A NOTE ON PIASECKI'S P-MEASURE

Zbigniew Świtalski

Os. Chrobrego 35/49

60-681 Poznań, Poland

We announce here a theorem on representation of Piasecki's P-measure by a usual probability measure on a σ-Boolean algebra.

Key word : Fuzzy probability measure.

Let $\mathfrak{S}$ be a family of fuzzy subsets of a universum Ω /i.e. mappings $\Omega \longrightarrow [0,1]$ / containing 0_Ω , 1_Ω ,not containing $\left\{\frac{1}{2}\right\}_\Omega$ and closed under countable union and complement. Such families are called by Piasecki [1] soft fuzzy σ-algebras.

Definition [1]. By W-empty fuzzy subset of Ω we will mean a mapping $\mu : \Omega \longrightarrow [0,1]$ such that $\mu \leqslant \mu'$ /$\mu' = 1 - \mu$/.

Definition [1]. By W-universum in Ω we will mean a mapping $\mu : \Omega \longrightarrow [0,1]$ such that $\mu \geqslant \mu'$.

Definition [1]. Two fuzzy subsets μ and ν of Ω are called W-separated sets if $\mu \leqslant \nu'$.

Definition [1]. Let $\mathfrak{S}$ be a soft fuzzy σ-algebra. A mapping $p : \mathfrak{S} \longrightarrow R^+ \cup \{0\}$ is called a P-measure on $\mathfrak{S}$ if

1/ $p(\mu) = 1$ for any W-universum μ ,

2/ $p(\bigcup_n \mu_n) = \sum_n p(\mu_n)$ for each countable family of pairwise W-separated sets from σ .

The representation theorem for the P-measure runs as follows.

<u>Theorem.</u> For any soft fuzzy σ-algebra σ and any P-measure p on σ there exists a σ-Boolean algebra $\mathcal{A}$, a σ-De Morgan algebras homomorphism $h: \sigma \longrightarrow \mathcal{A}$ and a usual probability measure $p_1: \mathcal{A} \longrightarrow R^+ \cup \{0\}$ such that the following diagram commutes

$$\begin{tikzcd} \sigma \arrow[rr, "h"] \arrow[dr, "p"'] & & \mathcal{A} \arrow[dl, "p_1"] \\ & R^+ \cup \{0\} & \end{tikzcd} \qquad /1/$$

<u>Proof.</u> We will give only a brief sketch of the proof. Let W_0 be the class of all W-empty subsets from σ and let $\sim$ denotes the following relation in σ :

$$\mu \sim \nu \iff (\mu \wedge \nu') \vee (\nu \wedge \mu') \in W_0 .$$

The relation $\sim$ is a tolerance on σ /reflexive and symmetric/ and may be not transitive. Let $\approx$ be the transitive closure of $\sim$. From the results of Piasecki [1] it can be checked /this part of the proof is omitted here/ that $\approx$ is consistent with the operations of taking countable sum /intersection/ and complement. We have also $\mu \vee \mu' \approx 1$, $\mu \wedge \mu' \approx 0$ and

$$\mu \approx \nu \implies p(\mu) = p(\nu) .$$

Hence $\mathcal{A} = \mathfrak{S}/\approx$ is a $\mathfrak{S}$-Boolean algebra and, if we define $h(\mu) = [\mu]$, $p_1([\mu]) = p(\mu)$, then h is a $\mathfrak{S}$-De Morgan algebras homomorphism, p_1 is a usual probability measure on $\mathcal{A}$ /i.e. $p_1(1) = 1$ and $p_1(\bigcup_n \mu_n) = \sum_n p_1(\mu_n)$ for any countable family μ_n of pairwise disjoint elements from $\mathcal{A}$ / and the diagram /1/ commutes. This ends the proof.

By the above theorem we can derive all the properties of P-measure established by Piasecki [1] in a very easy way. For example the continuity from below of p /Theorem 3.4. in [1] / is a consequence of continuity from below of p_1 and commutation of h with countable unions, namely

$$p(\bigcup_n \mu_n) = p_1 h(\bigcup_n \mu_n) = p_1\left[\bigcup_n h(\mu_n)\right] = \lim_{n\to\infty} p_1\left[h(\mu_n)\right] = \lim_{n\to\infty} p(\mu_n).$$

Other results of [1] can be obtained analogously with the help of /1/.

<u>Reference</u>

[1] K.Piasecki, Probability of fuzzy events defined as denumerable additivity measure, Fuzzy Sets and Systems 17/1985/ 271-284.