

Possibility theory and fuzzy logic in inference systems - A perspective view -

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Since the late seventies, there have been many attempts at using fuzzy sets (Zadeh [46]) and their further developments, especially possibility theory (Zadeh [48], Dubois and Prade [15]) in knowledge engineering and especially for the management of uncertainty in expert systems. The amount of existing works can be assessed by looking at recent edited volumes such as Gupta et al. [23], Prade and Negoita [35], Sanchez and Zadeh [38]. In fact, applications of fuzzy sets to diagnosis or rule-based control started very early in the history of fuzzy sets, since Sanchez [37] presented the fuzzy relational approach to medical diagnosis problems in 1974 and Mamdani [32] started investigations on fuzzy controllers also around 1974, following the approach outlined by Zadeh [47]. The development of industrial applications of fuzzy control techniques [41] and the design of corresponding hardwares (e.g. [42]) have been very impressive in the recent years, especially in Japan. In the mid-seventies the expert system technology was still in infancy, and the importance of automated reasoning as a research field was not yet completely recognized. Since then the gap between expert system approaches and fuzzy set-based approaches has been reduced, and fuzzy set and possibility theory hold a significant position as one of the main trends for the management of uncertainty in knowledge bases.

However the development of fuzzy set-based uncertainty management techniques has not been homogeneous, partly because of various existing intuitions and views of fuzzy sets. The gap between fuzzy controllers where vague rules are modelled by fuzzy sets and fuzzy relations, and pattern-directed inference systems, where symbolic rules are equipped with certainty factors (as in MYCIN (Buchanan and Shortliffe [9])), is still there. The latter methodology has had a strong influence on further research, since it has been adopted in fuzzy set-oriented works (RUM [7] ARIES [2], PROTIS [40], FESS [24]) although the idea of modelling vagueness of rules by

means of fuzzy sets has been retained (as in MILORD [22], FLOPS [10], PROTIS [40]). Triggering fuzzy rules requires an extension of the pattern matching procedure, called fuzzy pattern matching [12] that was already partially used in Mamdani's works.

Our team has a rather long experience of these types of methods since several inference engines based on possibility theory have been developed in the last five years. SPII [29] is a fuzzy inference engine processing fuzzy and uncertain rules in the same setting, and based on Zadeh's approximate reasoning paradigm [49] that uses a generalized version of modus ponens. It has been applied in petroleum geology. DIABETO [11] is a similar tool that allows for interpolation techniques, and has been used in diabetology. TAIGER [21] is an inference engine handling possibility degrees attached to rules and facts, and that are computed by fuzzy pattern matching. SPII and TAIGER can perform computations on ill-known quantities and discuss their ranking in terms of degrees of possibility and necessity. OPAL [6] is a production scheduling system using fuzzy priority rules for sequencing operations on machines.

It is interesting to emphasize that several of these systems have been commercialized (FRIL [3, 4], REVEAL [27], FLOPS [10], TAIGER [21]).

However there is a strong need to classify and unify these various works which rely on the fuzzy set idea. Moreover, there is also a need to locate these models with respect to other numerical formalisms such as Bayesian networks or belief functions, as well as to incorporate into the possibilistic framework ideas that were developed in other non-standard logics, but also in heuristic, non-numerical techniques.

Fuzzy set-based automated reasoning techniques have been used to deal with three kinds of systems : deduction, diagnosis (or classification), and decision. Deduction systems are concerned with extensions of logical inference which account for imprecision or incompleteness of information. The aim of deduction is simply to make implicit knowledge explicit in a knowledge base. Diagnosis systems deal with causal reasoning. Knowledge bases for such problems take the form of directed causal networks, where rules express causality links. Reasoning is often abductive, i.e. intends to derive the causes from observed facts. In decision systems, rules express more or less strong recommendations aiming at solving some problem, very often a planning or control problem ; the "then" part of decision rules generally contains an action or a guideline, rather than the statement of a new fact. In the following, we mainly discuss deductive or abductive reasoning rather than decision-making systems.

There are six schools of thought in the existing fuzzy literature about automated

reasoning : i) certainty-factors-like approaches [2, 5, 10, 40], ii) resolution techniques extended to the multiple-valued logic underlying fuzzy sets [30, 26, 34, 39], iii) fuzzy logic based on an approximate reasoning method on possibility distributions [3, 8, 11, 22, 29, 31, 32, 33, 44, 45, 49], iv) possibilistic logic based on possibility and necessity measures [14, 16, 18, 21], v) reasoning on fuzzy quantifiers [50, 19] and vi) fuzzy relation-based abductive reasoning [37, 1, 17]. Certainty-factor-like approaches often rely on multiple-valued logic connectives, and are usually rather ad hoc due to an extensive assumption of local compositionality. Intermediary degrees of truth make sense only for fuzzy propositions involving vague predicates or quantifiers and whose truth is evaluated with respect to a state of complete knowledge ; see [18] for a discussion. These degrees of truth can be manipulated in a truth- functional way using fuzzy set operators and Lee [30] has proposed a resolution principle for this logic which is used in various fuzzy extensions of PROLOG [25, 26, 34, 39, 43]. By contrast, fuzzy logic based on possibility distributions [49], is a rigorous methodology which deals with the vagueness of the available knowledge. Each item of information is translated into a possibility distribution on suitable semantic scales. The reasoning methodology consists in combining the elementary possibility distributions by performing a generalized join, and projecting the overall distribution over scales of interest. The advantage of this approach is to provide natural ways of interfacing symbolic knowledge with numerical data. Possibilistic logic [16] is an extension of classical logic where propositions are weighted by degrees of possibility or necessity (certainty) that obey the laws of possibility theory. A semantic of weighted propositions can be established consistently with fuzzy logic. Possibilistic logic is a logic of partial ignorance where the incompleteness of information is expressed in terms of possibility distributions (as opposed to probability distributions, in the case of probabilistic logic). Its advantages lie in its ability to represent shades of ignorance in a more natural way than Bayesian probabilities, and in very simple calculation rules as opposed to more general upper and lower probability approaches (of which it can be viewed as an extreme particular case). The fuzzy quantifier approach [50] is a generalized logic of numerical quantifiers whose semantics are in terms of relative set cardinality or conditional probabilities. Fuzzy quantifiers translate linguistic proportions which often pervade natural language formulations of commonsense rules. A fuzzy quantifier is modelled as a possibility distribution restricting the values of an ill-known proportion. This approach suggested by Zadeh leads to combine probabilistic and possibilistic techniques in a natural way. It is a sound basis for a numerical view of default reasoning ; see also [20]. Lastly fuzzy diagnosis methods stand apart from the logical approaches. Fuzzy relations are used to express more or less strong causal links between disorders and symptoms. The derivation of disorders causing given symptoms is made by solving an equation in a non-standard algebra. These methods are in the spirit of recent works by Reggia et al. [36] although 10 years older. Besides, ill-known certainty factors or degrees of belief can be accommodated in terms of fuzzy numbers (e.g. [13]).

From an algorithmic point of view, these various approaches are not equally advanced. The most currently and easily implemented methods are the certainty-factor-like ones because they rely on local propagation and combination of uncertainty ; but their schemes suffer from various flaws, including paradoxical responses, and have been severely criticized. The other reasoning methodologies are either global, (i.e. no longer on a rule-by-rule basis), or, or if local, equivalent to global strategies. Global strategies consider a knowledge base as a whole to define the reasoning method. This way, dependencies between rules (which must be accounted for in uncertain environments) are dealt with. However using a global strategy is time-consuming on a computer. This fact, and the ad-hocery of certainty-factor-like approaches, make it important to derive local techniques which are valid in the sense that they would not lead to results contradicting those of the global strategies. In fuzzy logic with possibility distributions, deriving such local strategies at the semantic level for general knowledge bases seems to be very difficult although it has been achieved in special cases. In that respect, possibilistic logic looks much promising because refutation methods and the resolution principle can be extended in that framework, and proved complete. Moreover, a symbolic treatment of vague predicates looks possible, in accordance with fuzzy set semantics. As for the logic of fuzzy quantifiers, the algorithmic work is still to be done, and nobody has implemented it yet to our knowledge. Lastly fuzzy relational diagnosis methods are well-mastered from a mathematical and algorithmic point of views ; however, semantic aspects have been neglected so far, and multi-layered diagnosis problems have never been considered.

This analysis leads us to particularly emphasize three lines of research for the future : possibilistic logic, fuzzy quantifiers logic, and fuzzy relational diagnosis methods. These techniques could be related to other formalisms. First mathematical links exist between belief functions and possibility measures, the former being more general than the latter. It will be interesting to compare the paradigms that lead to each theory, and assess what part of the information is lost when a possibilistic model is used instead of one based on belief functions. Contrastedly the framework offered by a logic of fuzzy quantifiers is easily related to conditional probabilistic logic, of which it is an extension. Indeed a fuzzy proportion generalizes a pair of upper and lower probabilities. A comparative study of Bayesian diagnosis techniques and fuzzy relational ones would also be worthwhile.

The links between possibility and numerical quantifier logics on the one hand, and non-monotonic logics on the other hand are not well-known. However a recent comparative study [28] has laid bare some analogy of behaviors between numerical formalisms and symbolic ones, especially regarding non-monotonicity. This fact opens the way to further investigations that may lead to incorporate concepts and techniques of non-monotonic logics into numerical formalisms, or

conversely to enrich symbolic non-monotonic formalisms with shades of uncertainty.

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