THE EXTENSIONS OF A CLASS OF SEMI-CONTINUOUS FUZZY MEASURES

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Abstract

In this paper, we introduce the concepts of the σ-possibility measure and the CP-system and the ECP-system on a class of fuzzy sets,and show that such a σ-possibility measure must be a semi-continuous fuzzy measure. Furthermore, we establish a necessary and sufficient condition for that a σ-possibility measure on a class of fuzzy sets may be extended, and prove some extension theorems of such σ-possibility measures, and therefore, we solve the extension problem of a class of semi-continuous fuzzy measures.

Keywords: Fuzzy measure, σ-possibility measure, CP-system, ECP-system, Weak plump field.

§1 Introduction

On a class of classical sets, the possibility measure introduced by L.A.Zadeh[8] is a special semi-continuous fuzzy measure, and it is difficult to establish a general extension theory for the semi-continuous fuzzy measures though some extension problems of possibility measures have been better solved (cf.[1,2,6,7]), in order to find out as many extendable semi-continuous fuzzy measures as possible, Qiao[5] introduced and studied the σ-possibility measures. If these similar problems are discussed on a class of fuzzy sets, it is undoubted to arise a great many difficulties. In this paper, we shall introduce the concept of the semi-continuous fuzzy measure on a class of fuzzy sets and shall solve the extension problem of such a class of semi-continuous fuzzy measures.

Throughout this paper, suppose that L is an infinitely distributive complete lattice, in other words, the lattice L satisfies the following conditions:

(1) For any $H \subset L$, $\bigwedge_{h\in H} h$ and $\bigvee_{h\in H} h$ are existent in L;

(2) For any $H \subset L$, $a \in L$, we have

$$a \vee (\bigwedge_{h\in H} h) = \bigwedge_{h\in H} (a \vee h), \quad a \wedge (\bigvee_{h\in H} h) = \bigvee_{h\in H} (a \wedge h).$$

And let X be a nonempty set, $\mathfrak{F}_L(X) = \{\underset{\sim}{A} ;\ \underset{\sim}{A}: X \longrightarrow L\}$ be the class of all L-fuzzy subsets of X, $\underset{\sim}{\mathfrak{B}}$ and $\underset{\sim}{\mathfrak{B}}^*$ and $\underset{\sim}{\mathfrak{D}}$ be nonempty subclasses of $\mathfrak{F}_L(X)$, $\{\underset{\sim}{A}_n\}$ be a finite or infinite sequence of L-fuzzy subsets of X, and we make the conventions:

$\sup_{t\in\phi}\{a_t;\ a_t \in [0,\infty)\} = 0$, $\bigcup_{t\in\phi}\{\cdot\} = \underset{\sim}{\phi}$, where $\underset{\sim}{\phi}$ is the smallest element of $\mathfrak{F}_L(X)$.

§2 Semi-continuous Fuzzy Measure and σ-Possibility Measure on Class of Fuzzy Sets

Definition2.1 A mapping $\underset{\sim}{\mu}: \underset{\sim}{\mathfrak{D}} \longrightarrow [0,a]$ (where a is an arbitrarily positive real number or $+\infty$) is called a semi-continuous fuzzy measure on $\underset{\sim}{\mathfrak{D}}$, if it satisfies the following conditions:

(SFM1) $\underset{\sim}{\mu}(\underset{\sim}{\phi}) = 0$, if $\underset{\sim}{\phi} \in \underset{\sim}{\mathfrak{D}}$;

(SFM2) For any $\underset{\sim}{A}, \underset{\sim}{B} \in \underset{\sim}{\mathfrak{D}}$, if $\underset{\sim}{A} \subset \underset{\sim}{B}$, then $\underset{\sim}{\mu}(\underset{\sim}{A}) \leq \underset{\sim}{\mu}(\underset{\sim}{B})$;

(SFM3) Whenever $\{\underset{\sim}{A}_n\} \subset \underset{\sim}{\mathfrak{D}}$, $\bigcup_{n=1}^{\infty} \underset{\sim}{A}_n \in \underset{\sim}{\mathfrak{D}}$, $\underset{\sim}{A}_n \subset \underset{\sim}{A}_{n+1}$, $n=1,2,\cdots$, then $\underset{\sim}{\mu}(\bigcup_{n=1}^{\infty} \underset{\sim}{A}_n) = \lim_{n\to\infty} \underset{\sim}{\mu}(\underset{\sim}{A}_n)$.

It is easy to see that the fuzzy measure studied in [3,4] is a semi-continuous fuzzy measure.

Definition2.2 A mapping $\underset{\sim}{\pi}: \underset{\sim}{\mathfrak{D}} \longrightarrow [0,a]$ (where $0 < a \leq \infty$) is said to be a σ-possibility measure on $\underset{\sim}{\mathfrak{D}}$, if and only if the following holds:

(σP1) $\underset{\sim}{\pi}(\underset{\sim}{\phi}) = 0$, if $\underset{\sim}{\phi} \in \underset{\sim}{\mathfrak{D}}$;

(σP2) $\underset{\sim}{\pi}$ is σ-fuzzy additive, that is, for any $\{\underset{\sim}{A}_n\} \subset \underset{\sim}{\mathfrak{D}}$, if

$\bigcup_n A_n \in \mathfrak{L}$, then $\pi(\bigcup_n A_n) = \sup_n \pi(A_n)$.

When L={o,1}, such π is a σ-possibility measure introduced in[5], and therefore, it is a generalization of the possibility measure.

The condition (σP2) implies the monotonicity condition:

(σP3) For any $A, B \in \mathfrak{L}$, if $A \subset B$, then $\pi(A) \leqslant \pi(B)$.

In the following, we shall discuss the relations between σ-possibility measures and semi-continuous fuzzy measures.

Theorem2.3 An arbitrary σ-possibility measure on $\mathfrak{L}$ is a semi-continuous fuzzy measure on $\mathfrak{L}$.

Proof. Suppose that π is a σ-possibility measure on $\mathfrak{L}$, it is clear that π satisfies the conditions (SFM1) and (SFM2). Now we prove that π meets the condition (SFM3) as well.

For any increasing sequence $\{A_n\} \subset \mathfrak{L}$ with $\bigcup_n A_n \in \mathfrak{L}$, by using conditions ($\sigma$P3) and ($\sigma$P2), we have $\pi(A_n) \leqslant \pi(A_{n+1}), n=1,2,\cdots$, and $\lim_{n\to\infty} \pi(A_n) = \sup_n \pi(A_n) = \pi(\bigcup_n A_n)$.

That is, π is a semi-continuous fuzzy measure on $\mathfrak{L}$.

The following theorem gives a necessary and sufficient condition for that a semi-continuous fuzzy measure on $\mathfrak{L}$ turns into a σ-possibility measure on $\mathfrak{L}$.

Theorem2.4 Let μ be a semi-continuous fuzzy measure on $\mathfrak{L}$, $\mathfrak{L}$ be closed under finite unions. μ is a σ-possibility measure on $\mathfrak{L}$, if and only if μ satisfies $\mu(A \cup B) = \mu(A) \vee \mu(B)$ whenever $A, B \in \mathfrak{L}$.

Proof. The necessity is obvious. Now we prove the sufficiency.

Evidently, μ satisfies the condition (σP1).

Let $\mu(A \cup B) = \mu(A) \vee \mu(B)$ for any $A, B \in \mathfrak{L}$. If $\{A_n\} \subset \mathfrak{L}$ and $\bigcup_n A_n \in \mathfrak{L}$, denote $B_m = \bigcup_{n=1}^{m} A_n$, m=1,2,$\cdots$, then $\{B_m\}$ is an increasing sequence in $\mathfrak{L}$ and $\bigcup_n A_n = \bigcup_m B_m$, thus we have

$$\underset{\sim}{\mu}(\bigcup_n \underset{\sim}{A}_n)=\underset{\sim}{\mu}(\bigcup_m \underset{\sim}{B}_m)=\lim_{m\to\infty}\underset{\sim}{\mu}(\underset{\sim}{B}_m)=\sup_m \underset{\sim}{\mu}(\underset{\sim}{B}_m)=\sup_m [\sup_{1\leqslant n\leqslant m}\underset{\sim}{\mu}(\underset{\sim}{A}_n)]=\sup_n \underset{\sim}{\mu}(\underset{\sim}{A}_n).$$

That is to say, $\underset{\sim}{\mu}$ is a σ-possibility measure on $\underset{\sim}{\mathfrak{D}}$.

§3 A Extendable Necessary and Sufficient Condition

Let $U(\underset{\sim}{\mathfrak{D}})=\{\bigcup_n \underset{\sim}{A}_n;\ \{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{D}}\}$, and we make the convention: $\underset{\sim}{\phi}=\bigcup_{t\in\phi}\{\cdot\}\in U(\underset{\sim}{\mathfrak{D}})$, then $U(\underset{\sim}{\mathfrak{D}})$ is the smallest class which is both including $\underset{\sim}{\mathfrak{D}}$ and closed under arbitrary countable infinite unions. In the section, we shall establish a necessary and sufficient condition for that a σ-possibility measure on $\underset{\sim}{\mathfrak{D}}$ may be extended to $U(\underset{\sim}{\mathfrak{D}})$ uniquely.

Definition3.1 A mapping $\underset{\sim}{\mu}:\underset{\sim}{\mathfrak{D}}\longrightarrow[o,a]$ (where $o<a\leqslant\infty$) is called consistent, if for any $\underset{\sim}{A}\in\underset{\sim}{\mathfrak{D}}$, whenever $\underset{\sim}{A}\subset\bigcup_n \underset{\sim}{A}_n$, where $\{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{D}}$, we have $\underset{\sim}{\mu}(\underset{\sim}{A})\leqslant\sup_n \underset{\sim}{\mu}(\underset{\sim}{A}_n)$.

Theorem3.2 A σ-possibility measure $\underset{\sim}{\pi}$ on $\underset{\sim}{\mathfrak{D}}$ may be extended to a σ-possibility measure $\underset{\sim}{\pi}'$ on $U(\underset{\sim}{\mathfrak{D}})$ uniquely, if and only if $\underset{\sim}{\pi}$ is consistent.

Proof. Necessity: Let $\underset{\sim}{\pi}$ may be extended to a σ-possibility measure $\underset{\sim}{\pi}'$ on $U(\underset{\sim}{\mathfrak{D}})$ uniquely.

For any $\underset{\sim}{A}\in\underset{\sim}{\mathfrak{D}}$, if $\underset{\sim}{A}\subset\bigcup_n \underset{\sim}{A}_n$, where $\{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{D}}$, then $\underset{\sim}{A}$, $\bigcup_n \underset{\sim}{A}_n\in U(\underset{\sim}{\mathfrak{D}})$. Thus

$$\underset{\sim}{\pi}(\underset{\sim}{A})=\underset{\sim}{\pi}'(\underset{\sim}{A})\leqslant\underset{\sim}{\pi}'(\bigcup_n \underset{\sim}{A}_n)=\sup_n \underset{\sim}{\pi}'(\underset{\sim}{A}_n)=\sup_n \underset{\sim}{\pi}(\underset{\sim}{A}_n).$$

That is, $\underset{\sim}{\pi}$ is consistent.

Sufficiency: Let $\underset{\sim}{\pi}$ be consistent. For any $\underset{\sim}{B}\in U(\underset{\sim}{\mathfrak{D}})$, then there exists $\{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{D}}$ such that $\underset{\sim}{B}=\bigcup_n \underset{\sim}{A}_n$. We define

$$\underset{\sim}{\pi}'(\underset{\sim}{B})\triangleq\sup_n \underset{\sim}{\pi}(\underset{\sim}{A}_n).$$

This definition is unambiguous. In fact, if $\underset{\sim}{B}=\bigcup_m \underset{\sim}{B}_m$, where $\{\underset{\sim}{B}_m\}\subset\underset{\sim}{\mathfrak{D}}$, then $\underset{\sim}{A}_n\subset\underset{\sim}{B}=\bigcup_m \underset{\sim}{B}_m$ for any $\underset{\sim}{A}_n$, thus $\underset{\sim}{\pi}(\underset{\sim}{A}_n)\leqslant\sup_m \underset{\sim}{\pi}(\underset{\sim}{B}_m)$, and therefore, $\sup_n \underset{\sim}{\pi}(\underset{\sim}{A}_n)\leqslant\sup_m \underset{\sim}{\pi}(\underset{\sim}{B}_m)$. Analogously, we may show the converse inequality. Consequently,

$$\sup_n \underset{\sim}{\pi}(\underset{\sim}{A}_n)=\sup_m \underset{\sim}{\pi}(\underset{\sim}{B}_m).$$

When $\underset{\sim}{B}\in\underset{\sim}{\mathfrak{D}}$, we have $\underset{\sim}{\pi}'(\underset{\sim}{B})=\sup_n \underset{\sim}{\pi}(\underset{\sim}{A}_n)=\underset{\sim}{\pi}(\bigcup_n \underset{\sim}{A}_n)=\underset{\sim}{\pi}(\underset{\sim}{B})$.

Furthermore, we are going to show that $\underset{\sim}{\pi}'$ is a σ-possibility measure on $U(\underset{\sim}{\mathfrak{D}})$.

(1) By the conventions: $\bigcup_{t\in\Phi}\{\cdot\}=\underset{\sim}{\Phi}$, $\sup_{t\in\Phi}\{a_t;\ a_t\in[0,\infty]\}=0$, then $\underset{\sim}{\pi}'(\underset{\sim}{\Phi})=\sup_{t\in\Phi}\{\cdot\}=0$, namely, $\underset{\sim}{\pi}'$ satisfies the condition (σP1).

(2) For any $\{\underset{\sim}{B}_n\}\subset U(\underset{\sim}{\mathfrak{D}})$, $\underset{\sim}{B}_n=\bigcup_m \underset{\sim}{A}_m^{(n)}$, where $\{\underset{\sim}{A}_m^{(n)}\}\subset\underset{\sim}{\mathfrak{D}}$, $n=1,2,\cdots$, since $\bigcup_n \underset{\sim}{B}_n=\bigcup_{nm}\bigcup \underset{\sim}{A}_m^{(n)}$, then

$$\underset{\sim}{\pi}'(\bigcup_n \underset{\sim}{B}_n)=\sup_{m,n}\underset{\sim}{\pi}(\underset{\sim}{A}_m^{(n)})=\sup_n[\sup_m \underset{\sim}{\pi}(\underset{\sim}{A}_m^{(n)})]=\sup_n \underset{\sim}{\pi}'(\underset{\sim}{B}_n).$$

That is to say, $\underset{\sim}{\pi}'$ satisfies the condition (σP2) too. Thus, $\underset{\sim}{\pi}'$ is a σ-possibility measure on $U(\underset{\sim}{\mathfrak{D}})$.

Finally, we prove the uniqueness of $\underset{\sim}{\pi}'$.

If $\underset{\sim}{\pi}''$ is another extension of $\underset{\sim}{\pi}$ on $U(\underset{\sim}{\mathfrak{D}})$, then for any $\underset{\sim}{B}=\bigcup_n \underset{\sim}{A}_n\in U(\underset{\sim}{\mathfrak{D}})$, where $\{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{D}}$, we have

$$\underset{\sim}{\pi}'(\underset{\sim}{B})=\sup_n \underset{\sim}{\pi}(\underset{\sim}{A}_n)=\sup_n \underset{\sim}{\pi}''(\underset{\sim}{A}_n)=\underset{\sim}{\pi}''(\bigcup_n \underset{\sim}{A}_n)=\underset{\sim}{\pi}''(\underset{\sim}{B}).$$

<u>Remark:</u> If take $L=\{0,1\}$, the above Theorem3.2 coincides with the Theorem2.2 given in[5].

§4 CP-system, ECP-system and Weak Plump Field

On a class of classical sets, Qiao[5,6] drew the concepts of the CP-system and the ECP-system from the class of all atoms of a set class introduced in[1,2,7]. In order to discuss some other extension theorems of σ-possibility measures given in §2, in the section, we shall give several similar concepts on a class of fuzzy sets.

<u>Definition4.1</u> A nonempty class $\underset{\sim}{\mathfrak{B}}$ of fuzzy sets is said to be an ECP-system, if the following conditions are satisfied:

(ECP1) The exchangeability. For any $\underset{\sim}{A}_1,\cdots,\underset{\sim}{A}_n\in\underset{\sim}{\mathfrak{B}}$, then there exist $\underset{\sim}{B}_1,\cdots,\underset{\sim}{B}_m\in\underset{\sim}{\mathfrak{B}}$ such that $\bigcap_{i=1}^n \underset{\sim}{A}_i=\bigcup_{j=1}^m \underset{\sim}{B}_j$;

(ECP2) The closeness for the partial covering. For any $\underset{\sim}{A}\in\underset{\sim}{\mathfrak{B}}$, if $\underset{\sim}{A}\subset\bigcup_n \underset{\sim}{A}_n$, where $\{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{B}}$, then there exists one subset $\{\underset{\sim}{A}_t;\ t\in T\}$ of $\{\underset{\sim}{A}_n\}$ such that $\underset{\sim}{A}\subset\bigcup_{t\in T}\underset{\sim}{A}_t\in\underset{\sim}{\mathfrak{B}}$.

If a class of fuzzy sets only satisfies the condition (ECP2), it is called a CP-system.

In the following, we shall always denote the ECP-system (resp. the CP-system) by $\underset{\sim}{\mathfrak{B}}$ (resp. $\underset{\sim}{\mathfrak{B}}^*$).

Evidently, if a class of fuzzy sets is closed under countable infinite unions, then it is a CP-system. An arbitrary fuzzy σ-algebra introduced in[3,4] is an ECP-system. The class of all atoms of a set class given in[1,2,7] is an ECP-system. If $\underset{\sim}{\mathfrak{B}}$ is closed under finite unions, then it is closed under finite intersections, but the converse proposition is not true.

In fact, if $\underset{\sim}{A}$,$\underset{\sim}{B}\in\mathfrak{F}(X)$, $\underset{\sim}{A}\cap\underset{\sim}{B}=\underset{\sim}{\phi}$, and if there is no inclusion relation between $\underset{\sim}{A}$ and $\underset{\sim}{B}$, then $\underset{\sim}{\mathfrak{B}}=\{\underset{\sim}{\phi},\underset{\sim}{A},\underset{\sim}{B}\}$ is an ECP-system, and it is closed under finite intersections, but it is not closed under finite unions.

<u>Definition4.2</u> A nonempty class of fuzzy sets is called a weak plump field, if it is closed under arbitrary finite intersections and arbitrary countable infinite unions.Denote the smallest weak plump field including $\underset{\sim}{\mathfrak{D}}$ by $W(\underset{\sim}{\mathfrak{D}})$.

<u>Proposition4.3</u> $W(\underset{\sim}{\mathfrak{D}})=\{\bigcup_{t\in T}\bigcap_{s\in S_t}\underset{\sim}{A}_s\ ;\ \underset{\sim}{A}_s\in\underset{\sim}{\mathfrak{D}}\}$, where T is an arbitrary finite or countable infinite index set, and S_t is an arbitrary finite index set whenever $t\in T$.

Proof. Write $\underset{\sim}{\mathcal{A}}=\{\bigcup_{t\in T}\bigcap_{s\in S_t}\underset{\sim}{A}_s\ ;\ \underset{\sim}{A}_s\in\underset{\sim}{\mathfrak{D}}\}$.

First of all, we verify that $\underset{\sim}{\mathcal{A}}$ is a weak plump field.

(1) Let $\{\underset{\sim}{B}_n\}\subset\underset{\sim}{\mathcal{A}}$, where $\underset{\sim}{B}_n=\bigcup_{t\in T_n}\bigcap_{s\in S_t}\underset{\sim}{A}_s^{(n)}$, $\underset{\sim}{A}_s^{(n)}\in\underset{\sim}{\mathfrak{D}}$, T_n is an arbitrary finite or countable infinite index set, S_t is an arbitrary finite index set, then

$$\bigcup_n\underset{\sim}{B}_n=\bigcup_n\Big(\bigcup_{t\in T_n}\bigcap_{s\in S_t}\underset{\sim}{A}_s^{(n)}\Big)=\bigcup_n\bigcup_{t\in T_n}\Big(\bigcap_{s\in S_t}\underset{\sim}{A}_s^{(n)}\Big)=\bigcup_{t\in T'}\bigcap_{s\in S_t}\underset{\sim}{A}_s^{(n)}\in\underset{\sim}{\mathcal{A}}\ ,$$

where $T'=\bigcup_n T_n$, namely, $\underset{\sim}{\mathcal{A}}$ is closed under countable infinite unions.

(2) Let $\underset{\sim}{D}_1=\bigcup_{t\in T}\bigcap_{s\in S_t}\underset{\sim}{A}_s, \underset{\sim}{D}_2=\bigcup_{j\in J}\bigcap_{i\in I_j}\underset{\sim}{B}_i\in\underset{\sim}{\mathcal{A}}$, where $\underset{\sim}{A}_s, \underset{\sim}{B}_i\in\underset{\sim}{\mathfrak{D}}$,

T and J are two arbitrary finite or countable infinite index sets, and for any $t\in T$, $j\in J$, S_t and I_j are two arbitrary finite index sets. Observe that L is an infinitely distributive complete lattice, we have

$$\underset{\sim}{D}_1\cap\underset{\sim}{D}_2=(\bigcup_{t\in T}\bigcap_{s\in S_t}\underset{\sim}{A}_s)\cap(\bigcup_{j\in J}\bigcap_{i\in I_j}\underset{\sim}{B}_i)=\bigcup_{j\in J}[(\bigcup_{t\in T}\bigcap_{s\in S_t}\underset{\sim}{A}_s)\cap(\bigcap_{i\in I_j}\underset{\sim}{B}_i)]$$

$$=\bigcup_{j\in J}[\bigcup_{t\in T}[(\bigcap_{s\in S_t}\underset{\sim}{A}_s)\cap(\bigcap_{i\in I_j}\underset{\sim}{B}_i)]]\in\underset{\sim}{\mathcal{A}} .$$

That is, $\underset{\sim}{\mathcal{A}}$ is closed under finite intersections.Therefore, $\underset{\sim}{\mathcal{A}}$ is a weak plump field.

Furthermore, since $\underset{\sim}{\mathfrak{D}}\subset\underset{\sim}{\mathcal{A}}$, then $W(\underset{\sim}{\mathfrak{D}})\subset\underset{\sim}{\mathcal{A}}$, and it is clear that $\underset{\sim}{\mathcal{A}}\subset W(\underset{\sim}{\mathfrak{D}})$. Consequently, $\underset{\sim}{\mathcal{A}}=W(\underset{\sim}{\mathfrak{D}})$.

Proposition4.4 $W(\underset{\sim}{\mathfrak{B}})=U(\underset{\sim}{\mathfrak{B}})$.

Proof. Using Proposition4.3 and the definitions of $\underset{\sim}{\mathfrak{B}}$ and $U(\underset{\sim}{\mathfrak{B}})$, it is easy to prove this conclusion.

§5 Extension Theorems of σ-Possibility Measures on Class of Fuzzy Sets

In the section, several extension theorems of σ-possibility measures will be proved, when $L=\{o,1\}$, these conclusions coincide with the relevant results presented in[5].

Theorem5.1 An arbitrary σ-possibility measure $\underset{\sim}{\pi}$ on $\underset{\sim}{\mathfrak{B}}^*$ may be extended to a σ-possibility measure $\underset{\sim}{\pi}'$ on $U(\underset{\sim}{\mathfrak{B}}^*)$ uniquely.

Proof. We show that $\underset{\sim}{\pi}$ is consistent.

Since $\underset{\sim}{\mathfrak{B}}^*$ is a CP-system, for any $\underset{\sim}{A}\in\underset{\sim}{\mathfrak{B}}^*$, if $\underset{\sim}{A}\subset\bigcup_n\underset{\sim}{A}_n$, where $\{\underset{\sim}{A}_n\}\subset\underset{\sim}{\mathfrak{B}}^*$, then there exists one subset $\{\underset{\sim}{A}_t;\ t\in T\}$ of $\{\underset{\sim}{A}_n\}$ such that $\underset{\sim}{A}\subset\bigcup_{t\in T}\underset{\sim}{A}_t\in\underset{\sim}{\mathfrak{B}}^*$, thus

$$\underset{\sim}{\pi}(\underset{\sim}{A})\leqslant\underset{\sim}{\pi}(\bigcup_{t\in T}\underset{\sim}{A}_t)=\sup_{t\in T}\underset{\sim}{\pi}(\underset{\sim}{A}_t)\leqslant\sup_n\underset{\sim}{\pi}(\underset{\sim}{A}_n).$$

That is to say, $\underset{\sim}{\pi}$ is consistent. Using Theorem3.2, $\underset{\sim}{\pi}$ may be extended to a σ-possibility measure $\underset{\sim}{\pi}'$ on $U(\underset{\sim}{\mathfrak{B}}^*)$ uniquely.

Theorem5.2 An arbitrary σ-possibility measure $\underset{\sim}{\pi}$ on $\underset{\sim}{\mathfrak{B}}$ may be extended to a σ-possibility measure $\underset{\sim}{\pi}'$ on $W(\underset{\sim}{\mathfrak{B}})$ uniquely.

Proof. By Proposition4.4 and Theorem5.1, it is easy to verify that the conclusion is true.

Theorem2.3 and Theorem5.1 and Theorem5.2 show that all of σ-possibility measures in the class of all semi-continuous fuzzy measures may be extended from $\underset{\sim}{\mathfrak{B}}^*$ (resp. $\underset{\sim}{\mathfrak{B}}$) to $U(\underset{\sim}{\mathfrak{B}}^*)$ (resp. $W(\underset{\sim}{\mathfrak{B}})$) uniquely. The following theorem gives a sufficient condition for that a semi-continuous fuzzy measure can be extended uniquely.

Theorem5.3 Let $\underset{\sim}{\mathfrak{B}}$ be closed under finite unions, $\underset{\sim}{\mu}$ be an arbitrary semi-continuous fuzzy measure on $\underset{\sim}{\mathfrak{B}}$ such that $\underset{\sim}{\mu}(\underset{\sim}{A}\cup\underset{\sim}{B})=\underset{\sim}{\mu}(\underset{\sim}{A})\vee\underset{\sim}{\mu}(\underset{\sim}{B})$ for any $\underset{\sim}{A}$,$\underset{\sim}{B}\in\underset{\sim}{\mathfrak{B}}$, then $\underset{\sim}{\mu}$ may be extended to a semi-continuous fuzzy measure $\underset{\sim}{\mu}'$ on $W(\underset{\sim}{\mathfrak{B}})$ uniquely.

Proof. It follows, from Theorem2.4 and Theorem5.2 and Theorem2.3, that this conclusion is true.

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