## A NOTE ON PIASECKI'S P-MEASURE

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We announce here a theorem on representation of Piasecki's P-measure by a usual probability measure on a 6-Boolean algebra.

Key word : Fuzzy probability measure.

Let 6 be a family of fuzzy subsets of a universum  $\Omega$  /i.e. mappings  $\Omega \longrightarrow [0,1]$  / containing  $0_{\Omega}$ ,  $1_{\Omega}$ , not containing  ${1 \choose 2}_{\Omega}$  and closed under countable union and complement. Such families are called by Piasecki [1] soft fuzzy 6-algebras.

Definition [1]. By W-empty fuzzy subset of  $\Omega$  we will mean a mapping  $\mu: \Omega \longrightarrow [0,1]$  such that  $\mu \leqslant \mu'/\mu' = 1 - \mu/$ .

Definition [1]. By W-universum in  $\Omega$  we will mean a mapping  $\mu: \Omega \longrightarrow [0,1]$  such that  $\mu \geqslant \mu'$ .

Definition [1]. Two fuzzy subsets  $\mu$  and  $\nu$  of  $\Omega$  are called W-separated sets if  $\mu \leqslant \nu'$ .

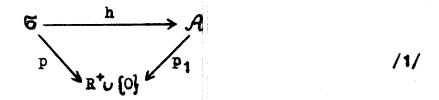
Definition [1]. Let 6 be a soft fuzzy 6-algebra. A mapping  $p: 6 \longrightarrow R^+ \cup \{0\}$  is called a P-measure on 6 if

 $1/p(\mu) = 1$  for any W-universum  $\mu$ ,

2/  $p(\bigcup_{n} \mu_{n}) = \sum_{n} p(\mu_{n})$  for each countable family of pairwise W-separated sets from 6.

The representation theorem for the P-measure runs as follows.

Theorem. For any soft fuzzy 6-algebra 6 and any P-measure p on 6 there exists a 6-Boolean algebra A, a 6-De Morgan algebras homomorphism  $h: 6 \longrightarrow A$  and a usual probability measure  $p_4: A \longrightarrow R^+ \cup \{0\}$  such that the following diagram commutes



<u>Proof.</u> We will give only a brief sketch of the proof. Let  $W_6$  be the class of all W-empty subsets from G and let  $\sim$  denotes the following relation in G:

The relation  $\sim$  is a tolerance on  $\mathcal{F}$  /reflexive and symmetric/ and may be not transitive. Let  $\approx$  be the transitive closure of  $\sim$ . From the results of Piasecki [1] it can be checked /this part of the proof is omitted here/ that  $\approx$  is consistent with the operations of taking countable sum /intersection/ and complement. We have also  $\mu \vee \mu' \approx 1$ ,  $\mu \wedge \mu' \approx 0$  and

$$\mu \approx \nu \implies p(\mu) = p(\nu)$$
.

Hence  $A = 6/\approx$  is a 6-Boolean and, if we define  $h(\mu) = [\mu]$ ,  $p_1([\mu]) = p(\mu)$ , then h is a 6-De Morgan algebras homomorphism,  $p_1$  is a usual probability measure on A/i.e.  $p_1(1) = 1$  and  $p_1(\bigcup \mu_n) = \sum_{n} p_1(\mu_n)$  for any countable family  $\mu_n$  of pairwise disjoint elements from A / and the diagram /1/ commutes. This ends the proof.

By the above theorem we can derive all the properties of P-measure established by Piasecki [1] in a very easy way. For example the continuity from below of p /Theorem 3.4. in [1] / is a consequence of continuity from below of  $p_1$  and commutation of h with countable unions, namely

 $p(\bigcup_{n}\mu_{n}) = p_{1}h(\bigcup_{n}\mu_{n}) = p_{1}[\bigcup_{n}h(\mu_{n})] = \lim_{n\to\infty} p_{1}[h(\mu_{n})] = \lim_{n\to\infty} p(\mu_{n}).$ Other results of [1] can be obtained analogously with the help of /1/.

## Reference

[1] K.Piasecki, Probability of fuzzy events defined as denumerable additivity measure, Fuzzy Sets and Systems 17/1985/ 271-284.