Fuzzy Detection of Radar Signals

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Abstract

The method of detecting radar signal in correlated clutters by the use of fuzzy subset theory is presented. The mathematical model of fuzzy detections is established first, and then the neartude criterion is proposed which is suitable for the above model, Based on the criterion and the concept of importance weighing, the fuzzy filter algorithm is derived. Two fuzzy detectors, FD-I and FD-II, are constructed. Monte-carlo simulations are made to evaluate the performances of these two fuzzy detectors.

Key words —— fuzzy detection, fuzzy detector, neartude criterion, importance weighing, fuzzy filler algorithm, clutter pattern, signal pattern.

1. Introduction

The problem of detecting radar signal in correlated clutter has been an important and significant subject in the field of modern radar signal processing. So far, There have existed lots of effect methods for restraining clutter, Some of which have been successfully used in some of radar systems. Just as pointed out in most of literatures, It has not been comp letely solved to detect radar signals in clutters since the enviroments of radar

targets are so complicated that we cannt precisely describe it with the aid of traditional mathamatics, and this description is indispensable for designing a detector.

Thus, it is very important to propose a method for restraining clutters which can avoid describing enviroments of radar targets precisely in some degree, has good detection performance, and can be realized easily in practices.

As a normal mathematical theory, fuzzy mathematics has greatly developed and almost permeated various fields of sciences and technologies. There are many examples of applications of fuzzy mathematics to the electronic systems such as commanication, radar and so on. It follows from the existed results that the applications of fuzzy subset theories to radar signal processing have broad prospects.

M.B.E. Fntmi and P.P. Wang[2] published an article titled "Fuzzy detector and fuzzy estimator in communication" in 1981 in which seven fuzzy detetors are proposed. Assuming that the interference is additive white Gaussian noise, and the signal-to-noise ratio is equal to -20 dB, they made the Monte-carlo simulation for these seven fuzzy detectors. The results obtained show that fuzzy detection is feasible.

The detection of radar signals in correlated clutters are studied by using fuzzy subset theory in this paper. The model, criterion, algorithm, and structure of fuzzy detection are proposed, and Monte-carlo simulations are made on the NOBUS-Z microcomputer. It is proved that the fuzzy detectors presented in this paper have good detection performance. In the case of strong-correlated clutters, the detection performance of FD-II is much better than that of correlative detector (SD).

2. Mathematical Model of Fuzzy Detections

Let $\underline{C} = [C_1\ C_2 \ldots C_N]^T$ be clutter vector; $\underline{r} = [r_1\ r_2\ \ldots\ r_N]^T$

observation vector, R observation space, $\underset{\sim}{R}_1 = (\underline{r}, u_{\underset{\sim}{R}_1}(\underline{r})/\underline{r} \in R)$, and $\underset{\sim}{R}_0 = (\underline{r}, u_{\underset{\sim}{R}_0}(\underline{r})/\underline{r} \in R)$. Where $\underset{\sim}{R}_1$ is a fuzzy subset of R whose elements contain signal "very possibly" and $\underset{\sim}{R}_0$ "very impossibly".

It is the key to fuzzy detection to calculate the grades of membership of $\underline{r}$ in $\underset{\sim}{R}_0$ and $\underset{\sim}{R}_1$ ($u_{\underset{\sim}{R}_0}(\underline{r})$, and $u_{\underset{\sim}{R}_1}(\underline{r})$). If $u_{\underset{\sim}{R}_0}(\underline{r})$ and $u_{\underset{\sim}{R}_1}(\underline{r})$ have been obtained, we can make a decision as follows:

$$H_1 \Big| u_{\underset{\sim}{R}_1}(\underline{r}) = u_{\underset{\sim}{R}_0}(\underline{r}) \vee u_{\underset{\sim}{R}_1}(\underline{r})$$

or

$$H_0 \Big| u_{\underset{\sim}{R}_0}(\underline{r}) = u_{\underset{\sim}{R}_0}(\underline{r}) \vee u_{\underset{\sim}{R}_1}(\underline{r})$$

and

$$u_{\underset{\sim}{R}_0}(\underline{r}) \neq u_{\underset{\sim}{R}_1}(\underline{r}).$$

The mathe matical model of fuzzy detection is shown in Fig. 2.1. It is principally made up of two parts:

(i) Fuzzy filter algorithm for calculating $u_{\underset{\sim}{R}_0}(\underline{r})$ and $u_{\underset{\sim}{R}_1}(\underline{r})$;

(ii) operator "V".

It is clear that this model is similar to that of brain's recognition and decision. In fact,

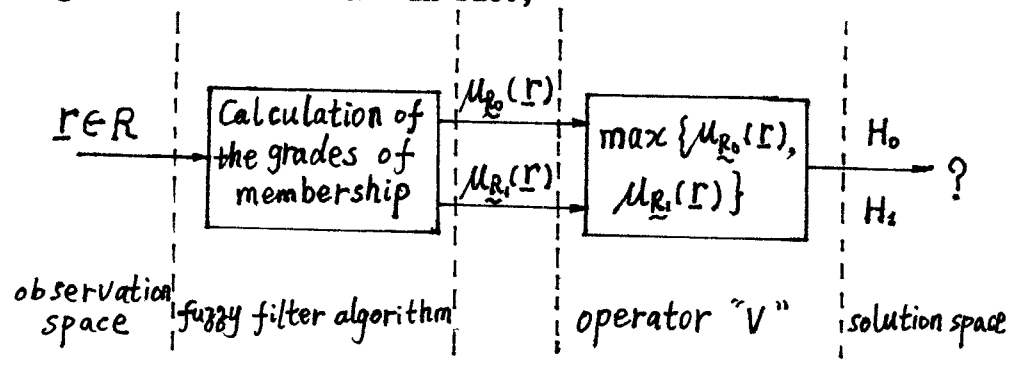

Fig. 2.1 The mathematical model of fuzzy detection

It is according to the characteristics of brain's recognition and decision that the model is found here. It is well known that a radar observer always asks himself about the problems: Does the observed signal $\underline{r} \in R$ take after the signal reflected by target or clutter? In order to answer the problems, the radar observer

must decide the grades of $u_{\underset{\sim}{R}_0}(\underline{r})$ and $u_{\underset{\sim}{R}_1}(\underline{r})$.

3. Fuzzy Filter Algorithm

It follows from Fig. 2.1 that the fuzzy filter algorithm is a key part of the model of fuzzy detection. The discusion here is based on the neartude criterion and the concept of importance weighing.

Definition 1. Let V be a universe, $\mathcal{F}(V)$ the family of all fuzzy subsets of V. The neartude on $\mathcal{F}(V)$ is a map from $\mathcal{F}(V)\times\mathcal{F}(V)$ to [0,1]:

$$\rho: \quad \mathcal{F}(V)\times\mathcal{F}(V) \longrightarrow [0,1],$$

which is satisfied with (i) $\forall\, \underset{\sim}{A}, \underset{\sim}{B} \in \mathcal{F}(V)$, $\rho(\underset{\sim}{A},\underset{\sim}{B})$ takes maximum value when $\underset{\sim}{A} = \underset{\sim}{B}$ and minimum when $\underset{\sim}{A} = \underset{\sim}{0}$ and $\underset{\sim}{B} = \underset{\sim}{1}$; (ii) $\forall\, \underset{\sim}{A},\underset{\sim}{B} \in \mathcal{F}(V)$, $\rho(\underset{\sim}{A},\underset{\sim}{B}) = \rho(\underset{\sim}{B},\underset{\sim}{A})$; (iii) $\forall\, \underset{\sim}{A}, \underset{\sim}{B}, \underset{\sim}{C} \in \mathcal{F}(V)$. If $\underset{\sim}{A} \subseteq \underset{\sim}{B} \subseteq \underset{\sim}{C}$ or $\underset{\sim}{A} \supseteq \underset{\sim}{B} \supseteq \underset{\sim}{C}$, then $\rho(\underset{\sim}{A},\underset{\sim}{B}) \geqslant \rho(\underset{\sim}{A},\underset{\sim}{C})$.

It can be seen from definition that the neartude $\rho(\cdot,\cdot)$ actually is a binary fuzzy relation in $\mathcal{F}(V)$ which is satisfied with (i), (ii) and (iii). It reflects the near relation between fuzzy subsets of V.

$\forall\, \underset{\sim}{A}, \underset{\sim}{B} \in \mathcal{F}(V)$, let $\underset{\sim}{A} \circ \underset{\sim}{B} \triangleq \sup\limits_{u\in V} (u_{\underset{\sim}{A}}(u) \wedge u_{\underset{\sim}{B}}(u))$ and $\underset{\sim}{A} \oplus \underset{\sim}{B} \triangleq \inf\limits_{u\in V} (u_{\underset{\sim}{A}}(u) \vee u_{\underset{\sim}{B}}(u))$, where $\underset{\sim}{A}\circ\underset{\sim}{B}$ is known as inner-product, $\underset{\sim}{A} \oplus \underset{\sim}{B}$ outer-product.

Lemma 1. $\forall\, \underset{\sim}{A} \in \mathcal{F}(v)$, we have:

$$\underset{\sim}{A} \circ \underset{\sim}{A} = \sup_{B\in\mathcal{F}(v)} \underset{\sim}{A} \circ \underset{\sim}{B}, \qquad \underset{\sim}{A} \oplus \underset{\sim}{A} = \inf_{B\in\mathcal{F}(v)} \underset{\sim}{A} \oplus \underset{\sim}{B}$$

Lemma 2. If $\underset{\sim}{B} \supseteq \underset{\sim}{A}$, then $\underset{\sim}{A} \circ \underset{\sim}{B} = \bar{\underset{\sim}{A}}$; If $\underset{\sim}{B} \subseteq \underset{\sim}{A}$, then $\underset{\sim}{A} \oplus B = \underline{\underset{\sim}{A}}$, where $\bar{\underset{\sim}{A}} = \sup\limits_{u\in V} (u_{\underset{\sim}{A}}(u))$, $\underline{\underset{\sim}{A}} = \inf\limits_{u\in V} (u_{\underset{\sim}{A}}(u))$.

From lemma 1 and lemma 2 we have:

Theorem 1. Let v be a finte set, ρ a map from $\mathcal{F}(V)\times\mathcal{F}(V)$ to [0,1], $\forall\, \underset{\sim}{A}, \underset{\sim}{B} \in \mathcal{F}(V)$,

$$\rho(\underset{\sim}{A}, \underset{\sim}{B}) = [\underset{\sim}{A}\circ\underset{\sim}{B}] \vee [1-(\underset{\sim}{A} \oplus \underset{\sim}{B})],$$

Then, ρ is a neartade on $\mathcal{F}(V)$.

Corollary 1. If $V = \{u_i\}$, then,

$$\rho(\underset{\sim}{A},\underset{\sim}{B}) = (u_{\underset{\sim}{A}}(u_i) \wedge u_{\underset{\sim}{B}}(u_i)) \vee (\overline{u_{\underset{\sim}{A}}(u_i)} \wedge \overline{u_{\underset{\sim}{B}}(u_i)}).$$

Let $M_{\underset{\sim}{r}}(\underset{\sim}{r}) = [\, u_1^r \;\; u_2^r \; \dots \;\; u_N^r]^T$ represent membership vector of $\underset{\sim}{r}$, $M_{\underset{\sim}{s}}(\underset{\sim}{s}) = [\, u_1^s \;\; u_2^s \; \dots \;\; u_N^s]^T$ membership vector of signal pattern, and $M_{\underset{\sim}{c}}(\underset{\sim}{c}) = [\, u_1^c \;\; u_2^c \; \dots \;\; u_N^c]^T$ membership vector of clutter pattern. From corollary 1 we obtain the neartade vectors as follows:

$$\underline{\rho}(\underset{\sim}{r},\underset{\sim}{s}) \triangleq \begin{bmatrix} \rho_1^s \\ \rho_2^s \\ \vdots \\ \rho_N^s \end{bmatrix} = \begin{bmatrix} \mu_1^s & \overline{\mu_1^s} & & & & \\ & & \mu_2^s & \overline{\mu_2^s} & & 0 \\ & 0 & & & \ddots & \\ & & & & \mu_N^s & \overline{\mu_N^s} \end{bmatrix}_{(N\times 2N)} \circ \begin{bmatrix} \mu_1^r \\ \overline{\mu_1^r} \\ \vdots \\ \mu_N^r \\ \overline{\mu_N^r} \end{bmatrix}$$

$$\triangleq R_s \circ [\mu_1^r \;\; \overline{\mu_1^r} \; \cdots \; \mu_N^r \;\; \overline{\mu_N^r}]^T,$$

$$\underline{\rho}(\underset{\sim}{r},\underset{\sim}{c}) \triangleq \begin{bmatrix} \rho_1^c \\ \rho_2^c \\ \vdots \\ \rho_N^c \end{bmatrix} = \begin{bmatrix} \mu_1^c & \overline{\mu_1^c} & & & & \\ & & \mu_2^c & \overline{\mu_2^c} & & 0 \\ & 0 & & & \ddots & \\ & & & & \mu_N^c & \overline{\mu_N^c} \end{bmatrix}_{(N\times 2N)} \circ \begin{bmatrix} \mu_1^r \\ \overline{\mu_1^r} \\ \vdots \\ \mu_N^r \\ \overline{\mu_N^r} \end{bmatrix}$$

$$\triangleq R_c \circ [\mu_1^r \;\; \overline{\mu_1^r} \; \cdots \; \mu_N^r \;\; \overline{\mu_N^r}]^T.$$

R_s and R_c are knows as fuzzy filter matrix.

Definition 2. $K_i \triangleq (u_i^s \vee u_i^c) - (u_i^s \wedge u_i^c)$ is called importance of "signal-clutter pattern pair" at ith information point. $\underline{K} = [K_1 \; K_2 \; \dots \; K_N]^T$ is called importance vector of information source.

Weighting the neartude vectors by $\underline{K}$, we have:

$$\underline{\rho}_{\underline{K}}(\underset{\sim}{r},\underset{\sim}{s}) = [\rho_{K_1}^s \; \rho_{K_2}^s \; \dots \; \rho_{K_N}^s]^T \quad ,$$

$$\underline{\rho}_{\underline{K}}(\underset{\sim}{r},\underset{\sim}{c}) = [\rho^{c}_{K_1}\ \rho^{c}_{K_2} \cdots \rho^{c}_{K_N}]^T,$$

where $\rho^{s}_{K_i} = K_i \cdot \rho^{s}_{i}$ and $\rho^{c}_{K_i} = K_i \cdot \rho^{c}_{i}$ $(i = 1, 2, \ldots N)$.

Now we can calculate the grades of membership of $\underline{r}$ in $\underset{\sim}{R}_0$ and $\underset{\sim}{R}_1$ as follows:

$$u_{\underset{\sim}{R}_0}(\underline{r}) = 1/[1 + (\sum_{i=1}^{N} \rho^{s}_{K_i} / \sum_{i=1}^{N} \rho^{c}_{K_i})]$$

and

$$u_{\underset{\sim}{R}_1}(\underline{r}) = 1/[1 + (\sum_{i=1}^{N} \rho^{c}_{K_i} / \sum_{i=1}^{N} \rho^{s}_{K_i})] .$$

The steps of the fuzzy fllter algorithm are:

a) Calculating memberships vector of $\underline{r}$ — $M_{\underset{\sim}{r}}(\underline{r})$;

b) Establishing fuzzy filter matrix — R_s and R_c;

c) Calculating importance vector of information source — $\underline{K}$;

d) Calculating neartude vectors — $\underline{\rho}(\underset{\sim}{r},\underset{\sim}{s})$ and $\underline{\rho}(\underset{\sim}{r},\underset{\sim}{c})$;

e) Calculating weighted neartued vectors — $\underline{\rho}_{\underline{K}}(\underset{\sim}{r},\underset{\sim}{s})$ and $\underline{\rho}_{\underline{K}}(\underset{\sim}{r},\underset{\sim}{c})$;

f) Calculating $u_{\underset{\sim}{R}_0}(\underline{r})$ and $u_{\underset{\sim}{R}_1}(\underline{r})$.

4. Two Fuzzy Detectors

In the light of model, criterion, and algorithm derived above. Two fuzzy detectors, FD-I and FD-II, are designed, here we choose the form of redar transmission signals as irregular "Coincidence no more than one" pulse sequence. Hence, the signal pattern is

$$M_{\underset{\sim}{s}}(\underline{s}) = [s_1\ s_2 \ldots s_N]^T,$$

where $s_i = 0$ or 1, $(i = 1, 2, \ldots, N)$, N is the ordinal number of code "1" which is final in this sequence.

FD-I is used for detecting a steady target in the weak-correlated clutter. The clutter pattern is $M_{\underset{\sim}{c}}(\underline{c}) = [0\ 0 \ldots 0]^T$, and $u^{r}_{i} = (r_i - r_{min})/(r_{max} + A - R_{min})$, $K^{I}_{i} = s_i$ $(i=1,2,\ldots N,$

$A \geqslant 0$).

FD-II is used for detecting a swerling-II target in strong-correlated clutter. The clutter pattern is

$$M_{\underset{\sim}{c}}(\underline{c}) = [1 \quad 1 \quad \ldots \quad 1]^T,$$

and $K_i^{II} = \tilde{s}_i$,

$$u_i^r = \begin{cases} ((1-A)/r_{max})r_i + A, & (r_i \geqslant (r_{max}/(A-1))\cdot A), \\ 0 & \text{, otheraise.} \end{cases}$$

$$(i = 1, 2, \ldots N, \quad A \leqslant 1).$$

where $r_{max} = \max_{i=1}^{N} \{r_i\}$, $r_{min} = \min_{i=1}^{N} \{r_i\}$.

5. Simulation and Concluding Remarks

The monte-carto simalation is made on Nobus-Z microcomputer to evalaste the performance of FD-I and FD-II. We choose Rayleigh distribution and weibull distribution as clutte models. The results of simulation are shown in Fig.1 to Fig. 4. We also compare the performance of FD-II to that of correlative detector under the same conditions, as shown in Fig. 5 and Fig. 6.

It follows from studies in this paper that:

a) It is teasible to detect radar signal in extremly complicated enviroments of clutters by using fuzzy subset theory since we neednot describe the statistic natures of clutters precisely to design fuzzy detector;

b) The fuzzy detection incorporates the characteristics of brain's recognition and decision and is nimbler than statistical detector;

c) The performance of fuzzy detector is better than that of correlative detector (SD) in strong-correlated chutters;

d) FD-I and FD-II have good detection performance in correlated clutter. We are convinced that the other fuzzy detectors can be

derived which have better detection performance than FD-I and FD-II.

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6. References

[1]. M.D. Srinath and P.K. Rajasekaran, An Introduction to statistical signal processing with Applications, John wiley and sons, 1979.

[2]. M.B.E. Fatmi and P.P. Wang, Fuzzy detector and fuzzy estimator in communication, In "Fuzzy set and Possibility theory", 1981.

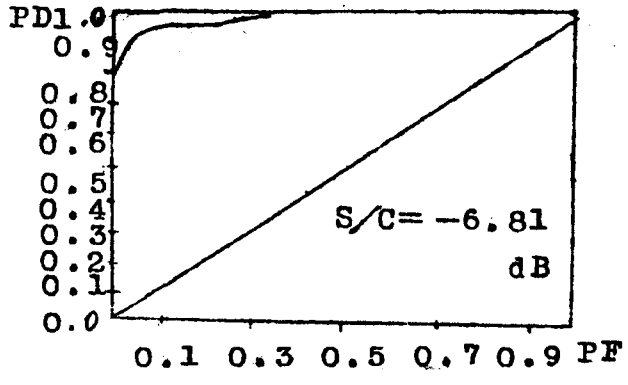

Fig.1 FD-I (Rayleigh clutter)

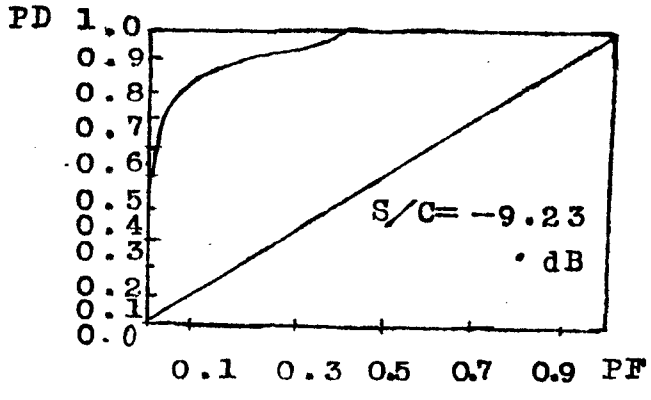

Fig.2 FD-I (Weibull clutter)

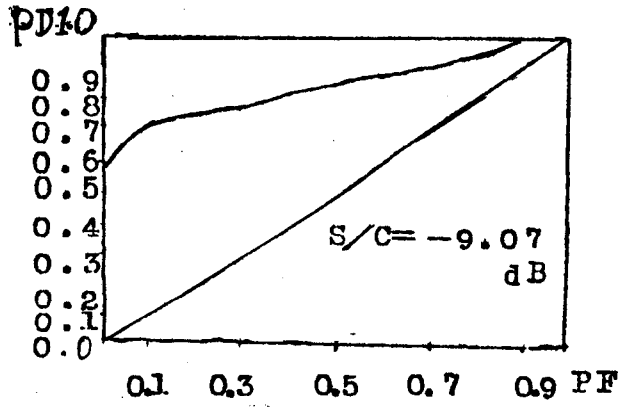

Fig.3 FD-II (Rayleigh clutter)

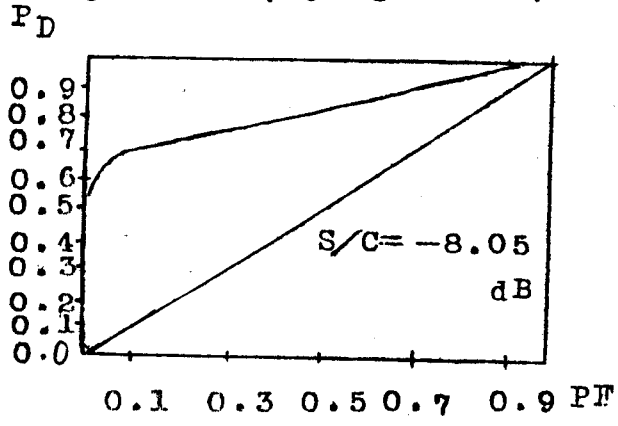

Fig.4 FD-II (Weibull clutter)

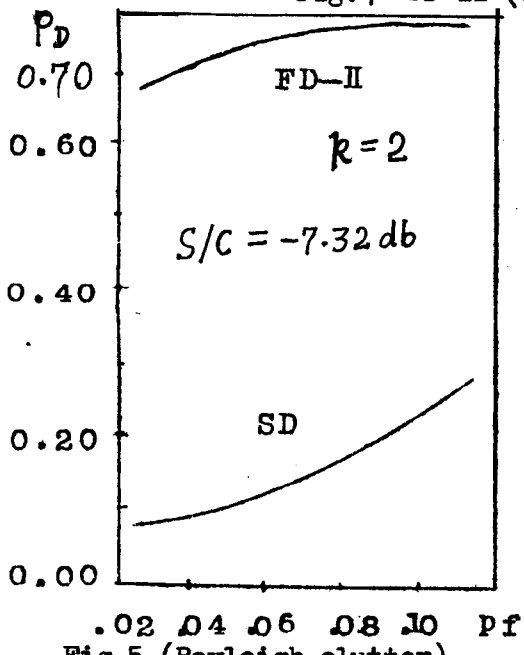

Fig.5 (Rayleigh clutter)

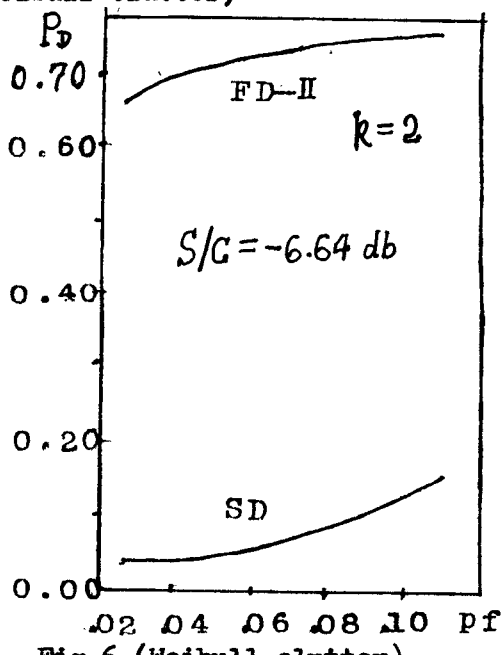

Fig.6 (Weibull clutter)