TOPOLOGIES ON THE PRODUCTS OF PARTICLE LATTICES

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-Summary-

Following [2] , the products of topological particle lattices are given here. Some properties of the topologies on the products of topological particle lattices are also given.

When $\{(L_i, \delta_i): i \in I\}$ is a family of topological particle lattices (refer to [1—3]), the topology $\otimes \delta_i$ on the product $\otimes L_i$ is the topology which induced by the set $\{p_j^{-1}(U_j): i \in I \text{ and } U_i \in \delta_i\}$. We shall use $\otimes(L_i, \delta_i)$ to denote the $(\otimes L_i, \otimes \delta_i)$. The p_j is the projecting from $\otimes L_i$ to L_j.

<u>Theorem 1</u>. In $\otimes(L_i, \delta_i)$, if $J \subseteq I$ and $W_j \in L_j$ for $j \in J$, then $(\bigwedge_J p_j^{-1}(W_j))^- = \bigwedge_J p_j^{-1}(W_j^-)$.

<u>Theorem 2</u>. If e is a particle in $\otimes(L_i, \delta_i)$, then $e^- = \langle p_i(e) \rangle$.

Theorem 3. A function $f: (L, \delta) \longrightarrow \otimes(L_i, \delta_i)$ is continuous iff each $p_j \circ f: (L, \delta) \longrightarrow (L_i, \delta_i)$ is continuous.

<u>Theorem 4</u>. If $\{(L_i, \delta_i): i \in I\}$ is a family of topological particle lattices, then both $f_{k,p}: (L_k, \delta_k) \longrightarrow \otimes(L_i, \delta_i)$ and $F_{J,p}: \otimes_J(L_j, \delta_j) \longrightarrow \otimes_I(L_i, \delta_i)$ (refer to [2]) are

continuous.

Definition 1. Let (L, δ) be a topological particle lattice. We have the following concepts:

(1) (L, δ) is a C_1 if it has a countable base of $\eta(p)$ for any particle p in (L, δ). It is called a C_2 if δ has a countable base.

(2) (L, δ) is a T_0, if when p and q are not less than a same particle in L, $P \in \eta(p)$ and $Q \in \eta(q)$ such that $q \leq P$ or $p \leq Q$. It is a T_1 , if for each $q \in P(L)$ there is a particle p in L such that for every particle r $r \not\leq p \Longrightarrow \exists R \in \eta(r)$ with $q \leq R$. (It is easy to see that (L, δ) is a T_1 iff each particle is a closed element in (L, δ)). It is a T_2, if $p \wedge q = 0$ implies that there are $U \in \eta(p)$ and $V \in \eta(q)$ such that $U \vee V = 1$.

Definition 2. (L, δ) is compact iff any one of the following condition holds:

(1) Any net has an accumulation particle in (L, δ).

(2) Any net has a subnet which converge to a particle.

(3) Any filter has an accumulation particle in (L, δ).

(4) For each $\delta_0 \subseteq \delta$, if $\bigwedge \delta_0 = 0$ there must be finite elements $B_1, \ldots, B_m$ in δ_0 such that $B_1 \wedge \ldots \wedge B_m = 0$.

The definition of connectedness is similar to that for CDTL's given in [3].

Theorem 5. If $\{(L_i, \delta_i)\}$ is a family of topological particle lattices, then we have the following results:

(1) $\bigotimes (L_i, \delta_i)$ is connected iff each (L_i, δ_i) is connected.

(2) $\bigotimes (L_i, \delta_i)$ is a C_1, iff there is a countable index set $I_0 \subseteq I$ such that each (L_i, δ_i) is a C_1 for $i \in I_0$ and the

others are trivial.

(3) $\otimes (L_i, \delta_i)$ is a C_2, iff there is a countable index set $I_0 \subseteq I$ such that for each $i \in I_0$ (L_i, δ_i) is a C_2 and the others are trivial.

(4) $\otimes (L_i, \delta_i)$ is a T_0 iff each (L_i, δ_i) is a T_0.

(5) $\otimes (L_i, \delta_i)$ is a T_1 iff each (L_i, δ_i) is a T_1.

(6) $\otimes (L_i, \delta_i)$ is a T_2 iff each (L_i, δ_i) is a T_2.

(7) $\otimes (L_i, \delta_i)$ is compact iff each (L_i, δ_i) is compact.

References

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