APPROXIMATE REASONING IN THE POSSIBILITY THEORY FRAMEWORK : where are we practically ?

Roger MARTIN-CLOUAIRE
Station de Biométrie et Intelligence Artificielle
INRA
Auzeville, B.P. 27
31326 CASTANET-TOLOSAN CEDEX
FRANCE

ABSTRACT : This paper discusses contemporary issues associated with the use of approximate reasoning techniques in knowledge-based systems relying on the theory of possibility. It aims at :
- providing lines along which comparison of existing systems can be done ;
- pointing out directions toward which future systems should evolve.

INTRODUCTION

Representational and inferential capabilities based on imprecise , uncertain, incomplete or inconsistent information are becoming more important in the design, implementation and operation of knowledge-based systems. Since the advent of the first expert systems several approaches for making inference from available information have been developed. This general problem has benifited from important contribution coming out of research conducted in the possibility theory framework. Unfortunately, the results from this contribution have not yet received much attention outside the fuzzy set community and may not have spread enough within this community itself. One of the reason may be that the great variety and complexity of the facets constituting this problem are confusing people. Therefore, clarifying the picture is a need and is modestly attempted here (see also Chapter 7 of [Zim87] for a survey).

This paper discusses contemporary issues associated with the use of approximate reasoning techniques in rule-based systems relying on the theory of possibility. We first examine the best known of these techniques with respect to their capabilities in processing "imperfect knowledge". Then we provide a perspective concerning some general approximate reasoning problems whose solutions have not yet received a satisfactory implementation (if any). Thus, the paper is pursuing a twofold goal :

- providing lines along which comparison of existing systems can be done ;
- pointing out directions toward which future systems should evolve.

EXISTING SYSTEMS

In this section, we propose a little review of some existing systems along a set of discriminating features. No claim of exhaustivity for either the systems or the features is made.

Differentiation between systems can be done by examining what kind of knowledge they are able to represent and process effectivelly. Indeed, beside the external aspect of rules and facts that constitute the knowledge base, one must observe how these systems behave in the typical four step process characterizing an inference because it is difficult to understand what is really represented without taking into account how information is used. The four steps under consideration are :

i) evaluation of the individual conditions of a rule with respect to corresponding facts ;

ii) aggregation of the above elementary evaluations ;

iii) deduction by the combination of step ii) with conditional information representing the

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antecedent-consequent relationship ;

iv) fusion of the result of step iii) with other related items of information.

It is convenient for the present discussion to consider deductive reasoning under the form of the basic pattern that follows :

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if p then q
  p'                      (1)
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           q'
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where p, q, p' and q' are propositions, p' is supposed to match p somehow and q' is what can be deduced from the pieces of information above the line. The sophistication of systems clearly depends on whether or not they support inferences where p, q and p' can be vague propositions. Let us recall that a proposition is uncertain when one cannot definitely state it is true of false. A proposition is imprecise if the single-valued parameter it aims to describe is incompletely specified in the sense that more than one value is possible. Vagueness is a particular kind of imprecision where the possible values of the parameter lack clear or crisp boundaries. Thus, uncertainty refers to the truth of the proposition whereas imprecision pertains to the content of the proposition or, in other words, its meaning.

Treating uncertainty

Some of the existing systems (e.g. RUM [Bon87], FESS [Hal86]) are addressing the issue of approximate reasoning as a problem of management of uncertainty only (where the proposition involved in pattern (1) are non-vague) because the applications they aim to do not require meaning computation. For more sophisticated systems, uncertainty management techniques are provided explicitly of implicitly as special cases of more powerful capabilities.

The uncertainty about the truth of a proposition is often represented by one (e.g. [Hal87] [Buc86]) or two numbers (e.g. [Mar85], [Bon87]) belonging to [0,1]. In the case two numbers are used (e.g. necessity and possibility degrees in SPII [Mar86]) one can express to what extent it is certain that p is true and to what extent the contrary of p (denoted ¬p) is true. It makes it possible to represent ignorance (which is a lack of support in favor of p as well as ¬p) and to distinguish it from total uncertainty (where the support in favor of p equals the support in favor of ¬p and is right in between total absence of support and full support). Using a single number makes this distinction impossible except if uncertainty has a probabilistic nature since, in such a case, the support in favor of p determines completely the support in favor of ¬p. Some authors have interpreted such a single number as an intermediate truth value. But this approach is strongly criticizable because it does not make sense to say that a precise proposition is more or less true. It can only be either true or false although one may not have full confidence in either its truth or its falsity.

The representation of the uncertainty on the inferential relationship between p and q in pattern (1) is another characteristic along which systems differ one from the other. Again, either one or two numbers may be used. The case of two numbers (e.g. [Leb86], [Bon87]) aims at expressing to what extent it is necessary (compulsory) and sufficient to have p true in order to deduce that q is true as well. The necessity aspect answers the purpose of deducing to what extent q is false when there is support in favor of ¬p. On the other hand, the sufficiency aspect serves in evaluating to what extent q is true when there is support in favor of p. Systems using a single degree (e.g. [Muk87], [Uma87]) usually deal with the sufficiency aspect only.

Representation issues concerning uncertainties on facts and rules have immediate consequences on the steps i) and iii) of the inference process. Steps ii) and iv) are basically two kinds of aggregation operations. Several existing systems (e.g. [Bon87], [Buc86]) provides many different possibilities (using families of t-norm and t-conorm operators) for performing these combinations of uncertain information. Some (e.g. [Leb87]) have so far adopted the safer position that consists in using 'min 'and 'max' operators only, thus keeping the meaning given to the the degrees of uncertainty globally in agreement with the underlying mathematical framework i.e. possibility theory.

Uncertain conclusion together with vagueness in conditions and facts

We consider here situations where p and p' in (1) are of the form X is A and X is A' respectively where A and A' are fuzzy sets. The step i) of the inference process has to be extended in order to accommodate the imprecision of information conveyed by p and p'. Basically, two approches are encountered. The first one [Uma87] involves the computation of the compatibility of A and A' i.e. CP(A ; A'). This compatibility is a fuzzy degree of truth (intermediate truth makes sense in this case) of X is A given that X is A'. The processing of such fuzzy degrees of truth (i.e fuzzy sets in [0,1]) in the four steps of inference may be very costly from a computational point of view. For this reason, their use is done in a highly constrained manner. For instance, Umano [Uma87] deals with fuzzy truth values represented by discrete possibility distributions involving few points. As another example, in MILORD [God87] only positive truth values are used (i.e. falsity is not represented). Moreover, any truth value is assumed to belong to a set of nine values (via an approximation if needed). All logical operations involved in the four steps of inference can then be expressed by "truth tables" on the nine truth value set. Although, in MILORD the p and p' of Pattern (1) may be fuzzy no meaning computation is really needed because p is expessed under the form "X is M is ∂" and p' by "X is M is ß" where ∂ and ß are fuzzy truth values. Thus, the only difference is on the truth value qualifying "X is M" so that computation can be carried on at the level of uncertainty only. If p where allowed to be of the form "X is N is ß" with N different from M then meaning processing (involving explicit consideration of the fuzzy sets M and N) would be necessary.

The second approach (see for instance [Mar85], [Sou86]) uses necessity and possibility degrees (i.e. N(A ; A') and Π(A ; A')) for expressing to what extent p' matches p. It is important to keep in mind that these two degrees convey information also contained in the fuzzy compatibility degrees considered in the other approach. Indeed, one can always compute N(A ; A') and Π(A ; A') from CP(A ; A'). Thus, the two approaches clearly share the same mold but the second one is not confronted to any inefficiency problem. When the similarity between the conditions and facts is expressed via necessity and possibility degrees the inference process can be carried on in the same way than with rules involving precise conditions only. Thus, this approach nicely copes with rules having both precise and vague conditions.

General framework for managing imprecision and uncertainty

Zadeh's general modus ponens corresponds to the pattern (1) where p , q, p' and q' are of the form "X is A", "Y is B","X is A'" and "Y is B'" respectively. Approximate reasoning in this scheme is seen as a problem of meaning computation or constraint propagation. Indeed the meaning of q' (i.e the restriction on the possible values of Y) is obtained by combining (via a non-linear programming technique) the restriction on the possible values of X with the semantic content of the rule expressing the X-Y relationship. This powerfull technique is used for instance in SPII [Leb87] and CARDIAG-2 [Adl86]. In the latter case, the involved fuzzy sets are on discrete universes and the X-Y relationship is explicitely defined by the user rather than induced from a loose specification conveyed by the rule. Usually, the fuzzy relation expressing the X-Y relationship is obtained by means of a multiple-valued implication connective -> such that $\mu_{X\text{-}Y}(s,t) = \mu_A(s) \rightarrow \mu_B(t)$. The generalized modus ponens permits to compute B' as $\mu_{B'}(t) = \sup_s \mu_{A'}(s) * \mu_{X\text{-}Y}(s,t)$ where * is a conjunction operator. The different possible choices for * and -> lead to many variants (see [Mar88] for some examples) but, if we want B' to be equal to B when A' is equal or included in A then the choice of -> cannot be done independently of the one concerning * [Dub84]. These choices of operators correspond to different interpretations of the rule " if X is A then Y is B". Other interpretations are possible if we consider the rule under it contraposive form (i.e. "if Y is not B then X is not A") since, in fuzzy logic, a rule is not necessarily equivalent to its contrapositive form. For instance, if the rule " if X is A then Y is B" is used in its contrapositive form together with the 'min' conjunction operator and the Gödel implication function then the conditional piece of knowledge actually represented tells something like "the more X is A the more certain the proposition Y is B" [Bui86].

The X-Y dependency is often described via a collection of rules rather than a single one. More specific results are obtained if the rules are combined before being used in inference [Mar88]. So far, no known system performs this prior combination.

The main reason why the generalized modus ponens has not been used more extensively in practical systems is that its straightforward implementation gives very poor performance. However, a satisfactory use of this technique can be reached with continuous parametrized possibility distributions and some approximation procedures. The technique developped in [Mar88] permits to see the generalized modus ponens as a deductive device running in a manner similar to the four step inference earlier mentioned in this paper although it does not if considered according to its basic definition.

Additional discriminating features

In the step ii) of inference with rules having precise conclusions one has to perform an aggregation of the levels of satisfaction of the individual conditions by the corresponding facts. A useful improvement is obtained if one take into account the notion of importance of conditions (e.g. that a condition is more important than another one or that a condition need not be fully satisfied in order to apply the rule). A technique for incorporating importance in conjunction and disjunction operations performed with necessity and possibility degrees has been developed by Dubois et al.[Dub88] and implemented in late versions of SPII [Mar86]. It is not clear in the literature whether other systems have a similar feature.

Conditions of rule may contain numerical predicates. Systems like FLOPS [Buc86] or SPII can evaluate to what extent a given fuzzy number is greater than another one.

Some inference engines allow numerical computation in the antecedent or consequent part of rules. Few fuzzy inference engines can do the same with fuzzy numbers. SPII [Leb86] supports computation in conclusions provided the involved variables are non-interactive.

Fusion of information coming from different sources is another problem that has received different solutions across existing systems. So far no implementation seems sufficiently satisfactory and some use very ad-hoc combination operations for procedural purpose only [Adl86].

Related to the problem of combination is the one of controling the search of the system toward a goal. In order to avoid exhaustive search some have introduced thresholding techniques that give quantitative flavor to degrees of uncertainty. Rules may be invoked according to the amount of their uncertainty or with respect to their specificity or any other criteria referring to imprecision and uncertainty.

Finally, another line of comparison of existing systems concerns their abilities in explaining results of uncertain reasoning. This often involves linguistic approximation techniques in order to ensure proper communication in linguistic terms.

3. GOING FARTHER

Representing more information

Future improvement of fuzzy inference engines should come from progresses in knowledge representation. It has been pointed out that the generic rule "if X is A then Y is B" can be interpreted in different ways. However, it is important to note that the generalized modus ponens, as has been considered so far in existing systems, is always oriented toward one particular understanding of the rule (we do not consider the contrapositive form here). Basically, it tells that the derived B′ restricts the values in the domain of Y which are in relation (via the rule) with at least one value in A′. As shown in [Dub87], there is a dual interpretation which permits to obtain the constraint on the values that are in relation (again via the rule) with all values of the domain of X more or less compatible with A′. Thus, the second acceptation of the rule gives a more restrictive possibility distribution (that is contained in the one resulting from the first one). Both interpretations may be useful in practice.

In addition to the above new rules one could also need, for instance, rules of the kind "the more X is A, the more Y is B" [Pra87]. Such a kind of rule describes in a qualitative manner the variation or the variable Y in terms of the variation of X.

So far, fuzzy sets used in most systems have a disjunctive meaning (i.e. they restrict the more or less possible values of a variable taking a single value). The conjunctive interpretation (i.e. where the described variable may have several values) should be taken into account too.

Better combination of imprecise and uncertain information

Given several items of information concerning a particular variable and coming from different sources one is confronted with the problem of synthesizing the pieces into a single item. Most of the time the combination is computed by performing a conjunction. However the combination process must depend on what has to be combined. In some cases, a disjunction or a compromized combination may be the good choice. In other cases the combination process must discard some items [Dub87b].

Future systems should handle the combination problem with more attention. This may have deep consequenses on system architecture because solving the problem of combination requires specific reasoning activities.

Merging different techniques

Knowledge-based systems often have to face problems of default reasoning. The techniques embedded in current systems make it possible to solve some of these problems but others are better handled by purely symbolic approaches or syllogistic reasoning methods [Zad87]. Future systems must provide a way for merging the different techniques needed for non-monotonic reasoning.

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