FUZZY CONVEXITY, PERIPHERIC CORE AND α-NEAR NUCLEUS OF AN EXCHANGE ECONOMY

PART 2

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3 - THE α-NEAR NUCLEUS

3.1. The establishment of the proposition (247A) suggest us - in the wording of (248) - that, in the eventuality of non-vacuity of the 1-cut of the peripheric core - the intra-

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muros core or the usual core of the economic game - it could be possible to resort to the "approximately balancing" allocations. In fact, it means that the membership level to the peripheric core must be less constraining than the unity which means the total membership, if we want to find a solution which approximately permits to balance the game, when the intra-muros core is empty. To the well-understanding of the concept of approximative equilibrium, it is necessary to consider the economy as a double collection of goods and agents, these two sets being finally managed by an outside agent which we call a planner. The agents, economical gamblers, say their preferences, cooperate to their own interest and, from these individual choices, a set of allocations arises which is, here, the solutions. If the free proposition of the agents is sufficiently pertinent - it means that the intra-muros core is non-empty - the planner is not necessary. On the contrary, if for the simple reason that the agents do not manage to define a balance solution, then the planner will choose the allocation whose membership level is the highest, in other words, the one which is the nearest of the unity (the allocation which is the nearest of the intra-muros core). Thus, we say this allocation to be approximately balancing. So, such an allocation always exists because of the proposition (247A) which insures us the non-vacuity of the peripheric core.

3.2.1. According to the proposition (2492), we call "α-near nucleus" any α-cut of the peripheric core $\underset{\sim}{CP}$ (with $\alpha \in (0.1)$).

3.2.2.1. We can remark that the intra-muros core (the usual one) defined in (252) is, in fact, the 1-near nucleus.

3.2.2.2. We can also remark that the 0-near nucleus is the exclusive support of the peripheric core because of the strict inequality written in the definition of the proposition (2491).

3.2.2.3. We can also remark that any α-near nucleus contains the intra-muros core.

3.2.2.4. It is obvious that, for a given peripheric core $\underset{\sim}{CP}$, the membership function being continuous ((235) and (253)), the more α is little (near 0), the more the cardinal of the α-near nucleus is big : $\frac{d(|\alpha\text{-near nucleus}|)}{d\alpha} \leq 0$

3.3.1. As the walrasian "commissaire-priseur" is endowed of an implicit utility function which decreases with the surplus (DEBREU (1982)), our planner increases its utility with the growth of α - that for an α-near nucleus which is non-empty. If we call "t" the planner utility function with :

$$t : [0.1] \longrightarrow R, \quad \alpha \longrightarrow t(\alpha)$$

, t a continuous function and $dt/d\alpha \geq 0$, the program of the planner is thus :
Max t(α) under the constraint : card(α-near nucleus) (i.e. |α-near nucleus|) ≠ 0. The α^* near nucleus where α^* corresponds to the α -solution of the program of the planner, contains the allocations "approximately balancing" or if α^* is equal to 1, the usual equilibrium.

3.3.2.1. The proposition (247A) insures us the existence of a solution for the program of the planner, and so the existence of an allocation which can balance the economic game, even approximately.

3.3.2.2. We remark that the intra-muros core can be a solution of the program of the planner because α is defined on [0.1].

3.3.3. We know that all the balancing solutions of the 1-near nucleus, of the intra-muros core are pareto-optimal but weak P.O. It means that any allocation which is prefered to them does not belong to RB($\mathcal{S}$). But the "approximately balancing" allocations are not weak P.O. because of the existence of a coalition weakly blocking ($\alpha^*<1$).

CONCLUSION

The theorical obtainment of a solution - trivialy pertinent (the intra-muros core is non-empty) or approximately balancing (resorting to the smallest of the α-near nucleus which are non-empty) - suggests us to envisage the cooperative fuzzy games under a more practical point of view. We already know the fuzzy core - in the sense of AUBIN - of a cooperative game represents the limit of the successive cores issued from the different swellings of the coalitions set (with the membership definition of a fuzzy coalition). This means that the fuzzy core - in the sense of AUBIN - is, in a way, the core of a limit economy. For that, we can see DEBREU - SCARF (1963), CORNWALL (1984) or HILDENBRAND (1976). In fact, if the number of agents composing society is sufficiently high, the fuzzy core - in the sense of AUBIN - will differ rather little from the 1-near nucleus - in other words - the intra-muros core. Thus, we can see, in these two steps, a convergence limit to the usual core of a cooperative game.

The very interest of the peripheric core - beyond its non-vacuity, proceeding from an "operational" solution because of the planner's intervention which always permits to balance the game, even approximately - is directly tied to the planner's part whose strategical importance was already emphasized in BILLOT (1987) where we presented some theorems about the aggregation of fuzzy preorders. The kind of cooperation which is, here, exhibited, delays however constrained -to be efficient -, because if there is no planner, the conditions of balancing of the game are identical to the classical one of the cooperative theory, even if the opportunity of a direct solution (intra muros core which is non-empty) with non-convex preorders is more easily appreciable.

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