On Contradiction Sets

SHI Jian-Sheng
Dept. of Chinese and Literature
Jilin University
Changchun City
Jilin Province

Abstract

My exploring in two ways: One is abstracting a new mathematical model from extensive contents of the objective material world; the other is taking the way different from all the previous ones, which is from fundamental mathematical research; and from this a new concept " contradiction sets" is put forward.It regards that the essence of fuzziness as a special representing form of contradiction.

Here the definition,classification,operation and its application of contradiction sets are given explicitly, particular attention being paid to compare it with classical sets and fuzzy sets at the same time of discussing the distinguishing characteristics of contradiction sets.

Therefore , "contradiction sets" is a new theory of sets.

Keywords

essence of fuzziness definition of contradiction sets vision-angle extensive-class distinguishing characteristics research of proposition's universe

1. Introduction

Man can discover the contradiction in a lot of substances in the material world. It believes that there would not be world without contradiction. It is usually contradictory as a whole, and not a pure and ideal one. For example,taking a group of people as the object of observation,we can classify it according to the ages, and are got child,young man and the adult and the old. And we also can classify it into taller,shorter and the medium according to the reight of the man bodies. It is not conjuncturing up but reality. An famous american writer of popular science, I. Athemouf has suggested such a fact in his books: No matter we view the world from the vast or ting,the law is always that the man discovered a neutron, then the anti-neutron and discovered a proton , then the anti-proton.We study the galaxy , and in the meantime,we also the anti-galaxy.The fact has been consented by many scientific departments such as phyzical chemistry etc, and some has been wiserly trying to make use of it serving the man self. But in the research of mathematics,the negative attitude to still be taken.

How do we know fuzziness? It is concerned with essence of fuzziness.The present writer had given a definition of fuzziness. I regard,the essence of fuzziness is that the limit among every contradiction aspects in entity is not clear.So, it is a contradiction phenomena.For example,the concept of "fatness",exists in a unity which includes a normal man and a thin man.Such,how much of fatness is more than a normal man,the concept of "fatness" could be established? This degree could not be mastered.Well,the fat man,the normal man and the thin man are of one contradiction unity.Only contrasting with the normal man and the thin man,the fat man exists.If there are no normal and thin men,the will be no fat man.But the divideing limit of these three contradictory aspects could not be easily made ,it is the so-called fuzziness which arises from the concept of "fatness".Again, the concept of "beautiful",it is concerned with "normal" and "ugly".If we only say "beautiful",then we could not get any information.So,the essence of fuzziness is a contradiction, a special representation form of contradiction.

The brilliant achivements of classical mathematics is famous for its accuracy and non-contradictoriness.Since Contor put forward the concept of "set",a great change has taken place. Contor's "set" is much of benefit to the classical mathematics,and it has been used in every part of mathematics as the basic theory.And it also caused the crisis and the absurd .One of the most serious crises was the appearance of "Russell's paradox" in 1903,the famous mathematician Hilbert called it as the terrible discovery to the mathematics. Many mathematicians seem to find out that it would not be good for the future,so they gave up their own work out went to study the contradiction of sets.All the studies started this way and they tried to avoid the appearance of the contradictionariness in set system. The hard work yielded some important results as on axioms sets etc.But they also found that the problem had not been soluted fundamentlly yet.All these causes the present writer to considerate that whether we can find another way to admit the contradictionariness in the set system:This is an astray which no one takes it and full of trakles.That is why I wants to study it.

2. Definition

In classical mathematics, Contor's "set" concept is usually described as:It is the whole of one pure event which we assumed . Now, the description about"set" becomes more ambiguous, even in one work we can find out more than ten kinds of definitions.But,no matter how complex a definition is,one point is still to emphasize the purity of the element's quality.

Therefore,the concept of classical set excludes the complex characteristics of the element's quality.Direct towards the method of making sets in classical set,I frankly present a new concept "contradiction set",admits the complexity and various characteristics of the element's quality, so it is different from the classical set.For describeing the different characteristics of these two kinds of set theory, we call classical set,the pure set,and the contradiction set, mixed set.

Definition: Arbitrarity one proposition,one general statement or one topic,if it is reasonable,we call it elaborate topic.

Definition: Under the specific condition,if one elaborate topic P exists, certainly one set exists in agreement with it,they are whole which is self-made system,containning a certain amount of elements. The common point of view,that the elements in this set could be divided into three kinds: A positive kind,a negative kind and a neutral kind,all of which constitute the set,we call this set the contradiction set. Among them we refer to the positive kind as A ;the negative kind as C ;and a neutral kind as B .

The existence of contradiction sets is a common fact. There are many examples in classical mathematics.For example, we can say the set of rational number is a contradiction set. It contains positive number,negative number and zero,three of them are contradictory with one another. The negative number is the negation of the positive number, meanwhile,it takes the positive number as the logical premise of its exitence, and vice versa. That is,there is no positive number,then no negative number either.Zero is one neutral number,but could not overlook its existence.The other example,we observe the set of real number from contradiction set,we discover that it is also a kind of contradiction set.It contains two parts: Rational number and non-rational number.Both of them are negative to each other.But either of them could be eliminated with.Otherwise there will be no set of real number.

Though in our life,we see objects being described as contradiction sets.Its condition of element arrangement in order are usually complex.But we may appeal to our imagination.Suppose it to be one plane which consists of three parts.We may suppose it to be one solid,then the three kinds of different elements cover the whole set.So we get indicating method which indicates elements of different quality in different order. We call it the "step method". We use " $\circ\!-$ " to stand for a contradiction set,we can indicate a contradiction set N as follows:

$$\overset{\circ\!-}{N} = \left\{ \begin{matrix} A \\ /B \\ /C \end{matrix} \right\}$$

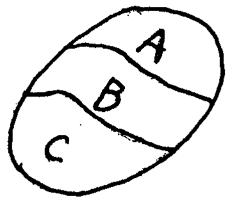
Here,we make use of the concept of membership function from fuzzy sets. Among the classes of A , B , C , we regard each of them as one part of the whole N . Using membership function to indicate them,in contradiction set $\overset{\circ\!-}{N}$, if an element of A kind possesses 30% in total elements, then A is 0.3;if an element of B kind possess 20% in grand total elements, B is 0.2;if an element of C kind possesses 50% in the grand total elements, C is 0.5;this $\overset{\circ\!-}{N}$ may be indicated:

$$\overset{\circ\!-}{N} = \left\{ \begin{matrix} \underline{0.3} \\ \underline{0.2} \\ 0.5 \end{matrix} \right\}$$

We see,though a form of a contradiction set changes greatly, the total membership function of every kind element in N is "1". For example, the transverse plus of membership function of every element in N

$0.3 + 0.2 + 0.5 = 1$.

The geometry intuition of a contradiction set is as follows:



Based on the form of contradiction set, we could divide the contradiction sets into four kinds.

We call the first kind a "standard type".

For example,

$$\overset{\circ}{N} = \left\{ \begin{array}{c} 0.3 \\ \hline 0.2 \\ \hline 0.5 \end{array} \right\}$$

In the character of a "standard type" set there are the elements of a positive kind, a negative kind and a neutral kind.

We call the second kind a "two-leaf type".

For example,

$$\overset{\circ}{S} = \left\{ \begin{array}{c} 0.3 \\ \hline 0.7 \end{array} \right\}$$

In the character of a "two-leaf type" set there are only two kinds of elements: The elements of positive kind and negative kind, but no neutral kind; or, the positive kind and neutral kind, but no negative kind.

We call the third a "one-leaf type".

For example,

$$\overset{\circ}{T} = \{1\}$$

In the character of a "one-leaf type" set there is only the element of positive kind,but no elements of a negative kind and a neutral kind.This is usually the so-called Contor's set.

We call the fourth kind a "extensive type".

For example,

$$\overset{\circ\!-}{R} = \left\{ \begin{matrix} 0.1 \\ / \\ 0.4 \\ / \\ 0.2 \\ / \\ 0.3 \end{matrix} \right\} \qquad \overset{\circ\!-}{W} = \left\{ \begin{matrix} 0.02 \\ / \\ 0.48 \\ / \\ 0.3 \\ / \\ 0.1 \\ / \\ 0.1 \end{matrix} \right\}$$

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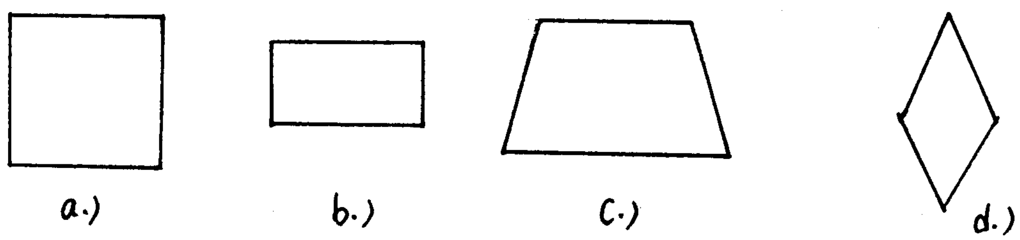
The character of an "extensive type" set is : The number of steps is more than that of the "standard type" steps.Of two examples mentioned above,one is a "four-leaf type" set,the other is a "five-leaf type" set. Here, our fuzzy sets belong to this kind of contradiction set.

The contradiction character of contradiction set arises from the composition of different element group which exclude one another,and from the method of producing contradiction set.This concerns the concept of "class" in contradiction set.If we regard that the concept of "class" in a classical set is a standard of "class" concept, then the special concept of "class" in contradiction set had to be called a concept of "extensive-class". There are two "class" concept in contradiction set, that are "class" and "extensive-class". The "same race" is defined same class in classical set. The significance of the concept is absolute. But the concept of "extensive-class" in a contradiction set is rather different.The significance of this concept of "extensive-class" is both absolute and relative.It is a unity of absolution and relativity.Before making the difinition of a "extensive-class" concept, we first make a definition of a "vision-angle" concept.

Definition: Under a promising condition, presening the standard scale of the eleborate topic P ,and we call it "vision-angle".In a simple word,it is the starting point of observing fact and material.In comparison, the "vision-angle" in the contradiction set and the limiting condition in the classical mathematics,there is something common and something different.They are all a limiting condition.But the limiting condition in the classical mathematics is a very special case.For example, a denominator could not be zero.But the "vision-angle" means more.With the "vision-angle" concept, we can easily make the definition of a "extensive-class"concept.

Definition: Under a promising condition, from a special "vision-angle",based on one or several common points of the whole things and face,we divide a classification, which call a "extensive-class".

Now,we give an example to explain the concept of a "extensive-class".There are four geomety graphs,which are a.) square; b.) rectangle; c.)trapezoid; d.) rhombus.



If we decide a "vision-angle" being a graph of four 90° angles,then a.) square and b.) rectangle constitute one "extensive-class";if we decide a "vision-angle" being a graph of four sides,then a.) square, b.) rectangle, c.) trapezoid and d.) rhombus constitude one "extensive-class".

Because the "extensive-class" itself is a wide definition, its content is extensive, contradiction set consituted in this way, naturally, its border is not fixed.

The contain of a contradiction set.

Definition: If any one of the elements of a contradiction set $\overset{\circ-}{M}$ is the element of a contradiction set $\overset{\circ-}{N}$, the contradiction set $\overset{\circ-}{M}$ is called the contradiction subaggregate of the contradiction set $\overset{\circ-}{N}$, and the contraction set $\overset{\circ-}{N}$ is called the extending contradiction set. It is recorded as follows:

$$\overset{\circ-}{M} \subseteq \overset{\circ-}{N} \quad \text{or} \quad \overset{\circ-}{N} \supseteq \overset{\circ-}{M}$$

Evidently, with the definition, we can proof $\overset{\circ-}{M} \subseteq \overset{\circ-}{M}$, which shows that any contradiction set is its contradiction subaggregate.

$\overset{\circ-}{M}$ is the contradiction subaggregate of $\overset{\circ-}{N}$. Also we can say that $\overset{\circ-}{M}$ is contained $\overset{\circ-}{N}$ or $\overset{\circ-}{N}$ contains $\overset{\circ-}{M}$.

Definition: If $\overset{\circ-}{M} \subseteq \overset{\circ-}{N}$, and in $\overset{\circ-}{N}$ at least one element do not belong to $\overset{\circ-}{M}$, then we call $\overset{\circ-}{M}$ the real contradiction subaggregate of $\overset{\circ-}{N}$, that is:

$$\overset{\circ-}{M} \subset \overset{\circ-}{N} \quad \text{or} \quad \overset{\circ-}{N} \supset \overset{\circ-}{M}$$

The equality of a contradiction set.

Definition: Supposing there are two contradiction sets, $\overset{\circ-}{M}$ and $\overset{\circ-}{N}$. If $\overset{\circ-}{M}$ contains $\overset{\circ-}{N}$, and $\overset{\circ-}{N}$ contains $\overset{\circ-}{M}$, we can say that $\overset{\circ-}{M}$ and $\overset{\circ-}{N}$ are equality.

Clearly, if the elements of two contradiction sets are all equality, then two contradiction sets are equality.

It is not difficult to proof that the relation of containing and equality in a contradiction set possesses the following quality of circulation:

(1) If $\overset{\circ-}{M} = \overset{\circ-}{N}$, $\overset{\circ-}{N} = \overset{\circ-}{W}$, then $\overset{\circ-}{M} = \overset{\circ-}{W}$;

(2) If $\overset{\circ-}{M} \subset \overset{\circ-}{N}$, $\overset{\circ-}{N} \subset \overset{\circ-}{W}$, then $\overset{\circ-}{M} \subset \overset{\circ-}{W}$;

(3) If $\overset{\circ-}{M} \subset \overset{\circ-}{N}$, $\overset{\circ-}{N} = \overset{\circ-}{W}$, then $\overset{\circ-}{M} \subset \overset{\circ-}{W}$;

3. Operation

The combine of the contradiction set.

Definition: Suppose there are two contradiction sets, $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$ of which the "vision-angle" has been settled, the set which consists of all elements is called combine set of the contradiction sets $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$. This is recorded $\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N}$.

All elements of $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$ belong to $\overset{\circ\!-}{M}$ or $\overset{\circ\!-}{N}$ (at least belong to one of them,or possibly belong to both of them at the same time.).So

$$\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N}$$
$$= \{ x \mid x \in \overset{\circ\!-}{M} \text{ or } x \in \overset{\circ\!-}{N} \}$$
$$= \{ x \mid \text{at least } x \text{ belong to one among } \overset{\circ\!-}{M}, \overset{\circ\!-}{N} \}$$

The product of the contradiction set.

Definition: After deciding the "vision-angle", the set which consists of all elements of $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$ is called the product set of contradiction sets $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$. This is recorded as $\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N}$.

The element belonging to two contradiction sets $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$ at same time is a common element of $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$, which belongs to both $\overset{\circ\!-}{M}$ and $\overset{\circ\!-}{N}$, or we can say that it belongs to $\overset{\circ\!-}{M}$ and also belongs to $\overset{\circ\!-}{N}$. So there are:

$$\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N} = \{ x \mid x \in \overset{\circ\!-}{M} \text{ and } x \in \overset{\circ\!-}{N} \}$$

Definition: A set which does not contain any element is called an "empty set". This empty set can be indicated with the mark ϕ.

For combine set, clearly it has the following qualitis:

(1) $\overset{\circ\!-}{M} \cup \overset{\circ\!-}{M} = \overset{\circ\!-}{M}$;

(2) $\overset{\circ\!-}{M} \cup \phi = \overset{\circ\!-}{M}$;

(3) $\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N} \supseteq \overset{\circ\!-}{M}$.

For product set,clearly it has the following qualitis:

(1) $\overset{\circ\!-}{M} \cap \overset{\circ\!-}{M} = \overset{\circ\!-}{M}$;

(2) $\overset{\circ\!-}{M} \cap \phi = \phi$;

(3) $\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N} \subseteq \overset{\circ\!-}{M}$.

For contradiction sets $\overset{\circ\!-}{M}$, $\overset{\circ\!-}{N}$, $\overset{\circ\!-}{W}$ ......, they have the following qualitis:

The law of commutation

$$\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N} = \overset{\circ\!-}{N} \cap \overset{\circ\!-}{M},$$
$$\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N} = \overset{\circ\!-}{N} \cup \overset{\circ\!-}{M};$$

the law of association

$$(\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N}) \cap \overset{\circ\!-}{W} = \overset{\circ\!-}{M} \cap (\overset{\circ\!-}{N} \cap \overset{\circ\!-}{W}),$$
$$(\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N}) \cup \overset{\circ\!-}{W} = \overset{\circ\!-}{M} \cup (\overset{\circ\!-}{N} \cup \overset{\circ\!-}{W});$$

the law of distribution

$$(\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N}) \cap \overset{\circ\!-}{W} = (\overset{\circ\!-}{M} \cap \overset{\circ\!-}{W}) \cup (\overset{\circ\!-}{N} \cap \overset{\circ\!-}{W}),$$
$$(\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N}) \cup \overset{\circ\!-}{W} = (\overset{\circ\!-}{M} \cup \overset{\circ\!-}{W}) \cap (\overset{\circ\!-}{N} \cup \overset{\circ\!-}{W});$$

the law of absorb

$$(\overset{\circ\!-}{M} \cup \overset{\circ\!-}{N}) \cap \overset{\circ\!-}{M} = \overset{\circ\!-}{M},$$
$$(\overset{\circ\!-}{M} \cap \overset{\circ\!-}{N}) \cup \overset{\circ\!-}{M} = \overset{\circ\!-}{M}.$$

4. Distinguishing characteristics

We have brifly described the contradiction set, its relations and operations, from which we can infer the following distinguishing characteristics of the contradiction set:

(1) The characteristics of contradiction.

The theory of contradiction set openly admits that contradiction undoubtly exists in a set. Since it is decided by the method of making contradiction set, this contradiction characteristics could not be avioded. We can see that the positive element,the negative element and the neutral element in a "standard type" contradiction set are excluding acoording to their qualitis. A contradiction set admits the legality of excluding. Comparing with the classical set, the contradiction set is a opposite one.

In classical mathematics, the non-contradiction characteristics is emphasized very much in a system, so is that in classical set. Hilbert, the representer of formlizm,has ever said that the so-called formlizm, in one word, is that there is no contradiction in axioms system. The excluding of the contradiction quality is decided by its method of making a set and guiding thought.

In 1965, professor L. A. Zadeh peresented "Fuzzy sets" concept. In his fuzzy set , there are no elements of complexity quality,of course,no contradiction quality elements. Contrasting with the method of classical set be making, Zadeh's method is not a revolutional one. It still belong to the classical method.

(2) The characteristics of the whole.

In contradiction sets, the characteristics of the whole or characterictics of structure stability is emphasized. Considering the close state of certain contradiction set, generally speaking, the elements of the set are in contact with and restrain one another. They all display their own function. For example, if the elements of a positive kind increase,then the elements of negative kind and neutral kind will correspondingly reduce, and vice versa.But the interval of their moving is not changing, and this is " 1 " (an empty set is a speciffic example of a contradiction set). It is important that contradiction set pay much attention to the unit" 1" as a whole.

Classical set takes two-value logic as its foundation, and the membership function only have two value " 0 " and " 1 ". If a set exists, we can get a membership function " 0 ".So there is no an idea of the whole.

The selecting value of membership function in fuzzy set is any value in among the region of " 0 " and " 1 ", but not unit " 1 ". We never heard of the idea of the whole in the research of fuzzy set . Also,among the elements in fuzzy set did not exist the relation of restraining one another.

(3) The characteristics of comparison.

The characteristics of comparison in contradiction sets arises from the people's subjectivity's factor. A chinese anceint poety said, " for the same thing, when seeing from transverse , it is continuous ; and when seeing from vertical, it is lofty; when seeing from every point is different." IN contradiction set, we can see that the definition of the value of membership function and the " vision-angle" contains the subjectivity's factor.The mathematics is no longer a tool of pure objective science. The definition of our concept is flexible,so is the definition of the operation. According to the appraising standard of classical mathematics,it certainly contains the fuzzy quality. Classical mathematics can not tolerated the subjectivity's factor. Classical mathematics emphasized the principle of objective quality, for example, $2 + 3 = 5$. This is pure objective.

A discussion on subjectivity's factor in fuzzy set is optional. Someone think that the apperance of the value of membership function shows that subjective factor has already entered mathematics, while other deny firmly this theory. Now we can not appraise which one is correct.

(4) The principle of analysis.

The principle of analysis means that in the relation and operation of contradiction set we should pay greatly attention to condition of the element's classification in the contradiction set. For example, a containing relation among contradiction sets , it must be established on the containing relation of the state of the classification of the element in the sets. But among elements of different kind, there is no any containing relation. Of cause, there is no the combining operation of the contradiction sets either.

For example, the combining operation of contradiction sets $\overset{\odot\!-}{M}$ and $\overset{\odot\!-}{N}$.

$$\overset{\odot\!-}{M} = \left\{ A_i \big/ B_i \big/ C_i \right\} \qquad \overset{\odot\!-}{N} = \left\{ A_j \big/ B_j \big/ C_j \right\}$$

$$\overset{\odot\!-}{M} \cup \overset{\odot\!-}{N} = \left\{ A_i \cup A_j \big/ B_i \cup B_j \big/ C_i \cup C_j \right\} = \left\{ A_p \big/ B_p \big/ C_p \right\}$$

(among them $A_p = A_i \cup A_j$, $B_p = B_i \cup B_j$, $C_p = C_i \cup C_j$)

But the principle of analysis is completely unnecessary in the classic set and fuzzy set.

(5) The principle of experience.

The principle of experience contains two aspects. They are : Pure experience and the formula of experience. The principle of experience is of importance in the definition and operation of the contradiction set. For example, we have mentioned above " the rationality " of an elaborate topic. The standard of rationality is experience and formula of experience. For another example, the deciding of the vision-angle and the membership function is base on experience and formula experience, as an important factor.

But the things are quit different in classical set. People have paid great attention to the experience in the earlier period of classical set, but when it developed to thre stage of axioms's system,people just paid their attention to the principle of deduction and the consistency of axioms8s system.

In this point,fuzzy set and classical set have no difference, Both of them pay great attention to the principle of the no-contradiction deduction of logic deduction. In consideration of this, fuzzy set do not exceed the range of classical set.

5. Application.

(1) In the study of the diverge material and fact.

(2) In describing about the theory which is developing.

(3) In the study of the language of nature science and social science.

(4) In the field of classic mathematics, there are also many applications.For example, studying for the universe of proposition. The so-called universe of proposition,it means a set that consists of possibly makes the proposition be true or false. Usually it is recorded as I . The so-called true universe of proposition P is a set that consists of all conditions making P true. It is marked P . The false universe of proposition P is a set that consists of all conditions making P false. It is marked $\bar{P}$. Clearly, we may deduce, from the definition, that the universe of proposition I equals the combine set of true universe P and false universe $\bar{P}$,that is $I = P \cup \bar{P}$. But it is difficult to describe the set I in the classical set. It is just a " two-leaf type " contradiction set, that is : $\overset{\frown}{I} = \left\{ \frac{P}{\bar{P}} \right\}$. In this set, the true universe of propositon P is the element of positive kind, and the false

universe of proposition $\overline{P}$ is the element of negative kind. Both of them together constitute the contradiction set $\overset{\circ}{I}$.

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Shi Jian-sheng
Dept. of Chinese and Literature
Jilin University
Changchun city
Jilin province
China