ON ITERATIVE FORMULA OF THE NUMBER OF LOWER SOLUTIONS FOR FUZZY RELATION EQUATION ON FINITE SET

Sun Rong-guang

(Department of Mathematics, Henan University, Kaifeng, Henan, The People's Republic of China)

Chen Guo-xun

(Department of Computer Science, Zhengzhou University, Zhengzhou, Henan, The people's Republic of China)

Yan Jia-jie

(Department of Mathematics and Mechanics, Zhengzhou Technical Institute, Zhengzhou, Henan, The People's Republic of China)

AbStract

In 1982, E. Czogala and the others asked the question that if the fuzzy relation equation on finite set has any solution, how many lower solutions does it have? And he has got a rough estimate of their upper bound (see[1]). In 1984, Wang Pei-zhuang and the others [2] has got a precise formula calculating this number of lower solutions of a finite fuzzy relation equation which satisfies $b_1 > b_2 > \cdots > b_m$. In 1985, Wang Jin-yi [3] got an iterative formula of lower solutions number for fuzzy relation equation on finite set which satisfies $b_{j-1} \geq b_j$ $(j=2,3,\cdots m)$. In this paper, we first point that the iterative formula in [3] is not suitable for general case. Next, the condition that the j-dimensional path is eliminated when $b_{j-1}=b_j$ in [3] is sufficient but is not necessary, which theoretical proof is given. Finally, we have corrected the iterative formula in [3]. And we got a precise formula calculating this number of lower solutions of a fuzzy relation equation on finite set for general case.

Let $(x_1,\cdots, x_n)\circ(a_{ij})_{n\times m}=(b_1,\cdots, b_m)$ $(b_1 \geq b_2 \geq \cdots \geq b_m > 0)$ (1.0)

be a fuzzy relation equation.

Define: $d_{ij} \triangleq \begin{cases} 0; & a_{ij}<b_j; \text{ or } a_{ij}>b_j \text{ and } \exists k \in \{1,2,\cdots,m-j\} \text{ make } b_j>b_{j+k} \\ & \text{and } a_{i\,j+k}>b_{j+k} \\ 1, & \text{otherwise} \end{cases}$

$$V_j \triangleq \sum_{j=1}^{n} d_{ij} \qquad (j=1,\cdots,m)$$

$$D_j \triangleq \{i \mid d_{ij}=1\} \text{ with } l_j \in D_j \quad (j=1,2,\cdots,m)$$

Call: $\overline{y_p}=(l_1,\cdots,l_p)$ after p-dimensional path and define $\overline{Y}_p \triangleq \{l_1,\cdots,l_p\}$

Suppose that all l-dimensional paths are effective paths, and L_1 denotes the all l-dimensional paths.

If L_{j-1} denotes set of j-1—dimensional effective paths, then the set L_j of j-dimensional effective paths is obtained according to the following method.

We select arbitrarilly for $\overline{y_{j-1}}=(l_1,\cdots,l_{j-1}) \in L_{j-1}$.

(1) If $\overline{Y}_{j-1}\cap D_j \neq \phi$, then we only select arbitrarilly for an element $l_j \in (\overline{Y}_{j-1}\cap D D_j)$. And we extend $\overline{y_{j-1}}$ into a j-dimensional effective path $\overline{y_j}=(l_1,\cdots,l_{j-1},l_j) \in L_j$.

(2) If $\overline{Y}_{j-1}\cap D_j=\phi$, then

2.1) when $b_{j-1}>b_j$, we take $l_j \in D_j$ that reach as far as D_j. And we divide up $\overline{y_{j-1}}$ into $|D_j|$ j-dimensional effective paths.

2.2) When $b_{j-1}=b_j$, we have

2.21) if exist $\overline{y'_{j-1}}=(l'_1,\cdots,l'_{j-2},l'_{j-1}) \in L_{j-1}$ such that $l'_1=l_1,\cdots,l'_{j-2}=l_{j-2}$ $l'_{j-1}\in D_j$, then we take $l_j \in E_j$ that reach as far as $E_j \triangleq D_j \setminus (D_{j-1}\cap D_j)$. And we divide up $\overline{y_{j-1}}$ into $|E_j|$ j-dimensional effective paths.

2.22) if the condition in 2.21) is not satisfied, then we divide up $\overline{y_{j-1}}$ into $|D_j|$ j-dimensional effective paths by method in 2.1).

Here L_j be composed of all j-dimensional effective paths according to the above method.

Define: $u_j \triangleq$ in L_{j-1} number of paths which satisfies $\overline{Y}_{j-1}\cap D_j=\phi$.

De fine: $g_j \triangleq |D_{j-1}\cap D_j|$

Define: $h_j \triangleq$ number of the $\overline{y_{j-1}}=(l_1,\cdots,l_{j-1}) \in L_{j-1}$ such following three conditions must be satisfied:

i) $\overline{Y}_{j-1}\cap D_j=\phi$ (1.1)

ii) $b_{j-1}=b_j$ (1.2)

iii) exist $\overline{y'_{j-1}}=(l'_1,\cdots,l'_{j-1}) \in L_{j-1}$ with $l'_1=l_1,\cdots,l'_{j-2}=l_{j-2}$, $l'_{j-1}\in D_j$ (1.3)

In [3], the iterative formula of j-dimensional effective path's number $t_j \triangleq |L_j|$ is

$$t_j=\begin{cases} t_{j-1}+u_j(v_j-1), & b_{j-1}>b_j \\ t_{j-1}+u_j(v_j-1)+g_jh_j, & b_{j-1}=b_j \end{cases} \qquad (1.4)$$

($t_1 \triangleq v_1$, and $j=2,3,\cdots m$), where t_m is the number of lower solution in (1.0)

If the formula (1.4) served equation

$$(x_1, x_2, x_3, x_4) \circ \begin{pmatrix} 0.9 & 0.5 & 0.4 & 0.5 \\ 0.9 & 0.7 & 0.3 & 0.8 \\ 0.8 & 0.6 & 0.6 & 0.5 \\ 0.9 & 0.4 & 0.6 & 0.5 \end{pmatrix} = (0.8, 0.6, 0.6, 0.6,) \qquad (1.5)$$

as for calculation of number of lower solutions, then an erroneous result is obstained.

With the help of following theorems, the reason that (1.4) occurs error is pointed

Definition 1: The path is called a strange path in L_{j-1}, if the conditions (1.1), (1.2) and (1.3) are satisfied, $(j=2,\cdots,m)$.

Definition 2: In $\overline{y_p}=(l_1,\cdots,l_p)$, if an amount of l_j are equation, then the first l_j is retained. And we change others l_j into 0. Thus we get a new vector $y_p=(k_1, \cdots, k_p)$ $(k_j=l_j$ or $0)$. The y_p is called a regular path correspond to $\overline{y_p}$.

Theorem 1: Suppose that $\overline{y_{j-1}}=(l_1,\cdots,l_{j-1}) \in L_{j-1}$ is a strange path, if the path $\overline{y_j}=(l_1,\cdots,l_{j-1},l_j)$ is eliminated for every $l_j \in (D_{j-1} \cap D_j)$, then every lower solution of the fuzzy relation equation on finite set can not be missed. However, such conditions as the surplus paths are eliminated are sufficient but not necessary.

Theorem 2: Suppose that the regular j-dimensional path $y_j=(k_1,\cdots,k_j)$ satisfies

1) $\{k_1,\cdots,k_j\} \cap D_j=\phi$,

2) $b_{j-1}=b_j$

Thus y_j is eliminated iff $\exists k_{i_0} \neq 0$ (where $i_0 \in \{t,\cdots,j-1\}$, $t=m$ in $\{i \mid b_i=b_j\}$), $\forall i \in \{t,\cdots,j-1\}$ has always $(D_i \setminus \{k_{i_0}\} \cap \{k_1,\cdots,k_j\} \neq \phi$.

Supoose that f_j denotes number of j-dimensional path which satisfies conditions in theorem 2 and is eliminated.

Define: $\delta_j \triangleq f_j - g_j h_j$

Theorem 3: Let $\overline{f_j}=\begin{cases} 0 & b_{j-1}>b_j \\ g_j h_j+\delta_j & b_{j-1}=b_j \end{cases}$ Then the iterative formula of number t_j of j-dimensional paths is

$$t_j=t_{j-1}+u_j(v_j-1))-\overline{f_j}$$

where $t_1=v_1$ and $j=2,\cdots,m$. Also t_m is number of lower solution for (1.0)

This is a precise result.

Reference

[1] E. Czogala, J. Drewniak, W. Pedrycz, Fuzzy Relation Equations on a Finite Set, Fuzzy and Systems 7 (1982) 89--101

[2] Wang Pei-Zhuang, Luo Cheng-zhong, The Number of Lower Solutions for Finite Fuzzy Relation Equation, Fuzzy Math, 3 (1984) 63—70. (in Chinese)

[3] Wang Tin-yi, An Iterative Formula Finding the Number of Lower Solutions of Finite Fuzzy Relation Equations, IFSA—85 July 1 — 6, 1985. In Spain.