

$$\underset{\sim}{R} = e^{-t/\underset{\sim}{MTTF}} \tag{14}$$

4. Mean time between one fuzzy failure ($\underset{\sim}{MTBF}$)

$\underset{\sim}{MTBF}$ is the fuzzy mean life for a repairable device.

5. General fuzzy maintainability

We define the general fuzzy maintainability as the ability of a device recovering its purpose in varying degrees of success for the period of time intended under the repairing conditions encountered.

Major indexes of general fuzzy maintainability are the fuzzy maintainability ($\underset{\sim}{M}$), the fuzzy repair rate ($\underset{\sim}{\mu}$) and the fuzzy mean repair time ($\underset{\sim}{MTTR}$).

① Fuzzy maintainability ($\underset{\sim}{M}$)

We define the fuzzy maintainability as the probability of a device recovering its fuzzy performance for the period of time intended under the repairing conditions encountered.

In the same way as the fuzzy reliability we have

$$\underset{\sim}{M} = \mu_{\underset{\sim}{B}_i}(M)\, M \qquad (15)$$

where M is general maintainability of a device; $\underset{\sim}{B}_i$ is one fuzzy subsets of a device recovering its one fuzzy performance, i = 1, 2, ..., n.

② Fuzzy repair rate ($\underset{\sim}{\mu}$)

The fuzzy repair rate is expressed in terms of the probability of a device recovering its one fuzzy performance per unit of time.

In the same way as the fuzzy failure rate we obtain

$$\underset{\sim}{\mu} = \frac{d\underset{\sim}{M}}{d\tau} \cdot \frac{1}{1 - \underset{\sim}{M}} \qquad (16)$$

where is the repair time.

③ Fuzzy mean repair time ($\underset{\sim}{MTTR}$)

MTTR is mean repair time of a device from happening to one fuzzy failure to recovering one fuzzy performance.

In view of the definition of general fuzzy maintainability: At first, there is no mistaking what the fuzzy maintainability is aimed at. The values of the fuzzy maintainability indexes vary with $\underset{\sim}{B}_i$. Next, the fuzzy maintainability is development of the general maintainability.

REFERENCES

[1] Wang Peizhuang, Theory of Fuzzy Sets and Its Applications, Shanghai Science and Technology Press (1983) (in Chinese).

[2] Lu Shibo, Fuzzy Mathematics, Science Press (1983) (in Chinese).

[3] Li Tingjie and Gao He, Reliability Design, Beijing Institute of Aeronautics and Astronautics Press (1982) (in Chinese).