Locally P-Convex Fuzzy Topological Algebras and Locally Multiplicatively P-Convex Fuzzy Topological Algeras (Abstract)

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In this paper, wegive the definitions of fuzzy quasi pnorm, multiplicatively P-Convex (m-p convex for short) fuzzy set over an algebra. Two kinds of particular fuzzy topological algebras are discussed here by means of the family of fuzzy quasi P-norms.

Definition 1. Let X be an algebra over number field K (K is real number field or complex number field.). To every $x_\lambda \in \widetilde{X}$ ($\widetilde{X}$ is the classical set of all fuzzy points in X) there is a real number $\|x_\lambda\|_p \geq 0$, which satisfies

(1). for every $\lambda \in (0,1]$, $\|\theta_\lambda\|_p = 0$; (2). $\|kx_\lambda\|_p = |k|^p \|x_\lambda\|_p$, $k \in K$;

(3). $\|x_\lambda + y_\lambda\|_p \leq \|x_\lambda\|_p + \|y_\lambda\|_p$; (4). if $0 < \mu \leq \lambda \leq 1$, then $\|x_\lambda\|_p \leq \|x_\mu\|_p$;

(5). for any $\varepsilon > 0$, there exists $0 < \delta < \lambda$, such that $\|x_{\lambda-\delta}\|_p < \|x_\lambda\|_p + \varepsilon$;

(6). $\|x_\lambda \cdot y_\lambda\|_p \leq \|x_\lambda\|_p \|y_\lambda\|_p$; then $\|\cdot\|_p$ is called a fuzzy quasi P-norm over the algebra X, $(X, \|\cdot\|_p)$ is called a fuzzy quasi P-normed algebra.

If (1) is changed into: for every $\lambda \in (0,1]$, if $\|x_\lambda\|_p = 0$, then $x = \theta$; then $\|\cdot\|_p$ is called afuzzy P-norm over the algebra X, $(X, \|\cdot\|_p)$ is called a fuzzy P-normed algebra.

Definition 2. Let X be an algebra, and U be a fuzzy subset of X. U is called m-p convex if it is P-convex and idempotent, absolutely m-p convex if it is absolutely P-convex and idempotent.

Properties of m-p convex sets: 1. A idempotent set's P-convex hull is a m-p convex set. 2. Let U be an abselutely m-p convex set. Then λU is an absolutely m-p convex set, where $|\lambda| \leq 1$. 3. Let U_i $(i \in I)$ be a family of m-p convex sets. Then $U = \bigcap_{i \in I} U_i$ is a m-p convex set. 4. Let $U = \bigtimes_{\alpha \in D} U_\alpha$, so that $\{U_\alpha / \alpha \in D\}$ is a family of fuzzy m-p convex (absolutely m-p convex) sets, if and only if U is a fuzzy m-p convex (absolutely m-p convex) set.

Definition 3. Let (X,T) be a fuzzy topolpgical algebra. If there exists a family of absolutely P-convex fuzzy sets $\{A^\alpha / \alpha \in D\}$, such that for every $\lambda \in (0,1]$, $0 < \varepsilon_\lambda^{(n)} < \lambda$ and $\varepsilon_\lambda^{(n)} \to 0$, $\{A^\alpha \cap (1-\lambda+\varepsilon_\lambda^{(n)})^* / \alpha \in D,\ n=1,2,\ldots\}$ is a Q-neighborhood base of θ_λ, then (X,T) is called locally P-convex fuzzy t-opological algebra; if $\{A^\alpha / \alpha \in D\}$ is a family of absolutely m-p convex fuzzy sets, then (X,T) is called locally m-p convex fuzzy topological alg-ebra.

Theorem1. Let X be an algebra, and $\{\|\cdot\|_p^\alpha / \alpha \in D\}$ be a family of fuzzy quasi P-norme over the algebra X, then a fuzzy topology T' can definied by the family of fuzzy quasi P-norms, such that (X,T') is a locally m-p convex fuzzy topological algebra.

Lemma. A fuzzy quasi P-normed algebra is a locally M-p convex fuzzy topological algebra.

Theorem2. Let (X,T) be a locally m-p convex fuzzy topological algebra, then there exists a family of fuzzy quasi P-norms in X, and the fuzzy yopolgy T' decided by the family of fuzzy quasi P-norms is equivalen-t to T.

Theorem3. Let $(X_\alpha, T_\alpha)_{\alpha \in A}$ be a family of locally P-convex (m-p convex) fuzzy topological algebra, and (X,T) be the product topological space of it. Then (X,T) is alocally P-convex (m-p convex) fuzzy topological alge-bra.

Theorem4. Let (X,T) be a fuzzy topological algebra. Then T can be decided by a sequence of fuzzy quasi P-norms, if and only if (X,T) is Q-C and locally m-p convex fuzzy topological algebra.

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