FUZZY CONTINUOUS FUNCTION AND ITS PROPERTIES

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Abstract

In this paper, we'll introduce the concepts of fuzzy function and limit fuzzy function and fuzzy continuous function on the set of fuzzy numbers and give some of their elementary properties. We'll also discuss some important theorems of fuzzy continuous function on M-closed interval.

Keywords: Fuzzy number, Fuzzy distance, Fuzzy function, Fuzzy continuous function, Fuzzy uniformly continuous function.

1 Introduction

In [1, 2], we have introduced the fuzzy distance of two fuzzy numbers and the limit of the sequence of fuzzy numbers in in the fuzzy distance and given some elementary properties of the fuzzy distance and the limit of fuzzy numbers, we have discussed also some important theorems of fuzzy numbers. In this paper, we'll introduce the concept of fuzzy function on the set of fuzzy numbers, and discuss some important theorems of fuzzy continuous function on M-closed interval.

The paper is divided into three sections. In Section 2, we introduce two concepts of fuzzy function and limit of fuzzy function and discuss some elementary properties of fuzzy limit of fuzzy function.

In Seciion 3, we introduce the fuzzy continuous function and obtain some results similar to those of real-continuous function.

In the paper, all concepts and signs undefined may be found in [1, 2].

2 Limit of Fuzzy Function

Let $A \subset F^*(R)$, we introduce

<u>Definition 2.1.</u> Let f be a mapping from A to $F^*(R)$, and if it has the properties: whenever $\underline{x} \in A$, there exists uniquely $\underline{y} \in F^*(R)$ such that

$$f(\underline{x}) = \underline{y},$$

then f is called a fuzzy function defined on A, A is called domain of f. $B = \{ f(\underline{x});\ \underline{x} \in A \}$ is called range of f.

<u>Definition 2.2.</u> Let f, g be fuzzy functions on $A(A \subset F^*(R))$, we define

1) $(f + g)(\underline{x}) = f(\underline{x}) + g(\underline{x})$;

2) $(f - g)(\underline{x}) = f(\underline{x}) - g(\underline{x})$;

3) $(f * g)(\underline{x}) = f(\underline{x}) * g(\underline{x})$;

4) $(f / g)(\underline{x}) = f(\underline{x}) / g(\underline{x})$, with $g(\underline{x})$ is positive or negative fuzzy number[3];

5) $(a * f)(\underline{x}) = a * f(\underline{x}),\ a \in R$;

6) $(f \vee g)(\underline{x}) = f(\underline{x}) \vee g(\underline{x})$;

7) $(f \wedge g)(\underline{x}) = f(\underline{x}) \wedge g(\underline{x})$.

It is easy to see that if f, g are fuzzy functions on A, then $f + g$, $f - g$, $f * g$, f / g, $f \vee g$, $f \wedge g$ are fuzzy function on A.

Definition 2.3. Let $X \subset F^*(R)$, $\underline{a} \in F^*(R)$, for arbitrary given $\varepsilon > 0$, there exist at most finitely many fuzzy numbers $\underline{x} \in X$ don't satisfy

$$\rho(\underline{x}, \underline{a}) < \varepsilon,$$

then X is said to converge to $\underline{a}$, write $\underline{x} \longrightarrow \underline{a}$.

Obviously, if $X = \{\underline{x}_n\} \subset F^*(R)$, $N = \max\{n;\ \underline{x}$ don't satisfy $\rho(\underline{x}_n, \underline{a}) < \varepsilon\}$, then

$$\rho(\underline{x}_n, \underline{a}) < \varepsilon,$$

as $n \geq N+1$. That is to say

$$\lim_{n\to\infty} \underline{x}_n = \underline{a}$$

Definition 2.4. Let f be a fuzzy function on A, $\underline{a}$, $\underline{b} \in F^*(R)$ and for arbitrary given $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\rho(f(\underline{x}), \underline{b}) < \varepsilon,$$

as $0 < \rho(\underline{x}, \underline{a}) < \delta$, then $f(\underline{x})$ is said to converge to $\underline{b}$ as $\underline{x}$ $\underline{a}$, write

$$\lim_{\underline{x}\to\underline{a}} f(\underline{x}) = \underline{b}.$$

Theorem 2.1. $\lim_{\underline{x}\ \underline{a}} f(\underline{x}) = \underline{b}$ if and only if for every $\underline{x}_n \in X$, $\underline{x}_n \neq \underline{a}$, $n = 1, 2, \cdots$, and $\lim_{n\to\infty} \underline{x}_n = \underline{a}$ has $\lim_{n\to\infty} f(\underline{x}_n) = \underline{b}$.

Theorem 2.2. (Boundedness theorem) Let $\underline{b} \neq \underline{\infty}$, $f(\underline{x}) \neq \underline{\infty}$ for every $\underline{x} \in X$, if $\lim_{\underline{x}\to\underline{a}} f(\underline{x}) = \underline{b}$, then there exist $\underline{m}$, $\underline{M}(\neq \underline{\infty}) \in F^*(R)$

$\delta > 0$ such that

$$\underline{m} < f(\underline{x}) \leq \underline{M},$$

as $0 < \rho(\underline{x}, \underline{a}) < \delta$.

Theorem 2.3. (Keeping sign property theorem) If $\lim_{\underline{x}\to\underline{a}} f(\underline{x}) = \underline{b}$, and there exists $\delta > 0$ such that

$$f(\underline{x}) \leq \underline{c} \quad (f(\underline{x}) \geq \underline{c}),$$

as $0 < \rho(\underline{x}, \underline{a}) < \delta$, then

$$\underline{b} \leq \underline{c} \quad (\underline{b} \geq \underline{c}).$$

Theorem 2.4. (Cauchy criterion for convergence) Let $\{f(\underline{x}_n); \underline{x}_n \in \{\underline{x}_n\}$, for every $\{\underline{x}_n\} \subset X\} \subset A^*$, then $f(\underline{x})$ is convergent as $\underline{x} \longrightarrow \underline{a}$ if and only if for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\rho(f(\underline{x}_n), f(\underline{x}_m)) < \varepsilon,$$

as $0 < \rho(\underline{x}_n, \underline{a}) < \delta$, and $0 < \rho(\underline{x}_m, \underline{a}) < \delta$, for every $\underline{x}_n, \underline{x}_m \in X$.

3 Fuzzy Continuous Function of Fuzzy Numbers

Definition 3.1. Let $f(\underline{x})$ be a fuzzy function on $X(X \subset F^*(R))$ if for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\rho(f(\underline{x}), f(\underline{a})) < \varepsilon,$$

as $\rho(\underline{x}, \underline{a}) < \delta$, then $f(\underline{x})$ is called cintinuous in $\underline{a}(\underline{a} \in X)$, we write

$$\lim_{\underline{x}\to\underline{a}} f(\underline{x}) = f(\underline{a}).$$

Definition 3.2. Let $f(\underline{x})$ be a fuzzy function on X, $\underline{a} \in X$. If for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\rho(f(\underline{x}), f(\underline{a})) < \varepsilon,$$

as $\rho(\underline{x}, \underline{a}) < \delta$, $\underline{x} \leq \underline{a}(\underline{x} \in X)(\underline{x} \geq \underline{a})$, then $f(\underline{x})$ is called left(right) fuzzy continuous in $\underline{a}$.

Definition 3.3. Let $\underline{a}$, $\underline{b} \in F^*(R)$ and $\underline{a} < \underline{b}$, $f(\underline{x})$ is called fuzzy continuous on $[\underline{a}, \underline{b}]$, if 1) for every $\underline{x} \in [\underline{a}, \underline{b}]$, $\underline{x} \neq \underline{a}$, $\underline{b}$, $f(\underline{x})$ is fuzzy continous in $\underline{x}$; 2) $f(\underline{x})$ is left fuzzy continuous in $\underline{b}$ and right fuzzy continuous in $\underline{a}$.

Theorem 3.1. If $f(\underline{x})$ is fuzzy continuous on $[\underline{a}, \underline{b}]^*$, then $f(\underline{x})$ is bounded on $[\underline{a}, \underline{b}]^*$.

Theorem 3.2. If $f(\underline{x})$ is fuzzy continuous on $[\underline{a}, \underline{b}]^*$, then $f(\underline{x})$ has the greatest value and least value, i,e. there exist $\underline{x}$ $\underline{x}_1$, $\underline{x}_2 \in [\underline{a}, \underline{b}]^*$ such that

$$f(\underline{x}_1) = \sup_{\underline{x} \in [\underline{a}, \underline{b}]^*} \{f(\underline{x})\};$$

$$f(\underline{x}_2) = \inf_{\underline{x} \in [\underline{a}, \underline{b}]^*} \{f(\underline{x})\},$$

where $\{f(\underline{x}); \underline{x} \in [\underline{a}, \underline{b}]^*\} \in A^*$.

Definition 3.4. Let $f(\underline{x})$ be a fuzzy function on $X(X \subset F^*(R))$ if for every $\underline{x}', \underline{x}'' \in X$, $\underline{x}' \neq \underline{x}''$, with

$$f(\underline{x}'') \leq f(\underline{x}') \text{ or } f(\underline{x}') \leq f(\underline{x}''),$$

then $f(\underline{x})$ is called O-function.

Theorem 3.3. If $f(\underline{x})$ is continuous O-function on $[\underline{a}, \underline{b}]^*$, $\underline{m} = \inf_{\underline{x} \in [\underline{a}, \underline{b}]^*} \{f(\underline{x})\}$, $\underline{M} = \sup_{\underline{x} \in [\underline{a}, \underline{b}]^*} \{f(\underline{x})\}$, then for every $\underline{c} \in [\underline{m}, \underline{M}]^*$, there exist at least a $\underline{z} \in [\underline{a}, \underline{b}]^*$ such that

$$f(\underline{z}) = \underline{c}.$$

Definition 3.5. Let $f(\underline{x})$ be a fuzzy function on $X(X \subset F^*(R))$ if for any $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that

$$\rho(f(\underline{x}'), f(\underline{x}'')) < \varepsilon,$$

as $\rho(\underline{x}', \underline{x}'') < \delta$, for every $\underline{x}'$, $\underline{x}'' \in X$, then $f(\underline{x})$ is called fuzzy uniformly continuous on X.

Theorem 3.4. If $f(\underline{x})$ is continuous on $[\underline{a}, \underline{b}]^*$, it is fuzzy uniformly continuous on $[\underline{a}, \underline{b}]^*$.

References

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