A Theory of Fuzzy Frames
by

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PART 2

Abstract

The theory of Fuzzy Frames is developed with examples. It is related to existing notions in knowledge representation and fuzzy mathematics, and directions for further research are explored.

KEYWORDS: Fuzzy Set, Frame, Semantic Network, Property Inheritance, Closed World Assumption, Fuzzy Quantifier, Knowledge Representation, Nonmonotonic Logic, Relational Database, Fuzzy Relation, Truth Maintenance, Usuality.

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5 Fuzzy Frames, Fuzzy Quantifiers and Nonmonotonic Logics

In yet another example of a fuzzy framebase, we can now explore their application to one of the classical problems in inheritance, using non-standard quantifiers instead of nonmonotonic logic. Touretzky discusses (and dismisses) this approach by reference to the work of Altham [20], but seems to be unaware of Zadeh's more encouraging results for the representation of fuzzy quantifiers and their inference properties [16,17,5,3].

Zadeh's theory of dispositions and fuzzy quantifiers and test score semantics [16,17,5,3] lets us express one of the classical motivating problems of nonmonotonic logic as 'Most birds can fly'. This can be neatly expressed with fuzzy inheritance. Here is the framebase.

Flying-animal
IsA: Animal
Can-fly:true [fuzz]

Bird
IsA: Flying-animal [0.9]
Wings: 2

Penguin
IsA: Bird
Can-fly:false [fuzz]

Tweety
IsA: Bird [1], Penguin [1]
Can-fly: ?

The fuzzy sets involved are illustrated in Figure 5.1. The answer is that Tweety is a bird and can't fly. So far this is the same result as that suggested in McDermott and Doyle [7] - but we can do better: Penguins do sort of fly (they make fluttering movements when diving or running) and the fuzzy set shown in Figure 5.2 preserves this information in a way.

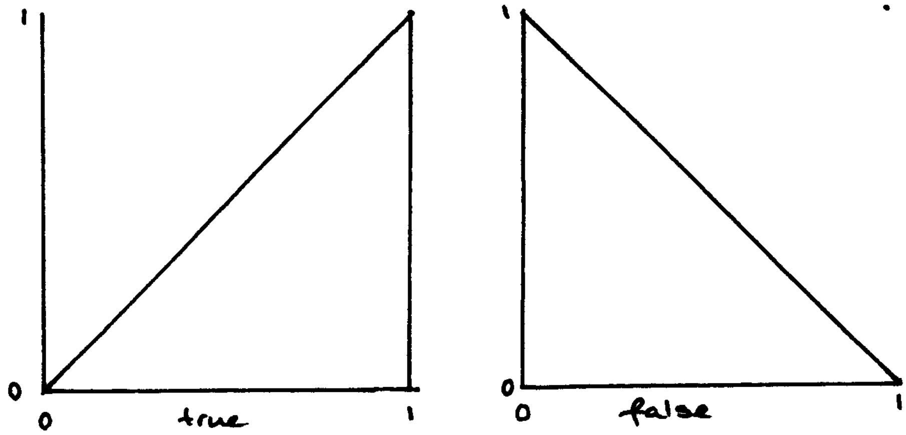
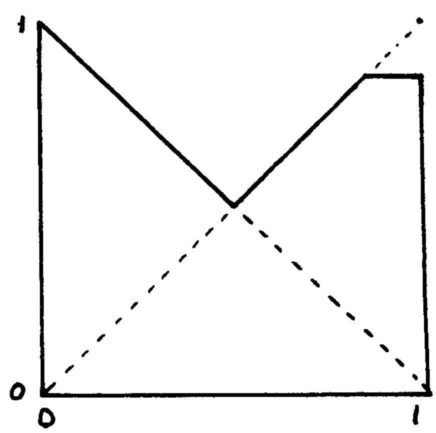


FIGURE 5.1 THE FUZZY SET 'true' IS GIVEN BY x=x AND 'false' by x=−x.



CAN TWEETY FLY?

MAXIMUM RULE: NO.

MOMENTS RULE:
NOT VERY MUCH.

FIGURE 5.2 FUZZY SET FOR THE COMPATIBILITY OF THE STATEMENT 'Tweety can fly'.

A possible generalisation springs to mind at this point. The numerical factor representing the degree of inheritance could be replaced by a linguistic variable (a fuzzy set or fuzzy number). This would mean that the truncation of inherited fuzzy sets would itself be fuzzy. We could call such objects 'Ultrafuzzy Frames' or '2-Fuzzy Frames'. However, finding a formal semantics then becomes much harder, and we suspect that the practical value of such a theory would be severely limited by its complexity. In fact, this generalisation would correspond much more closely to the interpretation of fuzzy quantifiers given in section 3.4. where fuzzy quantifiers are represented as fuzzy numbers. The inheritance mechanism of 2-fuzzy frames could indeed be modified to exploit the inference rules of approximate reasoning (e.g. the intersection-product syllogism given in 3.4). This is being investigated.

It does look very much as if we can interpret a fuzzy link as a most/some type fuzzy quantifier. In the hangglider/toy example this is not the most natural interpretation. IsA links may be used (or mis-used) for a variety of conflicting purposes. A good design theory would force us to state the interpretation of the inheritance links and not mix them up. An alternative is to permit fuzzy frames to have a number of 'typed' inheritance links. Then inheritance could take place through a manifold of different networks. This too is under investigation.

6. Design Theory for Fuzzy Frames - Some Problems

Fuzzy frames are not alone in raising general problems in terms of property inheritance. Touretzky [12] lists the analogous problems with crisp inheritance systems and suggests some very reasonable ways round them in terms of a lattice theoretic semantics. We can use Touretzky's hierarchical distance ordering to provide attenuation of inheritance in fuzzy frames. This will be explored in detail in a subsequent paper, due to lack of space herein. We also intend to investigate an analogous lattice theoretic formal semantics as part of our programme.

As a parenthetical remark at this point, it is worth observing that Touretzky's semantics generate the truth tables of Lukasiewitz logic and that this is precisely the multi-valued logic which corresponds to fuzzy logic [21,3]. Lukasiewitz motivation for the uncertain term was contingent statements (about the future); indeterminate values. The motivation behind Touretzky's system is the presence of links which indicate that no conclusion may be drawn (or value inherited). Other three-valued logics (of Bochvar and Kleene) offer interpretations in terms of meaninglessness and undecidability. This gives us the confidence to assert that the program we suggest is capable of being carried out, because the underlying logic of fuzzy set theory and *a fortiori* fuzzy frames turns out to be the same as that underpinning the classical case.

However, there are some problems that Touretzky's approach does not address. The problem that the hangglider is safe because toys are (because the typical commodity is) could be viewed as one of conflict resolution. Perhaps we could decompose the link using an additional class frame such as Dangerous-toy in such circumstances. Using an inferential distance ordering approach assumes that this has not only already been done, but that all possible such decompositions have been explicated in the framebase. Otherwise, a slight perturbation of the design could result in totally different behaviour under inheritance. What is therefore required is a procedure which determines whether a framebase in 'complete' in this sense. We need to develop a design theory for general framebases. This would appear to be a very difficult problem, since the recognition of a 'good' expansion is clearly a question of relevance. The absence of such a theory forces one to adopt some method of default reasoning under Reiter's closed world assumption (not our fuzzy version of it mentioned above).

Furthermore, there are some problems with the hang-glider example which can be dealt with in a variety of other ways. First of all, we can regard the problem of determining safety as one of conflict resolution. In the context of fuzzy inheritance this means that we could look for a mechanism which would recognise the presence of 'very bimodal' distributions in returned fuzzy sets and then prompt the user for a decision. The difficulty with this approach is centred on that of finding an universally acceptable measure of distance between fuzzy sets. It would, however, be worth exploring on an experimental basis.

A major problem in the area of design criteria arises when we have to decide whether the degree to which an object (or class) has a property is determined by inheritance or by slot filling (which in turn can result from inheritance). Thus the two frames

<u>Frame</u>		<u>Frame</u>
IsA: ?	or	IsA:Object-with-property-A [degree]
Property-A:extent		

are equivalent in one sense if not in all senses. The problem is to decide when to choose between the alternative formulations.

This is nothing more than the the class-property problem of traditional inheritance systems, and making the choice correctly remains a matter of skill and judgement at present. It is analogous to the problem of deciding what constitutes an 'entity' and an 'attribute' in entity modelling. However, there ought to be, one feels, some precedent in logic to assist with the decision. As a step towards a solution of the problem, we offer a discussion of the status of 'tautological frames' below.

This problem also raises the question of the status of the two kinds of fuzziness. The fuzziness subsisting in the way classes (or objects and classes) are related and the fuzziness inherent in predicates of description (attributes). It is necessary to ask if we are committing the unforgivable sin of mixing different kinds of uncertainties. This is clearly a danger with poor designs. On the other hand, it may be argued that more expressive power results from allowing the user to mix both forms as convenient.

What, we may ask, is the status of what we choose to call 'tautological' frames like the one below?

<u>Dangerous-object</u>
IsA: Object
Danger: not less than high

These frames are recognisable as having only one non-IsA slot with a meaning predicate and value corresponding exactly to the frame's name. The IsA slot also contributes nothing new. The specification of the frame asserts that a dangerous object is an object which has the property of being dangerous to the degree 'high' or more.

Certainly, in the hang-glider example, if we disallow all taulogical frames on principle then the only way the low value for safety can ever get into the slot is if we put it there. This indeed accords with intuition, because there is really no *a priori* reason (at least not within the confines of our given framebase) why we should think a hang-glider dangerous; it just IS. This is a very neat way of resolving the problem, which incidentally begs the same question for the design of crisp frame systems. On the other hand it is sometimes unnatural not to include such objects: An elephant is a grey object; a grey object is a drab object. It is not yet clear whether an example can be constructed which displays the same problem as the hang-glider one where there are no tautological frames present. This must remain a topic for further research at this stage.

One interesting angle on this problem is that the presence of a tautological frame in a design invariably indicates the presence of a personal construct e.g. Danger-Safety, so that the presence of tautological frames in a design naturally results from using the methods of knowledge elicitation derived from Kelly grids. Perhaps these can be used to arrive at good designs if a sensible method of factoring out the taulogical frames can be found and added to the design methodology.

Other structures which we might be tempted to disallow in a theory of non-redundant (or normal) forms for framebases include cycles. In the crisp case the example we discuss immediately below the problem does not arise, since a loop of the type below always represents equality and the network may be collapsed. In the case of Fuzzy Frames it is harder to decide whether there is a need for such relationships.

For example, the phrase 'Most men are avaricious' may be represented in the form:

<u>Man</u>	<u>Greedy-man</u>
IsA:Greedy-man [0.8]	IsA:Man [1]

Semantic net systems usually demand acyclicity. This is certainly computationally convenient, but no theory exists to say that it is strictly necessary for coherent inheritance. This is an open question for both crisp and fuzzy frames. The problem we point out above is, however, special to the fuzzy case. One way to remove the problem is to disallow the cycle and expand the network to include a new frame representing 'avaricious entity' (e.g. a petrol hungry car). Then a greedy man is avaricious [1] (or has the slot avaricious filled with 'high') and a man [1], and a man is avaricious [0.8] and there are no cycles. In doing this we have broken the no tautologies rule though. All these questions need a deep investigation for which some mathematical apparatus will need to be developed. For the time being experimental studies based on an implementation will best serve the needs of a sound design theory.

To end this rather speculative discussion of design principles, let us summarise the control regimes available within the theory of fuzzy frames.

1. The default regime: If a slot is filled don't inherit into it, if it isn't then combine the truncated inherited attributes with the maximum operator. For non-fuzzy variables the maximum of the certainty factors represented by the IsA values is attached to the inherited value. Multiple inheritance of non-fuzzy values may result in multi-valued slots (lists).

Different fuzzy logics may be used according to the application at hand. The maximum operator may then be replaced with the appropriate t-conorm.

2. The Fuzzy Closed World Assumption: Inherit all defined values and perform a union. Some theory of attenuation may be added. The control strategy precludes exceptions, but this is sometimes what is required. For example, when we reason that dogs have four legs because they are mammals and that humans are an exception, having only two, we are plumping for Naive Physics in contradistinction to mature Biology. Humans (normally) do have four legs, it is merely that two of them have become adapted to other purposes. From such a viewpoint we want an inheritance mechanism that propagates the mammality in despite of the human exception; takes account of both factors. For non-fuzzy fuzzy values there are some choices to be made. We will deal with this in detail in a forthcoming paper.

Each regime bifurcates because one of the maximum or moments methods of defuzzification must needs be selected, unless linguistic approximation is employed. This gives the system designer a choice from at least six control regimes. It also suggests the need for some experimental work.

7.Fuzzy Frames as a generalisation of fuzzy relations

This completes the exposition of the theory of Fuzzy Frames as a practical method of knowledge representation. We should now like to offer the reader some insight into the intuitions that led us to the notion and in doing so suggest the place of the theory within knowledge representation as a whole.

The first observation we make is that fuzzy relations generalise relations.

A relation is a subset of some cartesian product of sets. It can also be regarded as a function from that product into the truth set of classical logic 2={0,1}. A fuzzy relation is such a function where the codomain is the truth lattice of a multivalued logic; in the case we consider the unit interval fulfils this role. Of course a tabular (extensional) representation is also possible.

Now, it is well known (at least in mathematics) that there is a bijective correspondence between functions $A \times B \longrightarrow I$ and functions $A \longrightarrow I^B$. This also holds for n-dimensional cartesian products. The correspondence is given by assigning to every function f(a,b) the function which takes a point a to the function (fuzzy set) $g: B \longrightarrow I : b \longmapsto f(a,b)$. This proves that, in tabular representation, the form

Loves:	Person1	Person2	Degree
	John	Mary	0.9
	Mary	John	0.2
	Jill	Mary	0.4
	etc...		

corresponds to the form

Loves	Person	Possibility distribution
	John	$\pi1$
	Jill	$\pi2$
	etc.	

Thus, the syntax we use in fuzzy frames (ignoring the IsA slots for a moment) corresponds to an adequate syntax for fuzzy relations.

The next point to notice is that relations may form a category with arrows that preserve some desired properties of relations, such as projections or joins or both. There is no reason to think that, with a suitable notion of property-preserving arrow, fuzzy relations might not form a category. The exact definition of the arrows is not important for our argument - it would be if we wished to proceed to formal proofs. In this case there is an obvious embedding functor from relations to fuzzy relations:

(Rel) ----> (FuzRel)

This gives an exact meaning to the statement that fuzzy relations (fuzzy relational databases) generalise relations (relational databases).

If we now add the inheritance structure to a fuzzy relation we have fuzzy frames as a generalisation of fuzzy relations with a single tuple.

8. Fuzzy Frames as a generalisation of frames

In a similar way we argue that frame sets (sets of frames) also generalise relations.

The only differences between frames and relations are that frames do not require atomic values of attributes - there may be sub-slots (facets) and list valued entries, and (because of inheritance) we cannot predict in advance the number of attributes in the data dictionary definition of a frame. The first relaxation is equivalent to saying that the underlying logic is no longer first order (as with fuzzy relations). The second says that we are working in a potentially (countably) infinite cartesian product. Intuitively, this corresponds to the assertion that an object may be assigned ANY attribute in the world; blueness, hunger, pointedness, etc. Real objects only have a few relevant attributes, but in the frame model they MIGHT inherit anything.

Having said this, frames are normally thought of as representing single objects (or classes). This is the reason we talk about sets of frames. An n-tuple in a relation corresponds to one filling of a frame. Of course, there is absolutely no reason why frames should not be multi-valued. There are at least two interpretations of this. 1) Each 'filling' corresponds to a 'possible world'. 2) Each filling corresponds to a state on some world line.

Looked at in this way, frame sets may form a category - the arrows preserving the frame structure (and possibly having something to do with inheritance). Thus again we get the embedding functor:

(Rel) ----> (Frameset).

The practical application of this idea, on its own, is that truth maintenance systems and temporal logic systems acquire a very coherent implementation framework, and many of the techniques of relational database theory can be mapped across to this framework.

Fuzzy Frames

Adding the ability for framesets to hold fuzzy sets as slot values shows that fuzzy frames can be viewed as a generalisation of frames.

Obviously, we have only given a sketch of the idea here and much further work remains to be done.

9. Fuzzy Frames as a 'pushout'

Having two functors in the category of categories as described naturally leads to the question: What is their pushout? If we could construct it, we would have the *universal* generalisation of both frames and fuzzy relations. We conjecture at this point that Fuzzy Frames are a very reasonable candidate for this pushout and thus that Fuzzy Frames generalise all object knowledge representations.

If this result or even a much weaker version of it (that the diagram below commutes) could be established, then we would have a unified theory covering the following issues in knowledge representation:

The relational model of data
Fuzzy relations and fuzzy information retrieval
Frames
Inheritance systems
Nonmonotonic reasoning
Temporal reasoning
Viewpoints, possible worlds and modal logic systems

Fuzzy Frames

This is a bold claim. The arguments above are tentative. We, however, are convinced that, at the very least, it is worthy of further exploration. Our own programme of research includes the practical implementation of the ideas. We hope that academic researchers will take up some of the other suggested lines of research.

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  (Rel)     >--------->  (Fuzrel)
    v                       v
    |                       |
    |                       |
    V                       V
(Frameset) >--------> (Fuzframe)
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10. Discussion

This paper is the first in a series of papers on Fuzzy Frames. In it we have informally defined the syntax and semantics of fuzzy frames. This makes it possible to move on to the stage of implementation and application of this form of knowledge representation. A formal syntax is in preparation.

We have suggested several candidate applications. Among the most difficult of the others which suggest themselves is the application to the interpretation of natural language statements. We hope that other, more academic, researchers will take this up as a research topic. Our interest is in the practical knowledge engineering issues. Additionally, we have suggested a line of research for mathematicians interested in database theory and the abstract algebra of relations. We hope that this too will be taken up and solved by the academic community. A successful conclusion will have profound consequences for the whole of software science, and we hope that we have argued this clearly in this paper.

We have also begun to explore the design and methodological issues surrounding fuzzy frames. A number of unsolved problems in this area were suggested as topics for further research. In general, we believe that this research must be predicated on applications. However, the exploration of toy problems offers some scope for the development of sound design principles and methods.

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