# **Book Review**

# Fuzzy Set Analysis for Behavioral and Social Sciences

M. SMITHSON Springer Verlag, 1987

This is the first book about fuzzy sets solely devoted to behavioral and social sciences. As such it contrasts with previous treatises and monographs by its unusually applied scope, and its concern to discuss the practical usefulness of the mathematics of fuzzy sets in real world problems. The major topics of the book are measurement and data analysis techniques. It contains 7 chapters, and only the first one is about fuzzy set theory itself. This chapter reviews basic notions including fuzzy set theoretic operations, and touches upon fuzzy logic and possibility theory. Chapter 2 is a good survey about the ability of fuzzy sets to represent concepts; it provides very relevant discussions about various objections to fuzzy set theory raised by psychologists, such as the excluded middle law controversy. It also gives an extensive account of published experiments in the empirical validity of fuzzy connectives.

Chapter 3 considers the important problem of membership measurement. It stresses that although some attempts at defining measurement techniques have emerged in the eighties, the situation of membership measurement has not drastically evolved. However the author notices that even in the probabilistic setting controversies about measurement are still going on. An interesting subsection deals with the necessity of the unit interval as a measurement scale. Another one suggests to relate the question of fuzzy connectives with the theory of conjoint measurement.

Chapter 4 discusses the use of fuzziness indices as well as fuzzy cardinality to assess the internal structure of linguistic categories. Chapter 5 is about fuzzy similarity relations and their use in clustering and discriminant analysis. Chapter 6 deals with the question of interpreting statistically observed dependencies between variables in terms of fuzzy set theoretic operations. It suggests that it may be useful to use the basic fuzzy intersections, unions and exclusive "or" to define new non-linear regression models. Chapter 7 tries to use many-valued implication operations in the study of statistical prediction models and the induction of "if... then..." rules from empirical data about a sample of objects. The conclusion of the book emphasizes the following points: fuzzy

set theory sheds new light on the problem of translating numerical models into words; it proposes new non-conventional methods for solving problems in the social sciences.

This book is certainly a valuable and original attempt. If some reservation is to be made, it will be on the question of relating fuzzy set theory to frequencies as obtained from statistical experiments (such as polls which are a very widely used tool in social sciences). Many pages in the book try to apply fuzzy set operations so as to interpret contingency tables, much in the spirit of previous works by Gaines, Bandler and Kohout and others from what can be called the "English fuzzy school". Whether it is sensible to apply fuzzy sets to the processing of classical contingency tables is debatable, and can be viewed as a risky approach since it may lead to a confusion between membership values and probabilities. The relationship between possibility measures and probability measures is now completely understood from a mathematical point of view, by viewing a fuzzy set as a convex combination of nested sets, while a probability distribution is a convex combination of disjoint sets. However, a full-fledged statistical interpretation of membership functions based on this property and a paradigm of repeated experiments yielding imprecise outcomes is still to be developed (see Dubois and Prade [1] for a sketchy attempt). Frequentist and personal membership functions may be useful in the social sciences, but I doubt whether the latter may help in the study of contingency tables in a well-founded way. They however can serve to model contingency tables based on fuzzy categories, as done by Smithson in chapter 3 with fuzzy Gutman scaling. This idea relates to fuzzy random variables following Kwakernaak [2], Kruse and Meyer [3] which were motivated by problems in the social sciences; however these works are not considered in the book, neither are "frequentist" membership functions based on consonant random sets.

In fact this book is much thought-provoking by the philosophical issues it considers, and it also proposes practical techniques (including computer programs) to solve data analysis problems. But the way fuzzy sets are applied may make a few readers believe with some reason that only preliminary steps ahead have been taken for the specification of a convincing, and systematic approach, based on fuzzy sets, to behavioral and social sciences. But the author himself acknowledges this state of facts in many pages of his -otherwise excellent- monograph.

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- [1] Dubois D., Prade H. Fuzzy sets and statistical data. Europ. J. Operational Research, 25, 345-356, 1986.
- [2] Kwakernaak H. Fuzzy random variables. Information Sciences, Part I, 15, 1-15, 1978, Part II, 17, 253-278, 1979.
- [3] Kruse R., Meyer K.D. Statistics with Vague Data. D. Reidel Dordrecht, 1987.

#### A review of

# Préférence et Utilité Floues - Applications à la Théorie de l'Équilibre Partiel du Consommateur -

Antoine B. BILLOT (Presses Universitaires de France, Paris, 1987)

This thin book (86 pages) presents an attempt to diminish the gap between highly normative and idealistic models in economy such as the ones proposed by Arrow or Debreu, and the complex and nuanced reality of the world, by using a fuzzy set-based approach. He takes the consumer equilibrium problem as an illustrative example.

The book is organized in three chapters. The first one gives the necessary background, by introducing possibility and necessity measures and providing a comparison of their axiomatics with probability theory (perhaps the fact of using both possibility measures and Nahmias' valuations is prejudicial to a clear distinction between fuzzy sets and possibility measures). The connection between Shackle [1]'s non-probabilistic modelling of uncertainty and possibility theory, is recalled and stressed. The chapter ends by pointing out that the ideas of imprecision and uncertainty are latent in works by Marchal and Arrow. The second chapter is devoted to a presentation of fuzzy preference relations and a method, proposed by Ponsard [2], for deriving a utility function from a fuzzy preference relation, is analyzed in detail. The third chapter discusses the consumer's equilibrium problem in a fuzzy context, allowing both a fuzzy utility and a fuzzy constraint; it leads to a fuzzy mathematical programming problem. In each chapter, the author makes efforts for giving the economic meaning of his mathematical model.

This book should contribute to show the interest of using new mathematical tools such as fuzzy sets and possibility theory in order to develop less normative models (with a less stringent view of the supposed rationality of the individual) in economy. It may also help to a better diffusion of the pionnering work made by Professor Claude Ponsard [2] and his colleagues during the last fifteen years on the modelling of various aspects of economics by means of fuzzy sets. However, this small book provides neither a complete account of what has been made in this

research group, nor an overview of non-purely probabilistic approaches to utility or preference; see Dubois and Prade [3] for a survey of other recent proposals, and Roubens and Vincke [4] on preference modelling.

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## References

- [1] Shackle G.L.S. Decision, Order and Time in Human Affairs. Cambridge University Press, Cambridge, U.K., 2nd edition, 1961.
- [2] Ponsard C. Fuzzy mathematical models in economics. Fuzzy Sets and Systems. Special issue on Mathematical Modelling, 1988, to appear.
- [3] Dubois D., Prade H. Recent models of uncertainty and imprecision as a basis for decision theory: towards less normative frameworks. In: Intelligent Decision Support in Process Environments (E. Hollnagel, G. Mancini, D.D. Woods, eds.), Vol. 21 in Computer and Systems Sciences Series, Springer Verlag, Berlin, New York, 1986, 3-24.
- [4] Roubens M., Vincke P. Preference Modelling. Lecture Notes in Economics and Mathematical Systems n° 250, Springer Verlag, Berlin, New York, 1985.