PRODUCTS OF PARTICLE LATTICES

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Following [1], we introduce the product of a family of particle lattices. Some useful properties are given here.

Let $\{L_i: i\in I\}$ be a family of particle lattices [1]. We use (p_i) to denote the element p in $\prod P(L_i)$ which satisfying $p(i) = p_i$ for $i\in I$ (P(L) is the collection of all particles in L). And $<p_i>$ denotes the subset $\{(q_i): q_i\in P(L_i)$ and $q_i\leq p_i\}$ of $\prod P(L_i)$, and $\otimes_I$ denotes the set $\{<p_i> : p_i\in P(L_i)$ for $i\in I\}$

Definition 1. The product $\otimes L_i$ of a family of particle lattices $\{L_i: i\in I\}$ is the collection of the elements which satisfying the following condition:

$B \in \otimes L_i$ iff $p_k \leq \bigvee\{q_k: (q_i)\in B$ and $q_i\geq p_i$ for $i\neq k\}$ implies that $(p_i)\in B$ for each $k\in I$.

The order $\leq$ in L is the "$\subseteq$".

Proposition 1. If $B\in\otimes L_i$, $(p_i)\in B$ and $q_i\leq p_i$ for each i in I, then $(q_i)\in B$.

Theorem 1. $\otimes L_i$ is a particle lattice, $\otimes_I\subseteq P(\otimes L_i)$, and $B = \bigvee\{<p_i> : (P_i)\in B\}$ holds fot each $B\in\otimes L_i$.

Definition 2. The mapping $p_j\colon \bigotimes L_i \longrightarrow L_j$ defined by $p_j(E) = \bigvee\{q_j\colon (q_i)\in E\}$ is called the projecting mapping from $\bigotimes L_i$ to L_j.

It is easy to see that such p_j's are functions (refer to [2, Def. 1.1 and Th. 1.2]). And we have the following results.

Theorem 2. For $\bigotimes L_i$, if U_i and V_i are two elements in L_i for each $i\in I$ and $\bigwedge_I p_i^{-1}(U_i) \leqslant \bigwedge_I p_i^{-1}(V_i)$, then $U_i\leqslant V_i$ holds for each $i\in I$.

Theorem 3. For $\bigotimes L_i$, if $p_k^{-1}(U_k) \leqslant \bigvee_J p_j^{-1}(V_j)$ holds for $k\in J\subseteq I$ (in which the V_j is not the greatest element in L_j for $j\in J$), then $U_k\leq V_k$.

Theorem 4. If e is a particle in $\bigotimes L_i$, then for any $j\in I$ $p_j(e)\in P(L_j)$, and $e\leqslant\bigvee_{k=1}^m p_{i_k}^{-1}(W_{i_k})$ iff $\langle p_i(e)\rangle\leqslant\bigvee_{k=1}^m p_{i_k}^{-1}(W_{i_k})$ for any $i_k\in I$ and $W_{i_k}\in L_{i_k}$.

Given a $k\in I$ and a $p_i\in P(L_i)$ for $i\neq k$ and $i\in I$, we define a mapping $f_{k,p}\colon L_k \longrightarrow \bigotimes L_i$ as follows:

$$f_{k,p}(U) = \{(q_i)\colon q_k\leq U \text{ and } q_i\leqslant p_i \text{ for } i\neq k\}$$

where $U\in L_k$.

Let J be a subset of the index set I. Given a $(p_j)_{J^c}$ in the product $\prod_{J^c} P(L_j)$, the mapping $F_{J,p}\colon \bigotimes_J L_j \longrightarrow \bigotimes_I L_i$ is defined as follows:

$$F_{J,p}(W) = \{(q_i)\colon (q_j)_J\in W \text{ and } q_i\leqslant p_i \text{ for } i\in I\sim J\}$$

for each $W\in \bigotimes_J L_j$. (Here, $J^c = I\sim J$)

Theorem 5. The two mappings $f_{k,p}\colon L_k\longrightarrow \bigotimes L_i$ and $F_{J,p}\colon \bigotimes_J L_j\longrightarrow \bigotimes_I L_i$ are functions, and the inverse of them satifying

$$f_{k,p}^{-1}(B) = \bigvee\{q_k\colon (p_i)\in B \text{ and } q_i\geqslant p_i \text{ for } i\neq k\} \text{ for } B\in\bigotimes L_i$$

and

$$F_{J,p}^{-1}(E) = \{(q_j)_J : (q_i) \in E \text{ and } q_i \geq p_i \text{ for } i \in I \sim J\} \text{ for } E \in \otimes_I L_i.$$

References

[1] X. D. Zhao, Fuzzy particle lattices and fuzzy topological particle lattices, Preprints of Second IFSA Congress, 40-43.

[2] X. D. Zhao, Connectedness on fuzzy topological spaces, FSS, 20-(1986), 223-240.