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FUZZY GAMES OF SADDLE POINT EXISTED

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ABSTRACT

The classical game theory,which is based on the double value logic theory, lose sight of much fuzzy information or grey information. In most problems of complicated game, the states of each side of the game systems are not certain. Much fuzzy information or grey information is contained with in the real game grocess. Hence, distortion of decision making used the classical game theory is considerable under fuzzy enviroment or fuzzy meaning.

In this paper, we introduce the fuzzy sets and possibility theory for dealing with the game problems under fuzzy circumstances or fuzzy states, The some rough models of fuzzy game theory for solving problems involving fuzzy number and imprecise variables are presented and the relevant illustrations are given.

KEY WORDS

Fuzzy game theory ; Fuzzy game ; Fuzzy matrix game ; Fuzzy profit matrix ; Imprecise variable

1. INTRODUCTION

The game theory is widely applied to military affairs, physical training , commercial production and so on . But the classical game

theory , which is based on the double value logic theory , lose sight of much fuzzy information and grey information . In most problems of complicated game , the states of each side of the game systems are not certain. Much fuzzy information or grey information is contained with in the real game grecess . For example , in the commercial production competition,the profit distribution of each side of competition is fuz- ziness and in the military competition, not only gains and losses ot the competition is fuzziness but also possibility applied tacticeses is not certain. Hence, distortion of decision making used the classical game theory is considerable under fuzzy enviroment or fuzzy meaning.

How do we consider the fuzzy information and the grey information ot game problems into the game processes and make theoretic decision accord with the actual game situation . This study task would be a im- portant study task of game theory reseach .

In this paper, we introduce the fuzzy sets and possibility theory for dealling with the game problems under fuzzy circumstances or fuzzy states.The some rough models of fuzzy game theory for solving probims involving fuzzy number and imprecise variables are presented and the relevant illustration are given .

2 . GAME PROBLEMS UNDER FUZZY
CIRCUMSTANCE AND FUZZY MEANING

In the most game problems,uncertain elements contained with the game grocesses are common occurrence , this uncertain elements are frequently shown fuzziness . Let us see a concrete example following:

Example 1 . A certain factory will determine a plan of the pro-

duct output of the latter half of the year . According to past expe-
rience and market forecast , the product   market   predicted   will be
three possibility :   a good sale   possibility as p(B1) and   a common
sale possibility as   p(B2)   and   a   bad   sale possibility as p(B3) .
We use three tacticeses possibility to be :   a   big batch process as
p(A1) and a middle batch process as p(A2) and a small batch   process
as p(A3) . Possible profits under the situation (Ai , Bj)   composed
each pure tactics can be estimated also, to see table 1 .

Table 1 .   product output decision table

		product market		
		B1 (good)	B2 (common)	B3 (bad)
		P(B1)	P(B2)	P(B3)
A1 (big)	P(A1)	VH	H	VH
A2(middle)	P(A2)	VL	L	H
A3(small)	P(A3)	M	VL	H

HERE :   VH----VERY HIGH ;   H----HIGH ;   M----MEDIUM ;

L----LOW ;   VL----VERY LOW

How do you make decision analysis and rational selection of   pro-
duct tacticses , let business profit be maximum ?

supose that the product tacticses are one side (I)   of   the game
problem and the natural states is other one(II).There are tow outstan-
ding characteristics in the   above--mentioned example : (1) the states
of the both side of the game system are fuzziness and one is presented
by   imprecise   variables   or fuzzy subsets .   The   description words :
"big , middle , small " of the product tactics (I) are imprecise lan-
-guage and the desicription words " good , common , bad " of the na--

-tural states (II) are also . There are not outstanding limit between tacticses ,the sircumstance of the game system is fuzziness call fuzzy circumstance .(2) The profits of each situation (Ai , Bj) are vague, and it must be described by fuzzy numbers or imprecise variables (i.e fuzzy subsets) not distinct numbers . The profits are vague to call fuzzy profit (or say fuzzy meaning).

Definition 1 . The game problems under fuzzy circumstance and fuzzy meaning are fuzzy game problems . The game theory dealling with the fuzzy game problems call fuzzy game theory .

Following we will discuss the fuzzy matrix game problems .

3 .FUZZY MATRIX GAME

Definition 2 . Call fuzzy matrix game G ,if $G = \{ \underset{\sim}{S}_1, \underset{\sim}{S}_2, \underset{\sim}{A} \}$

where $\underset{\sim}{S}_1 = \{ A_i \}$ $i = 1, 2, 3, \ldots\ldots, m$

$\underset{\sim}{S}_2 = \{ B_j \}$ $j = 1, 2, 3, \ldots\ldots, n$

$\underset{\sim}{A} = \{ \underset{\sim}{C}_{ij} \}m*n$

only as m ,n is limited positive integer and $\underset{\sim}{C}_{ij}$ is fuzzy numbers or imprecise variables (i,e. fuzzy subsets) . $\underset{\sim}{A}$ is called the fuzzy profit matrix .

Let us intruduce

$$\mu : \underset{\sim}{C}_{ij} \longrightarrow [0, 1]$$

and thus

$$\mu = \{ \mu_{ij} \}m*n$$

it is called the fuzzy profit membership fuction (or grade) matrix .

So the fuzzy matrix game $G = \{ \underset{\sim}{S}_1, \underset{\sim}{S}_2, \underset{\sim}{A} \}$ might become

$$G = \{ \underset{\sim}{S}_1, \underset{\sim}{S}_2, \mu \} .$$

Definition 3.Supose that there is a fuzzy game $G^\wedge = \{ \underset{\sim}{S}_1^\wedge, \underset{\sim}{S}_2^\wedge, \underset{\sim}{A}^\wedge \}$

and $G = \{S_1, S_2, A\}$, $G^\wedge$ is a G alternate state . Among wich $S_1^\wedge = \{A_i^\wedge\}$ is the $S_1 = \{A_i\}$ expand tactics set .and $S_2^\wedge = \{B_j^\wedge\}$ is the $S_2 = \{B_j\}$ expand tactics set , $A^\wedge = \{E_{ij}\}$, element E_{ij} is the profits of the situation $(A_i^\wedge, B_j^\wedge)$.

If the situation (A_i*, B_j*) make

$$E_1(A_i*, B_j*) = \max_i \min_j E_{ij}$$

$$E_2(A_i*, B_j*) = \min_j \max_i E_{ij}$$

to tenable . So we consider the situation (A_i*, B_j*) be a optimum (or satisfactory) situation of the fuzzy matrix game G .

The optimun (or satisfactory) situation (A_i*, B_j*) of various fuzzy matrix games might be determined by differentiate dealling with. The following we will put forward the way that find the optimum si- -tuation (A_i*, B_j*) thet the fuzzy matrix game have a saddle point .

4 . FUZZY GAMES OF SADDLE PIONT EXISTED

Let us consider a fuzzy matrix game $G = \{S_1, S_2, A\}$

where
$$A = \{C_{ij}\}_{m*n}$$

supose that elements C_{ij} have a special structure , we might imitate the method that is to find optimum situation with the classical game theory to find the optimum situation (A_i*, B_j*) of the fuzzy matrix game .

Definition 4 . supose that the fuzzy matrix game

$$G = \{S_1, S_2, A\} \quad or \quad G = \{S_1, S_2, \mu\}$$

where $S_1 = \{A_i\}$ $\qquad$ $i = 1, 2, 3, \ldots\ldots, m$

$$\underset{\sim}{S}2 = \{ \ Bj \ \} \qquad\qquad j = 1, 2, 3, \ \ldots\ldots, n$$

$$\underset{\sim}{A} = \{ \ \underset{\sim}{C}ij \ \}m*n$$

$$\mu = \{ \mu ij \}m*n$$

If situation (Ai*, Bj*) make equetion

$$\underset{i}{\max}\ \underset{j}{\min}\ \underset{\sim}{C}ij \ = \ \underset{j}{\min}\ \underset{i}{\max}\ \underset{\sim}{C}ij$$

and

$$\underset{i}{\max}\ \underset{\sim}{C}ij* \ = \ \underset{j}{\min}\ \underset{\sim}{C}i*j$$

or

$$\underset{i}{\max}\ \underset{j}{\min}\ \mu ij \ = \ \underset{j}{\min}\ \underset{i}{\max}\ \mu ij$$

and

$$\underset{i}{\max}\ \mu ij* = \underset{j}{\min}\ \mu i*j$$

be tenable , its relevant value mark VG call the value of the fuzzy ganme G . The situation (Ai*, Bj*) is a saddle piont of the game G and name the optimun (satisfactory) situetion .

It $\underset{\sim}{C}ij$ is fuzzy number , we might use the way that find maximum or minimum fuzzy number to find a saddle piont of the fuzzy maxtrix game .

Example 2 . Find a saddle piont of the G and the game value VG and the optimum situation . supose that

$$G = \{ \ \underset{\sim}{S}1 \ , \ \underset{\sim}{S}2 \ , \ \underset{\sim}{A} \ \ \}$$

where

$$\underset{\sim}{S}1 = \{ \ A1 \ \ , \ A2 \ , A3 \ , \ A4 \ \}$$

$$\underset{\sim}{S}2 = \{ \ B1 \ , \ B2 \ , \ B3 \ \}$$

$$\underset{\sim}{A} = \begin{bmatrix} \underset{\sim}{-7} & \underset{\sim}{1} & \underset{\sim}{-8} \\ \underset{\sim}{3} & \underset{\sim}{2} & \underset{\sim}{4} \\ \underset{\sim}{16} & \underset{\sim}{-1} & \underset{\sim}{-3} \\ \underset{\sim}{-3} & \underset{\sim}{0} & \underset{\sim}{5} \end{bmatrix}$$

Solution 1 . Decide whether or not a saddle piont exist. First we find the minimum fuzzy number of each rows of the matrix $\underset{\sim}{A}$.

$$\min_{j} \underset{\sim}{C}1j = \min_{j} \{ -\underset{\sim}{7} , \underset{\sim}{1} , -\underset{\sim}{8} \} = -\underset{\sim}{8}$$

$$\min_{j} \underset{\sim}{C}2j = \min_{j} \{ \underset{\sim}{3} , \underset{\sim}{2} , \underset{\sim}{4} \} = \underset{\sim}{2}$$

$$\min_{j} \underset{\sim}{C}3j = \min_{j} \{ \underset{\sim}{16} , -\underset{\sim}{1} , -\underset{\sim}{3} \} = -\underset{\sim}{3}$$

$$\min_{j} \underset{\sim}{C}4j = \min_{j} \{ -\underset{\sim}{3} , \underset{\sim}{0} , \underset{\sim}{5} \} = -\underset{\sim}{3}$$

next we find the maximum fuzzy number in each minimum fuzzy numbers.

$$\max_{i} \{ -\underset{\sim}{8} , \underset{\sim}{2} , -\underset{\sim}{3} , -\underset{\sim}{3} \} = \underset{\sim}{2}$$

so

$$\max_{i} \min_{j} \underset{\sim}{C}ij = \underset{\sim}{C}22 = \underset{\sim}{2}$$

use the same method , we find

$$\min_{j} \max_{i} \underset{\sim}{C}ij = \underset{\sim}{C}22 = \underset{\sim}{2}$$

and

$$\max_{i} \underset{\sim}{C}i2 = \min_{j} \underset{\sim}{C}2j = \underset{\sim}{2}$$

So situation (A2 , B2) is a saddle piont and VG = $\underset{\sim}{2}$ and the optimum situation is (A2 , B2) .

Supose that we definite profit mumbership grades

$$\mu ij = \frac{\underset{\sim}{C}ij + | \underset{\sim}{C}ij* |}{\max \{ \underset{\sim}{C}ij + | \underset{\sim}{C}ij* | \}}$$

here $| \underset{\sim}{C}ij* | = \begin{cases} | \min \underset{\sim}{C}ij| & \text{if } \min Cij < 0 \\ \underset{\sim}{0} & \text{if } \min Cij >= 0 \end{cases}$

thus we can find a saddle piont according to the profit mumbership grades. According to the above definition , the fuzzy profit matrix $\underset{\sim}{A}$ can become

$$\mu = \begin{bmatrix} 0.041 & 0.375 & 0 \\ 0.458 & 0.416 & 0.5 \\ 1 & 0.291 & 0.209 \\ 0.209 & 0.333 & 0.541 \end{bmatrix}$$

Solution 2.We use profit membership function matrix to determine a saddle piont of the fuzzy game problem .

First we obtain

$$\max_i \min_j \{\mu_{ij}\} = \max\{0, 0.416, 0.209, 0.209\} = 0.416 = \mu_{22}$$

next obtain

$$\min_j \max_i \{\mu_{ij}\} = \min_j \{1, 0.416, 0.541\} = 0.416 = \mu_{22}$$

and $\max \mu_{i2} = \min \mu_{2j} = 0.416 = \mu_{22}$

So we obtain the saddle piont to be (A2,B2) and the result is same with solution 1 . the profit membership grade of optimum situation is 0.416 that express a specific value of VG with the maximum gains.We consider this expression still more express the gains grade of optimum situation .

It C_{ij} is imprecise variable (fuzzy subsets), we might imitate the classical method to obtain the saddle piont of the game,according to the variables preference order provided .

Example 3. Obtain the VG and the optimum situation (Ai* , Bj*), supose that the fuzzy matrix game

$$G = \{S_1, S_2, A\}$$

where $S_1 = \{A1, A2, A3\}$

$$S_2 = \{B1, B2, B3\}$$

$$A = \begin{bmatrix} VH & H & VH \\ VH & L & M \\ L & VL & H \end{bmatrix}$$

Solution : We introduce priority order that is

$$VL \propto L \propto M \propto H \propto VH .$$

So we have

$$\min_j C1j = \min_j \{VH ,H ,VH \} = H$$

$$\min_j C2j = \min_j \{VH ,L , M \} = L$$

$$\min_j C3j = \min_j \{L , VL ,H \} = VL$$

thus

$$\max_i \min_j Cij = \max_i \{ H , L ,VL \} = H$$

use the same method , we find

$$\min_j \max_i Cij = H$$

and

$$\min_j C1j = \max_i Ci2 = H$$

So we obtain the saddle piont to be (A1 , B2) that is the optimum situation of the game G and VG = H .

Supose that we introduce the specific values of among imprecise variables, the fuzzy gain matrix that is prencented by imprecise variables might transform the gains membership function matrix

$$\mu = \{ \mu ij \}m*n .$$

Use the method

$$\max_i \min_j \mu ij = \min_j \max_i \mu ij$$

and

$$\max_i \mu ij = \min_j \mu ij$$

determene whether or not the saddle piont existed .

5 . CONCUDING REMARKS

In the above--mentioned sections, we have put the basic concepts of fuzzy game and the method finding the optimum situation of the fuzzy

matrix game,it is one of the fundamental contents of fuzzy game theory.

The method finding the optimum situation is based on the real application,its result is directly perceived through the senses. The theory about fuzzy games and the method finding the optimum situation of other fuzzy probles will be put separately .

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