FUZZY MODEL OF INEXACT REASONING

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1. Introduction

Vast portion of social, psychological, medical etc. experience suffers from so little data and so much imperfect knowledge that a rigorous probabilistic analysis is not possible. Nevertheless, it is instructive to examine models for the less formal aspects of decision making. The nature of such nonprobabilistic and unformalized reasoning processes is examinated e.g. in [2] . Shortliffe´s model of inexact reasoning in medicine was very succesfull (MYCIN, [2]) in the microbiological area and it is potentially applicable to many other domains. Of course, this model corresponds to the character of the microbiological data. For a general use, we generalize the Shortliffe´s model. Also, we give a fuzzy interpretation of the Shortliffe´s model and the generalized model, too.

2. Shortliffe´s model of inexact reasoning

It would be desirable to have such measures of evidential strength, which satisfy the following Törneböhm´s axioms [3] :

A1. If E implies H, then $C(H,E) = \max$.

A2. If E implies not H, then $C(H,E) = \min$.

A3. $C(H\&E,E) = C(H,E)$.

A4. If H and E are independent, then $C(H,E) = 0$.

Here $C(H,E)$ is a measure of evidential strength quantifying the influence of the knowledge of the information E to the verification of the hypothesis H. Note that the conditional probability P(H/E) satisfies A1, A2 and A3. It is not possible to construct an exact measure satisfying all these axioms.

The nature of investigated decision making together with the famous Paradox of the Ravens (C. Hempel, see e.g. [2]) led Shortliffe to create some new terms for the measurement of evidential strength. His notation is as follows.

(1) measure of "Belief", MB(H,E) = a means " the measure of increased Belief in the hypothesis H, based on the information E, is a "

(2) measure of "Disbelief", MD(H,E) = b means " the measure of increased Disbelief in the hypothesis H, based on the information E, is b "

(3) certainty factor CF combines the MB and MD,
$CF(H,E) = MB(H,E) - MD(H,E)$.

For a simple hypothesis h and a simple information e we have:

$$MB(h,e) = \begin{cases} 1 & \text{if } P(h) = 1 \\ \dfrac{\max\{P(h/e),P(h)\} - P(h)}{\max\{1,0\} - P(h)} & \text{otherwise ,} \end{cases}$$

$$MD(h,e) = \begin{cases} 1 & \text{if } P(h) = 0 \\ \dfrac{\min\{P(h/e),P(h)\} - P(h)}{\min\{1,0\} - P(h)} & \text{otherwise .} \end{cases}$$

Here P(h) denotes a priori probability, P(h/e) a conditional probability. Note e.g. for $P(h) < P(h/e)$ we have $MD(h,e) = 0$,

$$MB(h,e) = \frac{P(h/e) - P(h)}{1 - P(h)} = \frac{\text{real increament of belief}}{\text{max. possible increament of bel.}} .$$

Proposed MB, MD and CF = MB - MD for simple h and e satisfy the axioms A1, A2 and A4. The conventions adopted for combining MB and MD (for CF we have always CF = MB - MD) allow us to satisfy A3. We present some of these combining rules. For all details see [2] .

I. Increamentaly acquired evidence

a) $MB(H,E_1\&E_2) = MB(H,E_1) + MB(H,E_2).(1 - MB(H,E_1))$,
$= 0$ if $MD(H,E_1\&E_2) = 1$,

b) $MD(H,E_1\&E_2) = MD(H,E_1) + MD(H,E_2).(1 - MD(H,E_1))$,
$= 0$ if $MB(H,E_1\&E_2) = 1$.

II. Conjunctions of hypotheses

a) $MB(H_1\&H_2,E) = \min\{MB(H_1,E),MB(H_2,E)\}$,

b) $MD(H_1\&H_2,E) = \max\{MD(H_1,E),MD(H_2,E)\}$.

III. Disjunctions of hypotheses

a) $MB(H_1\vee H_2,E) = \max\{MB(H_1,E),MB(H_2,E)\}$,

b) $MD(H_1\vee H_2,E) = \min\{MD(H_1,E),MD(H_2,E)\}$.

IV. Strength of evidence

a) $MB(H,S) = MB'(H,S).CF^+(S,E)$,

b) $MD(H,S) = MD'(H,S).CF^+(S,E)$.

Here the evidence S is not known with certainty, but only with CF(S,E) based upon prior information E; MB′ (MD′) is the MB (MD) for H, when S is known to be true; $CF^+(S,E) = \max\{0,CF(S,E)\}$.

3. <u>Fuzzy operations</u>

For the fuzzy sets, we can propose a variety of models for background fuzzy operations, which correspond to the logical conjunction and disjunction. The aspect of maximal likelyhood is a basis for a model "ML". Let $A = \sum_{x\in U} m_A(x)/x$ and $B = \sum_{x\in U} m_B(x)/x$ be two fuzzy sets. Then

"ML model": $m_{A\cap B} = \min\{m_A,m_B\}$,
$m_{A\cup B} = \max\{m_A,m_B\}$.

We propose to use the following notation for these operations:

$\underset{ML}{A \cap B}$, $\underset{ML}{A \cup B}$.

The model "I" is based on the aspect of independence.

"I model": $m_{A \cap B} = m_A . m_B$,

$m_{A \cup B} = m_A + m_B - m_A . m_B = 1 - (1 - m_A).(1 - m_B)$.

We propose to use the notation

$\underset{I}{A \cap B}$, $\underset{I}{A \cup B}$.

Every convex combination of two models "ML" and "I" can be taken as a model for the fuzzy operations. We present a general model combining both the aspect of maximal likelyhood and independence, which will be called model "G".

"G model": $m_{A \cap B} = \max\{m_A, m_B\}.m_A.m_B + (1 - \max\{m_A, m_B\}).\min\{m_A, m_B\}$,

$m_{A \cup B} = \max\{m_A, m_B\}.(m_A + m_B - m_A.m_B) +$

$+ (1 - \max\{m_A, m_B\}).\max\{m_A, m_B\}$.

We propose to use notation $A \underset{G}{\cap} B$, $A \underset{G}{\cup} B$. If we use original Zadeh´s notation for fuzzy convex combination (see e.g. [4]) we get

$A \underset{G}{\cap} B = (A \underset{I}{\cap} B,\ A \underset{ML}{\cap} B,\ A \underset{ML}{\cup} B)$, resp. $A \underset{G}{\cup} B = (A \underset{I}{\cup} B,\ A \underset{ML}{\cup} B,\ A \underset{ML}{\cup} B)$.

Note, that the models "ML" and "I" are commutative and associative, but the model "G" is only commutative. Its nonassociativity corresponds to real decision making e.g. in medicine, psychology etc.

4. Shortliffe´s model in fuzzy terms

Let $\mathcal{H}$ be a set of all possible hypotheses, $\mathcal{E}$ a set of all possible pieces of evidence. We can suppose $\mathcal{E} \subset \mathcal{H}$. Let

$$\mathcal{U} = \mathcal{H} \times \mathcal{E}$$

be our universal space. Denote by HMB the cylindric fuzzy set

$$HMB = \sum_{E \in \mathcal{E}} MB(H,E)/E$$

for a $H \in \mathcal{H}$. Here $m_{HMB}(E) = MB(H,E)$. Then, denote HMB(E) the single fuzzy set

$$HMB(E) = MB(H,E)/(H,E) .$$

Similarly we denote

$$HMD = \sum_{E \in \mathcal{E}} MD(H,E)/E \quad , \quad HMD(E) = MD(H,E)/(H,E) \quad ,$$

$$MBE = \sum_{H \in \mathcal{H}} MB(H,E)/H \quad , \quad MDE = \sum_{H \in \mathcal{H}} MD(H,E)/H \quad ,$$

$$CFE = \sum_{S \in \mathcal{E}} CF^{+}(S,E)/S$$

$$HMB/E = \sum_{S \in \mathcal{E}} MB(H,S)/S \quad , \quad HMD/E = \sum_{S \in \mathcal{E}} MD(H,S)/S \quad .$$

Here the evidence S is not known with certainty, but only with CF(S,E) , based upon prior evidence E.

Then the Shortliffe´s combining rules can be present in the form:

I. a) $MB(E_1 \& E_2) = MBE_1 \underset{I}{\cup} MBE_2$, b) $MD(E_1 \& E_2) = MDE_1 \underset{I}{\cup} MDE_2$.

II. a) $(H_1 \& H_2)MB = H_1MB \underset{ML}{\cap} H_2MB$, b) $(H_1 \& H_2)MD = H_1MD \underset{ML}{\cup} H_2MD$.

III. a) $(H_1 \vee H_2)MB = H_1MB \underset{ML}{\cup} H_2MB$, b) $(H_1 \vee H_2)MD = H_1MD \underset{ML}{\cap} H_2MD$.

IV. a) $HMB/E = F(CFE,HMB)$, b) $HMD/E = F(CFE,HMD)$.

Here F is a fuzzy extension operator,

$$F(A,K) = \underset{ML}{\cup} m_A(x).K(x), \text{ where } K(x) = F(1/x,K) .$$

5. <u>General model of inexact reasoning</u>

In the proposed model we replace the fuzzy operations of the type "ML" and "I" by those of the type "G". As an example we can give the combining rules for increamentaly acquired evidence:

I. a) $MB(E_1\&E_2) = MBE_1 \underset{G}{\cup} MBE_2$, b) $MD(E_1\&E_2) = MDE_1 \underset{G}{\cup} MDE_2$.

Other properties of Shortliffe's model (e.g. for extreme values of MB or MD) we remain unchanged, see [2] .

Original Shortliffe's model, namely for increamentaly acquired evidence, is associative one. The theory of associative models of inexact reasoning based upon the certainty factors CF is developed in [5] . Our proposed model is nonassociative one. In the nonassociative domains we use described rules for more then two objects at once - e.g. for the union $\underset{G}{\cup} A_i$ of n fuzzy sets A_i we get $\underset{G}{\cup} A_i = (\underset{I}{\cup} A_i, \underset{ML}{\cup} A_i, \underset{ML}{\cup} A_i)$. For the membership function it means that for $A = \underset{G}{\cup} A_i$ we have

$$m_A = \max_i\{m_{A_i}\}.(1 - \prod_i (1 - m_{A_i})) + (1 - \max_i\{m_{A_i}\}).\max_i\{m_{A_i}\} .$$

If the nature of analysed problem is associative, we use the combining rules for more special H or E consecutively step by step (so we get "near-associative" model).

Our proposed model is also archimedean one (for more details about archimedean and nonarchimedean concepts of the rules of type I. , i.e. the combining rules for increamentaly acquired evidence, see [5]), so as Shortliffe's model did. Let us define another model for basic fuzzy operations, we denote it H:

$$A \underset{H}{\cap} B = (A \underset{ML}{\cap} B,\ A \underset{I}{\cap} B,\ (A \underset{ML}{\cup} B)_{0,5}) ,\quad A \underset{H}{\cup} B = (A \underset{ML}{\cup} B,\ A \underset{I}{\cup} B,\ (A \underset{ML}{\cup} B)_{0,5}) .$$

Here $C_{0,5} = \{ x \in U,\ m_C(x) \geq 0,5 \}$, i.e. $m_{C_{0,5}}(x) = 0$ for $m_C(x) < 0,5$ and $m_{C_{0,5}}(x) = 1$ for $m_C(x) \geq 0,5$.

Then the model of inexact reasoning which use for the combining rules for increamentaly acquired evidence the unions of H type is nonarchimedean.

The proposed models of inexact reasoning can be taken as a mathematical basis of more general expert consultation programs as those, which are used till now. Such a first application, using the basic fuzzy operations of G model, is described in [1] .

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