

## OPTIMAL FUZZY PROCESSING FOR RADAR DATA

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## ABSTRACT

The problem of processing radar data by means of fuzzy subset theories and possibility theories is considered. In this paper, fuzzy model of radar observations is founded and maximal clearness criterion is proposed. On the basis of this criterion we derive the structure of minimum fuzziness estimators. The computer simulation results show that it is a feasible and effective to process radar data with the aid of fuzzy techniques.

## INTRODUCTION

It is important and significant to obtain target information precisely in radar applications. Radar measures various target parameters by observing the amplitude and the phase of target echoes at given time, given frequency and given space. For improving radar's ability to acquire target information we must get a series of amplitude and phase samples which contain target information. Lees has ever indicated that we can get various sorts of target informations by differentiating amplitude and phase samples. For instance, we obtain azimuthal information by using space differential of phase sequences and shape information by using space differential of amplitude sequences. Lees's expositions are brilliant. According to this fact, the sample sequence above can be treated as a target information sequence directly:  $r_i = \bar{\theta} + e_i$  ( $i=1, 2, \dots, n$ ), where  $\bar{\theta}$  stands for the

true value of a target parameter,  $e_i$  is a additive error.

In the traditional probabilistical approach,  $\{e_i\}_{i=1}^n$  are usually modeled as a random vector which is described by a probability distribution function (p.d.f.). This is reasonable for large samples. However, this is not the case for small samples. It is easily seen that for small  $n$ ,  $\{e_i\}_{i=1}^n$  can't be described by a sole p.d.f.. That is to say, in the case of small samples, an uncertainty on distribution function will take place when the probabilistical approaches are used. In this paper, a nonnormal modeling method is presented to overcome this gap. We consider  $e_i (i=1, \dots, n)$  as a fuzzy error described by a possibility distribution function under the condition of small samples. In the light of this view, fuzzy model for radar observations is founded and maximal clearness criterion is proposed. We also derive the structure of minimum fuzziness estimators. The experimental results show that the nonnormal modeling method presented in this paper is reasonable and effective.

#### MODEL AND CRITERION

Let  $\underline{r} = [r_1, \dots, r_n]^T \in \mathbb{R}^n$  be a data vector,  $\underline{e} = [e_1, \dots, e_n] \in \mathbb{R}^n$  a error vector and  $\bar{\theta}$  the unknown true value of a target parameter. Then, fuzzy model for radar observations is as follows:

$$r_i = \bar{\theta} + e_i \quad (i=1, 2, \dots, n) \quad (1)$$

where  $\{e_i\}_{i=1}^n$  are \*-independent and constant distributed. That is,

$$\begin{aligned} \text{poss}(e_i = x) &= f(x/\sigma), \quad \forall x \in \mathbb{R}, \sigma > 0, 1 \leq i \leq n, \\ \text{poss}(\underline{e} = \underline{x}_1, \dots, e_n = x_n) &= \prod_{i=1}^n f(x_i/\sigma), \quad (\underline{x}_1 \dots x_n)^T \in \mathbb{R}^n. \end{aligned}$$

$$\underline{f} \in \mathbb{F} = \{f/f: \mathbb{R} \rightarrow [0, 1]; f(0) = 1; f(x) = f(-x), \forall x \in \mathbb{R}; x_1 > x_2 > 0 \Rightarrow f(x_1) \leq f(x_2)\}.$$

It is well known that an estimator  $T$  is a mapping which from every  $n$  and every  $r$  yields a parameter in  $\mathbb{H}$ .

i.e.,  $T: \mathbb{R}^n \rightarrow \mathbb{H}$ . In terms of above fuzzy models,  $T$  induces a fuzzy subset  $\mathcal{Q}_n^T$  of  $\mathbb{H}$  for every  $n$ . From the extension principle we can find the membership function of  $\mathcal{Q}_n^T$  easily. The minimum fuzziness element of  $\mathcal{Q}_n^T$  is one for which the membership function  $\mathcal{Q}_n^T(\cdot)$  is maximal. This enables us to create an optimal data processing method for seeking a  $\theta \in \mathbb{H}$ , the minimum fuzziness element of  $\mathcal{Q}_n^T$  induced by estimator  $T(\underline{r})$  is just  $\bar{\theta}$  (true value of a target parameter). This idea is expressed by so-called maximal clearness criterion:

Suppose that  $\underline{r} \in \mathbb{R}^n$ ,  $M(\underline{r}, \theta) = \prod_{i=1}^n f(r_i - \theta / \sigma)$ . If  $M(\dots)$  takes maximum when  $\theta = T(\underline{r})$  for  $\underline{r} \in \mathbb{R}^n$ , then  $T(\underline{r})$  is called minimum fuzziness estimation of  $\theta$ .

#### MINIMUM FUZZINESS ESTIMATOR AND RECURSIVE ALGORITHM

$\wedge$ -operation is the most useful binary operator in fuzzy algebraic system. In this paper, the structure of minimum fuzziness estimators is derived under the condition of  $* = \wedge$ . Firstly, we have a lemma as follows:

**Lemma 1:** Suppose that  $* = \wedge$ ,  $f \in F$  and  $d(\underline{r}, \theta) = \prod_{i=1}^n |r_i - \theta|$ . If  $\underline{r} \in \mathbb{R}^n$ ,  $d(\underline{r}, T(\underline{r})) = \inf_{\theta \in \mathbb{H}} d(\underline{r}, \theta)$ , Then  $T(\underline{r})$  is the minimum fuzziness estimator of  $\theta$ .

The proof is omitted because it is obvious. From this lemma, we can derive the structure of minimum fuzziness estimators.

**Theorem 1:** Suppose that  $* = \wedge$ ,  $f \in F$ ,  $d(\underline{r}, \theta) = \prod_{i=1}^n |r_i - \theta|$ . Let  $T(\underline{r}) = (r_{\min} + r_{\max}) / 2$ ,  $\forall \underline{r} \in \mathbb{R}^n$ , (2) where  $r_{\min} = \min_{i=1}^n \{r_i\}$ ,  $r_{\max} = \max_{i=1}^n \{r_i\}$  ( $\min = \wedge$ ,  $\max = \vee$ ), then,  $T(\underline{r})$  is the minimum fuzziness estimation of  $\theta$ .

The ware which realizes the operation corresponding to the eq. (2) is called minimum fuzziness estimator whose structure is shown in Fig. 1.

It is clear that the minimum fuzziness estimator consists of maximal calculator, minimum calculator and average calculator. For reducing operations and saving memory capacities,  $T(\underline{r})$  is changed into another form.

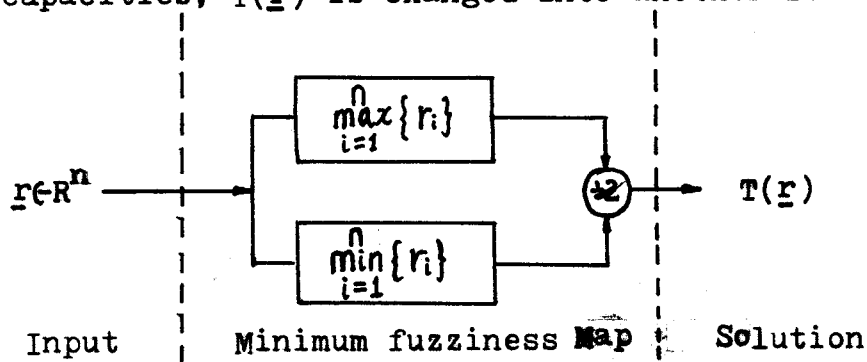


Fig.1 Minimum fuzziness estimator.

Suppose that the  $T(\underline{r}) = t_n$  when  $\underline{r} \in R^n$ ,  $r_{\min} = \min_{i=1}^n \{r_i\}$ ,  $r_{\max} = \max_{i=1}^n \{r_i\}$ , we have:

$$t_n = \begin{cases} t_{n-1} & , \quad r_{\min}^{(n-1)} \leq r_n \leq r_{\max}^{(n-1)} \\ (r_n + r_{\max}^{(n-1)})/2 & , \quad r_n \leq r_{\min}^{(n-1)} \\ (r_n + r_{\min}^{(n-1)})/2 & , \quad r_n \geq r_{\max}^{(n-1)} \end{cases} \quad (3)$$

Thus, the estimator need only store  $t_{n-1}$ ,  $r_{\max}^{(n-1)}$  and  $r_{\min}^{(n-1)}$  and need not store all  $n$  samples regardless of how large  $n$  is. When the  $n$ th sample is observed, we can get  $t_n$  by a addition operation and a shift operation.

#### SIMULATION AND CONCLUSION

The average relative error (GRE) is used for evaluating the performance of minimum fuzziness estimators in this paper. Let  $t_n^{(i)}$  be the  $i$ th estimation of the target parameters when the length  $n$  data samples are observed,  $N_e$  the experimental number of times, we define,

$$GRE = \frac{1}{N_e} \sum_{i=1}^{N_e} \left| \frac{t_n^{(i)} - \bar{\theta}}{\bar{\theta}} \right| \quad (4)$$

To evaluate the GRE performance of minimum fuzziness

estimators(FME) we have done the computer simulation experiments in which  $\theta$  represents the range of the ship targets. The experimental results for various observation length( $n$ ) and signal-noise-ratio (S/N) are listed in table1. (where  $N_e=100$  and  $\bar{\theta}=50$  miles).

Table1 Average relative error on FME

S/N \ GRE \ n	15	25	37
6 dB	0.044	0.042	0.042
S/N \ GRE \ n	20	30	40
0 dB	0.092	0.09	0.08
S/N \ GRE \ n	33	37	40
-4 dB	0.139	0.130	0.129

It can be seen from table1 that FME has good ability to suppress the measure noise and improve the measure precision. For example, by  $GRE=0.04$  we mean that the range error is equal to 2.2 miles ( $50 \times 0.044$ ), this performance is better than that of the average filter (TME), in the case of S/N equaling 6 dB and data length equaling 15. Under equal conditions, we compare the performance of FME with that of TME. The results are shown in Fig.2 and Fig.3. It is clear that FME's GRE are smaller than TME's GRE. Thus we conclude FME's performance is better than TME's performance under equal conditions.

It is our principal goal to process radar data under the condition of small samples by means of a possibility theory. In this paper, the model, criterion and structure are given to realize this goal. From the paper's researches we have several preliminary views;

(1) It is an effective and reasonable modeling method to describe the performance of radar measure errors with the aid of a possibility theory. That is to say, the uncertainty on measure error is suitably described by a possibility theory in the case of small samples;

(2) The structure of minimum fuzziness estimator is robust in the possibility distribution family  $F$ ;

(3) The minimum fuzziness estimator is simple on structure and easy on practical implement. The performance of FME is better than that of TME.

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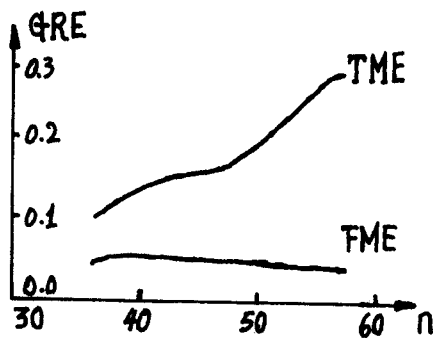


Fig.2 S/N=6dB

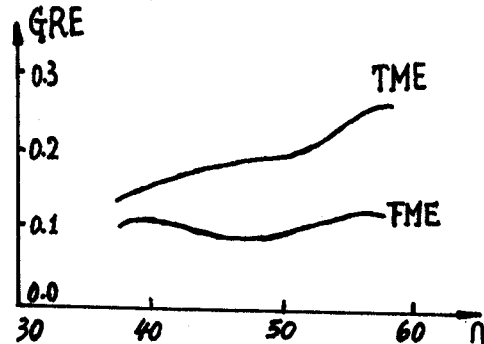


Fig.3 S/N=-4dB

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