EQUIVALENT CONDITIONS UNDER WHICH THE NORMAL HYPERGROUPS ARE QUOTIENT GROUPS

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This paper is a continuous approach paper (3). Three equivalent conditions under which the normal hypergroups are quotient groups are discussed here.

KEYWORDS: Hypergroup, Normal hypergroup, Quotient group, Subgroup, Group.

In Ref (1) the concept of hypergroup was proposed. Let G be a group, for any A, $B \in g \subset 2^G - \{\phi\}$, we induce an operation $A \cdot B = \{ ab \mid a \in A, \ b \in B \}$, if the g is a group with respect to the operation "$\cdot$", g is called a hypergroup on G. A normal hypergroup is a hypergroup whose unit element contains the unit element of the G. Let g be a normal hypergroup on G, for any $A \in g$, write $\bar{A} = \{ a \in A \mid a^{-1} \in A^{-1} \}$, in Ref(3), we had proved that $\bar{g} = \{ \bar{A} \mid A \in g \}$ is a quotient group and $g \cong \bar{g}$. In this paper, we shall prove three equivalent conditions under $g = \bar{g}$.

THEOREM Let G be a group, if g is a normal hypergroup on G, then following conditions are equivalence.

(1) $g = G_o/_E$; (here $G_o = \cup \{ A \mid A \in g \}$)

(2) $G_o < G$ and for any $A, B \in g$, $A \cap B = \phi$;

(3) $E < G$; (here E is unit element of g)

(4) For any $a \in A \in g$, we have $a^{-1} \in A^{-1}$.

PROOF (1) $\Longrightarrow$ (2); It is apparent.

(2) $\Longrightarrow$ (3); For any $a, b \in E$, we have $ab \in E^2 = E$. Assume $a^{-1} \notin E$, from $Go < G$, we know there is $A \in g$ such that $a^{-1} \in A$ and $A \neq E$. For any $c \in A^{-1}$, we have $a^{-1}c \in AA^{-1} = E$. Thus $aa^{-1}c \in E^2 = E$, i.e. $c \in E$. So $c \in A^{-1} \cap E$. This means $A^{-1} = E$. Thus $AA^{-1} = AE = A$, i.e. $A = E$. This is at variance with $A \neq E$. So $a^{-1} \in E$. Therefore $E < G$.

(3) $\Longrightarrow$ (4); For any $b \in A^{-1}$, from $a \in A$, we have $ab \in AA^{-1} = E$. Since $E < G$, so $(ab)^{-1} = b^{-1}a^{-1} \in E$. Thus $bb^{-1}a^{-1} \in A^{-1}E = A^{-1}$. Therefore $a^{-1} \in A^{-1}$.

(4) $\Longrightarrow$ (1) Firstly it is easy to prove $E < Go < G$. Secondly, for any $a \in Go$, there is $A \in g$ such that $a \in A$, from $a^{-1} \in A^{-1}$ and $AEA^{-1} = E$, we have $aEa^{-1} \subset AEA^{-1} = E$. Therefore $E \triangleleft Go$. Finaly, for any $a \in A$, it is clear that $aE \subset AE = A$. Conversely, for any $b \in A$, from $a^{-1} \in A^{-1}$, we have $a^{-1}b \in A^{-1}A = E$. Thus $aa^{-1}b \in aE$, i.e. $b \in aE$. So $A \subset aE$. Therefore $A = aE$.

From above the results we have $g = Go/E$.

In special, if $G = Go = \cup \{ A \mid A \in g \}$, then above Theorem is equivalent conditions under $g = G/E$.

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