

## AN EXPERT SYSTEM WITH THINKING IN IMAGES

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## Abstract

In this paper we present a new method of modelling human thinking in images, which had been applied to the expert systems for diagnosis of faults in aircraft and blood diseases. In this method, first, the knowledge and experience of experts are described by the theory of fall shadows random sets in the factors space. Secondly, to gain their knowledge and experience, we put them to the psychology test according to the theory of fall shadows statistics. Thirdly, we consider thinking in images as a process of comparing, analysis and recognition. Finally, we apply consciousness-tree to modelling the complex process.

Keywords: Expert systems, thinking in images, fall-shadows of random sets, set-valued statistics, consciousness-tree

## 1. INTRODUCTION

An expert system is an information processing system, which could store human knowledge and experience as model human thinking process for processing knowledge and information. So far, it has made great progress in modelling human logical thinking. Some high-level programming languages, as LISP, PROLOG and so on, which are applied to artificial intelligence, have been offered.

According to professor Gian Xieshen's points of view, however, modes of human thinking may be divided into logical thinking, thinking in images and inspirational thinking. We consider that the main study direction of artificial intelligence would be thinking in images and inspirational thinking.

Logical thinking is a process of using concepts for judgment and inference. The inference process must abide by some rules and certain program. In the real world, human experts apply logical thinking as well as thinking in images. For instance, when an engineer with abundant experience makes a decision for some problem, his thinking process is that he recognizes a matter by making use of image according to his experience. His foundation of decision-making is not some concepts and rules, but some practical matters and impressions of imagery in human brains. This is what we mean by saying thinking in images. It is very much different from logical thinking.

Generally, human recognizes matters by thinking in images from an integral impression. Thinking in images has features, that is, fuzziness and integrality. A doctor does not diagnose patient's illness according to a single symptom of temperature, blood pressure, or weight and pure reasoning rules, but according

to all the symptoms of the patient's illness and his own (or even tragedy). For this reason, we should apply some method of fuzzy mathematics for representing the process of thinking in images.

In this paper, we present a new idea and method of modelling human thinking in images, i.e., the basic theory of knowledge representation, knowledge acquisition and application of knowledge. We use the method to build up a STIM (Shell with Thinking in Images) expert system, which has been applied to the expert systems for diagnosis of faults in aircraft and blood diseases, and gained satisfactory results.

## 2. KNOWLEDGE REPRESENTATION

In this section, we offer a new mathematical method, which describes knowledge of thinking in images.

### 2.1 Factors space

Factors space would be useful for mathematically describing the real world [1]. In this paper we are going to use some theory on factors space.

**DEFINITION 2.1** A factors space is a family of sets  $\{X_f\}_{f \in F}$  where  $F$  is a complete Boolean algebra  $F = (F, \vee, \wedge, c)$  satisfying

$$1) \quad X_{\emptyset} = \emptyset; \quad (2.1)$$

2) if  $\{f_t\}_{t \in T}$  are independent, i.e., for any  $t_1, t_2 \in T$ ,  $t_1 \wedge t_2 = \emptyset$ , then

$$\bigvee_{t \in T} f_t = \prod_{t \in T} X_{f_t} \quad (2.2)$$

where 0 and 1 are the smallest and the largest element of  $F$  respectively, and  $\prod_{t \in T}$  is Cartesian product, allows that

$$\{\emptyset\} \times \prod_{t \in T} X_{f_t} \quad (2.3)$$

We call the elements of  $F$  the factors,  $X_f$  the range of factor  $f$  and  $X_1$  is the whole range or state space.

An image of object can be described as point in a finite or finite-dimensional factors space. In doctor's image, for instance, a patient's state may be described as a point in multidimensional factors space where the factors are sex, weight, temperature, blood pressure, etc.

For  $f, g \in F$ ,  $g \leq f \leq h$  define

$$\uparrow_f^h A \triangleq A \times X_{(h \wedge f^c)} \quad (2.4)$$

and also

$$\downarrow_g A = \{x \mid x \in X_g \text{ \& } (\exists y \in X_{(f \wedge g^c)}) \times (x, y) \in A\} \quad (2.5)$$

we call  $\uparrow_f^h A$  the cylindric extension of  $A$  from  $f$  to  $h$  and  $\downarrow_g A$  the projection of  $A$  from  $f$  to  $g$ .

**DEFINITION 2.2** For any  $A \subseteq X_1$ , define

$$\tau(A) \triangleq \wedge \{f \mid \uparrow_f A = A\} \quad (2.6)$$

We call  $\tau(A)$  the rank of  $A$ . If  $f \geq \tau(A)$ , we say that  $A$  is clear in

$\lambda_f$  other, we may say that A is unclear in X. If  $f \leq \tau(A)$ , we offer a concept of clear degree.

DEFINITION 2.3 Let  $\delta$  be a function from  $\sigma$ -algebra to  $[0,1], \forall A \in \mathcal{F}$ ,  $\delta$  is a clear degree satisfying the conditions:

$$1. \delta_A \emptyset = 0; \delta_A(f) = 1, \quad f \geq \tau(A) \quad (2.7)$$

$$2. \text{If } h \wedge g = 0, \text{ then } \delta_A(h \vee g) = \delta_A(h) + \delta_A(g) + \lambda \delta_A(h) \delta_A(g) \quad (1 < \lambda < 1) \\ h, g < \tau(A); \quad (2.8)$$

$$3. \text{If } \forall n \in \mathbb{N}, f_i \in \mathcal{F} \text{ and } f_i \text{ is monotonic } (f_1 \leq f_2 \leq \dots \leq f_n \leq \dots \text{ or } f_1 \geq f_2 \geq \dots \geq f_n \geq \dots), \text{ then } \lim_{i \rightarrow \infty} \delta_A(f_i) = \delta_A(\lim_{i \rightarrow \infty} f_i) \quad f_i < \tau(A) \quad (2.9)$$

The clear degree is a  $\lambda$ -fuzzy measure. Equiprobability of fuzziness can be described by the clear degree.

DEFINITION 2.4 Let  $(X_f, \mathcal{B}_f, P_f) (f \in F)$  a factors field if  $(X_f)$

$f \in F$  is a factors space; for every  $f \neq 0 (X_f, \mathcal{B}_f, P_f)$  is a probability field; and  $(f_t) (t \in T)$  are independent then  $(X_{\prod_{t \in T} f_t}, \mathcal{B}_{\prod_{t \in T} f_t}, P_{\prod_{t \in T} f_t})$

is the product probability field of  $(X_f, \mathcal{B}_f, P_f) (f \in T)$ .

For any  $\mathcal{B} \subseteq \mathcal{P}(X_f)$ , if  $f \leq h$ , we define

$$\uparrow_h \mathcal{B} = \{h \cap C \mid C \in \mathcal{B}\} \quad (2.10)$$

If  $g \leq f$ , we define

$$\downarrow_g \mathcal{B} = \{g \cap C \mid C \in \mathcal{B}\} \quad (2.11)$$

We can easily prove that:

### 2.2 Set-valued Statistics and Fall-shadows of Random Sets

Set-valued Statistics were proposed by Wang Peizhuang [2]. The classical statistical experiments, which are familiar to us, are mainly used to measure some physical quantities, and are rarely based upon the psychological reflections of human beings. The set-valued statistics is however closely connected with the psychological process. There is a knowledge problem here. A large number of experiments in physical psychology have proved that there exists very precisely a power function law between the changes in psychological reflection quantity which is obtained by various sense organs (such as vision organ, hearing organ, taste organ, touch organ) and the changes in various physical stimulation quantities in the outside world (such as bright degree, loud degree, sweet degree, fragrant degree). This shows that a scientific psychological measurement method may objectively reflect the real world. As to those objects which cannot be measured by physical or chemical or any other measurement means, the psychological measurement is really an important quantitating method. So we may say that bringing psychological measurement into set-valued statistics is not a fault but an advantage.

Image of a kind of object for experts can be described as a subset in the factors space. A large number of images of experts are random sets in the factors space. Now we define the measurable structure and "random variable" on  $\mathcal{P}(U)$ . Denote

$$\hat{A} = \{B | B \in \mathcal{P}(U), \exists \omega \in B\} \quad (2.13)$$

$$\hat{U} = \{u | u \in U\} \quad (2.13')$$

Given a  $\sigma$ -field  $\mathcal{B}$  containing  $U$ ,  $(\mathcal{P}(U), \mathcal{B})$  is a measurable space. A measurable mapping from some factors field  $(X_f, \mathcal{B}_f, \mathcal{P}_f)$  to  $(\mathcal{P}(U), \mathcal{B})$

$$\xi: X_f \rightarrow \mathcal{P}(U) \quad (2.14)$$

$$\xi^{-1}(B) = \{\omega | \omega \in B, B \in \mathcal{B}_f, \forall B \in \mathcal{B}\}$$

is called a random set on  $U$ , which is a random variable on  $\mathcal{P}(U)$ . In what follows, we use  $\mathcal{P}(X_f, \mathcal{B}_f; \mathcal{P}(U), \mathcal{B})$  to denote all such random sets.

DEFINITION 2.5 Let  $\xi \in \mathcal{P}(X_f, \mathcal{B}_f; \mathcal{P}(U), \mathcal{B})$ . Denote

$$\mu_\xi = \{u \in U | P(\omega | \xi)(\omega \in u), \quad u \in U\} \quad (2.15)$$

is called the fall-shadow of  $\xi$ . Knowledge representation may be used in the fall-shadows.

DEFINITION 2.6 For given random sets  $\xi: X_f \rightarrow \mathcal{P}(U)$  and  $\eta: X_f \rightarrow \mathcal{P}(U)$

$$\mu_{(\xi, \eta)}(x, y) = P(\omega | \xi(\omega) \ni x, \eta(\omega) \ni y) \quad (2.16)$$

is called the union fall-shadow of  $\xi$  and  $\eta$ . If  $\mu_\xi(u) > 0$ , denote

$$\mu_{\eta|\xi}(y) = P(\eta \ni y | \xi \ni x) \quad (2.17)$$

is called the conditional fall-shadow of  $\eta$  when  $\xi \ni x$ .

PROPOSITION 2.1 The necessary condition for independence of  $\xi$

$$\mu_{(\xi, \eta)}(x, y) = \mu_\xi(x) \mu_\eta(y) \quad (\forall x \in X, y \in Y) \quad (2.18)$$

Let  $\mu$  be a probability measure on the measurable space  $(U, \mathcal{B})$ , and the fall shadow  $\mu_\xi$  of random set  $\xi$  on  $U$  is integrable, denote

$$\bar{m}(\xi) = \int \mu_\xi(u) m(du) \quad (2.19)$$

We call  $\bar{m}(\xi)$  a class such self-valued statistics in which each  $\xi \in \mathcal{P}(X_f, \mathcal{B}_f; \mathcal{P}(U), \mathcal{B})$ , make independent above a certain  $\bar{m}(\xi)$ , self-approach

$$1, 2, \dots, n \quad \xi_i \in \mathcal{P}(U) \quad (i=1, \dots, n)$$

Example

$$\bar{m}(\xi) = \frac{1}{n} \sum_{i=1}^n \chi_{x_i} \quad (2.20)$$

It is called a law concerning the law of  $\xi$  on  $U$ . We call the self-approach function of  $\mu_\xi$ , containing the law a theorem.

THEOREM 2.2 The law of great numbers of fall-shadow  $\xi_i$  given

$$\xi_i \in \mathcal{P}(X_f, \mathcal{B}_f; \mathcal{P}(U), \mathcal{B}) \quad (i=1, 2, \dots, n)$$

are independent and have the same distribution  $\mu_{\xi_i}(u) = \mu(u)$ , then

$$\bar{\xi}_n(\omega) = \bar{w} = \frac{1}{n} \sum_{i=1}^n \chi_{\xi_i}(\omega) \quad (2.21)$$

where  $\chi_{\xi_i}(\omega) = 1$  if  $\omega \in \xi_i$  and 0 otherwise.

$$\bar{\xi}_n(\omega) \xrightarrow{\text{a.e.}} \mu(\omega) \quad (n \rightarrow \infty) \quad (2.22)$$

2.2. Fuzzy Entropy

The fuzzy entropy was first proposed by De Luca and Termini (1972). It is a fuzzy extension of the classical entropy. Let  $X$  be a universe of discourse and  $\bar{\mathcal{F}}(X)$  be the set of fuzzy subsets of  $X$ . A fuzzy entropy is a mapping  $H$  from  $\bar{\mathcal{F}}(X)$  to  $[0, \infty)$  satisfying the conditions:

- (1)  $H(A) = 0$  iff  $A$  is an ordinary subset of  $X$ ;
- (2)  $H(A) \leq H(A^*)$  where  $A^*$  is any sharpened version of  $A$ , i.e.,  $\mu_{A^*}(x) \leq \mu_A(x) \leq 1/2$  and  $\mu_{A^*}(x) \geq \mu_A(x) \geq 1/2$ ;

where  $\mu_{A^*}(x) = \mu_A(x)$  if  $\mu_A(x) \leq 1/2$  or  $\mu_A(x) \geq 1/2$ .

More recently, De Luca has proposed a general definition of fuzzy entropy:

$$H(A) = - \sum_{x \in X} \mu_A(x) \log \mu_A(x) - \sum_{x \in X} (1 - \mu_A(x)) \log (1 - \mu_A(x)) \quad (2.23)$$

This can be extended to evaluate a whole fuzzy partition, i.e., to give a measure of the total amount of ambiguity of a set of fuzzy subsets deciding to which of  $A_1, \dots, A_m$  an element  $x$  belongs. If  $(A_1, \dots, A_m)$  is a fuzzy partition, the fuzzy entropy is

$$H(A_1, \dots, A_m) = - \sum_{x \in X} \sum_{j=1}^m \mu_{A_j}(x) \log \mu_{A_j}(x) \quad (2.24)$$

where  $(A_1, \dots, A_m)$  is an ordinary partition of  $X$ ,  $\sum_{j=1}^m \mu_{A_j}(x) = 1$ .  $H(A_1, \dots, A_m)$  is maximum iff  $\forall i, \forall j, \mu_{A_j}(x) = 1/m$  (maximal ambiguity).

Let  $X_f \subseteq X$  be a factorial space,  $\mu_{A_j}(x) = 1/m$  for  $x \in X_f$  and 0 otherwise. Let  $(A_1^*, \dots, A_m^*)$  be a fuzzy partition satisfying the condition:

$$\mu_{A_j^*}(x) = \mu_{A_j}(x) \sum_{i=1}^m \mu_{A_i}(x) \quad (x \in X_f) \quad (2.25)$$

The fuzzy entropy of  $(A_1^*, \dots, A_m^*)$  is

$$H(A_1^*, \dots, A_m^*) = \sum_{x \in X_f} \sum_{j=1}^m \mu_{A_j^*}(x) \log \mu_{A_j^*}(x) \quad (2.26)$$

and the fuzzy entropy of  $(A_1, \dots, A_m)$  is

$$H(A_1, \dots, A_m) = \max_{x \in X_f} H(A_1^*, \dots, A_m^*) \quad (2.27)$$

where  $\max_{x \in X_f} H(A_1^*, \dots, A_m^*)$  is maximum fuzzy entropy, iff  $\forall x \in X_f, \mu_{A_j}(x) = 1/m$  and  $\sum_{j=1}^m \mu_{A_j}(x) = 1$ .  $H(A_1, \dots, A_m)$  is the fuzzy entropy of  $(A_1, \dots, A_m)$  in the factorial space  $X_f \subseteq X$ .



is the point estimate of the degree discussed.

The interval estimate and confidence method may be transformed into an algorithm. The more concentrated the points are, the more concentrated the participant is, we can find a functional relationship  $\delta = \theta$  between the concentration degree of the points and the confidence degree after putting the points, where  $\theta$  means the confidence degree and the red value of the estimation. For example, finding  $f(x_i, \theta_i)$  as a result from the confidence approach, we can transform it into  $f(x_i - f(\theta), x_i + f(\theta) \cap \Omega_i)$ , which can be regarded as a result  $A$  in the interval  $x \in [a, b]$  and  $\theta = \frac{f(a) + f(b)}{2}$ .

According to the set of problems,  $\mu_{A_j}$  can be defined as the following:  $\mu_{A_j} = \mu_{A_j}(\Omega)$ , where the information amount  $f_i$  of the objects  $A_j$  is  $f_i = (f_i \in \Omega), f_i \wedge f_j = \emptyset, i \neq j$ . Additionally,

the degree of the objects  $A_j$  for the fault  $\tau$  is  $\delta_{A_j} = \tau(A_j)$ , where  $\tau$  is the fault.

$$\delta_{A_j} = \frac{\sum_{f_t \in \tau(A_j)} f_t}{\sum_{f_t \in \tau(A_j)} f_t} \quad (3.4)$$

where  $\tau(A_j)$  is the fault of  $A_j$ . The system will give a wrong conclusion if the knowledge base is modified automatically.

For instance, if the image  $x_0$  inputs the computer, then the computer will give a wrong conclusion  $A_j$ , but the correct conclusion is  $A_k$ .

$$\mu_{A_j} \cdot \mu_{A_k} \cdot \mu_{A_l} \quad (3.5)$$

$$\mu_{A_j} = \mu_{A_k} \cdot \mu_{A_l} \quad (3.6)$$

where  $\mu_{A_j} = \exp(-\alpha |x_0 - x_j|^2)$ ,  $N(x_0, \mu_{A_j})$ ,  $\alpha$  is a constant of the degree according to the expert given.

4. AN EXPERT APPLICATION

In this section, we will provide the thinking in images while the process of thinking: comparing, analysis, synthesis, and apply, based on experience to modeling human thinking process.

1) Process of Thinking in Images  
 The process of thinking in images is a process of using image for comparing, analysis and recognition. For instance, when a doctor diagnoses illness, his thinking process is as follows: he

get a tentative image by examining the total symptoms of the patient's disease, then compare the tentative image with the typical image of kinds of disease according to their experience. The doctor makes a decision by means of analysis and recognition. The following is the mathematical description of comparing, analyzing and recognizing.

Comparing is a kind of divergent thinking. First, the doctor gets the space  $(f_0, f_0 \in F)$ . The computer gets an image  $(f_0, f_0 \in F)$  according to tentative examination patient's symptom, then compares the image  $f_0$  with typical experience of kinds of disease  $(A_j, j=1, \dots, m)$ . The process of comparing is usually divided into two steps: First, the computer projects sets  $A_j$  into the space of disease to tentative factors space  $(f_0, f_0 \in F)$ , then

calculates the clear degree  $\delta_{A_j}(f_0)$  of kinds of disease  $A_j$  according to the image  $f_0$  and the doctor's experience. Secondly, the doctor thinks of the image  $f_0$  has some address, so the comparing from integrality aspect, i.e., calculate the clear degree  $\delta_{A_j}(f_0)$  of kinds of disease  $A_j$  in tentative factors space

$(f_0, f_0 \in F)$ . The method is as follows:  
 Let  $(f_0, f_0 \in F)$  be a tentative factors space,  $(f_0, f_0 \in F)$  be the image and sets  $A_j$  ( $j=1, \dots, m$ ) be kinds of disease in the factors space of disease  $(f_j, f_j \in F)$  ( $f_j = \tau(A_j)$ ),  $f_0 < f_j$ , then

$$\mu_{A_j}(f_0) = \mu(\downarrow_f A_j) \quad (4.1)$$

where  $\downarrow_f A_j$  is the projection from the factors space  $(f_j)$  to  $(f_0)$ . If  $f_0 > f_j$ , then

$$\mu_{A_j}(f_0) = \mu(\uparrow_f A_j) \quad (4.2)$$

where  $\uparrow_f A_j$  is the cylindric extension from the factors space  $(f_j)$  to  $(f_0)$ . The clear degree of disease  $A_j$

$$\delta_{A_j}(f_0) = \sum_{f \in f_0} \delta_{A_j}(f) \quad (4.3)$$

is a kind of divergent thinking. First, according to the comparing calculation, we find out a disease or diseases, which is as follows

$$\mu_{A_w}(f_0) = \max_{1 \leq j \leq m} \mu_{A_j}(f_0) \quad (4.4)$$

where  $\mu_{A_w}(f_0)$  is the possible  $\mu_{A_w}(f_0)$  and the clear degree  $\delta_{A_w}(f_0)$  is the possible  $\delta_{A_w}(f_0)$  and  $\mu_{A_w}(f_0)$  and  $\delta_{A_w}(f_0)$  are all very large

then factor or result of diagnosis is disease  $A_w$ . If the value  $\mu_{A_w}(f_0)$  is very large and the  $\delta_{A_w}(f_0)$  is

very small, then extend the tentative factors space  $(f_0)$  to  $(f_1)$ , where  $f_0 < f_1$  and  $\delta_{A_w}(f_0) < \delta_{A_w}(f_1)$ .

Case 2: If the value  $|\mu_{Aw}(f_0) - \mu_{Ah}(f_0)|$  is ver. small, then we  
 move the tentative factors space  $(f_0)$  to  $(f_t)$ , where  $(f_t)$

$$|\mu_{Aw}(f_t) - \mu_{Ah}(f_t)| = \max_{i \in S} |\mu_{Aw}(f_t) - \mu_{Ah}(f_t)|$$

Since all the values  $\mu_{Aw}(f_0)$  and  $\delta_{Aw}(f_0)$  are all ver. small,  
 we move the tentative factors space  $(f_0)$  to  $(f_t)$ , where  
 $(f_t) = \bigcup_{i \in S} f_i$  and  $I(f_t) = \max_{i \in S} I(f_i)$ ,  $f_t \cap f_0 = \emptyset$ .

Since the value  $|\mu_{Aw}(f_0) - \mu_{Ah}(f_0)|$  is very small and  $\delta_{Aw}(f_0)$  is very  
 large, then the knowledge-base should be modified.

3.2.2. Degree of

The degree of synthesis decision making process is  
 defined as the possibility degree and the choice degree,  $\mu_{Aj}$  and  $\delta_{Aj}$ .

$$\mu_{Aj}(f_0) = \mu_{Aj} \cdot \delta_{Aj}(f_0)$$

$\mu_{Aw}(f_0) \rightarrow \lambda$  The result of decision is  $w$ .

### 3.2.3. Multiple Consciousness Process

Human thinking is conscious but the thinking process is  
 complex. In order to model human thinking process, there must be  
 some kind of process in the expert system. The computer  
 thinking process can be described by a neural network.

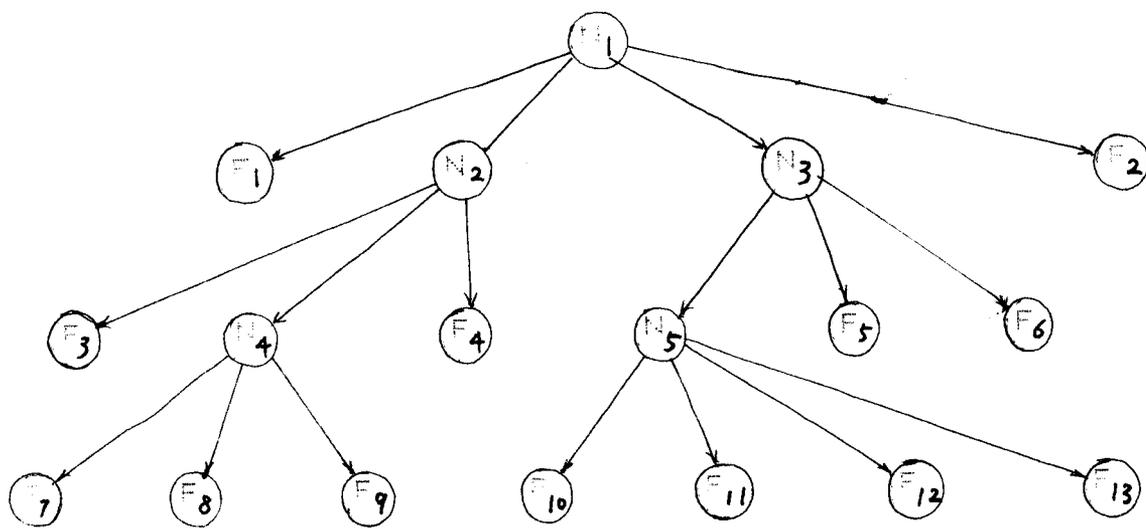
The consciousness tree is a above or below a variable  
 weight. The nodes on tree is a tree such that each node is a  
 multiple decision function with variable weights  $w_j$  and  
 inputs  $f_i$  (1, 2, 3), where  $f_i$  is the factors of input. Let  $w_j$   
 be the weight coefficients, we have

$$w_j = \sum_{i=1}^3 w_j \cdot f_i \quad (3.13)$$

Let  $w_j$  be the varying weight coefficients (see 3.1.1). Human  
 thinking is a multiple, the varying weights processing correspond  
 to human multiple thinking. Following are experiential rules  
 1, 2, 3 be finite factors, the varying weight

$$\begin{aligned} & 11 \quad 1 \times 2 \times 3 \\ & 12 \quad 1 \times 2 \times 3 \\ & 13 \quad 1 \times 2 \times 3 \end{aligned} \quad \begin{aligned} & \dots \dots \dots 12 \quad 2 \times 13 \quad 3 \\ & \dots \dots \dots 11 \quad 1 \quad 12 \quad 2 \quad 13 \quad 3 \\ & \dots \dots \dots 11 \quad 1 \quad 13 \quad 3 \\ & \dots \dots \dots 11 \quad 1 \quad 12 \quad 2 \quad 13 \quad 3 \\ & \dots \dots \dots 11 \quad 1 \quad 12 \quad 2 \\ & \dots \dots \dots 11 \quad 1 \quad 12 \quad 2 \quad 13 \quad 3 \end{aligned}$$

Let  $w_j = 11 \times 12 \times 13$  be the path of result  $w$  and  $f_j$   
 The result of decision is  $w = \max_{i \in S} f_j$



$N_i$  is a node and  $F_j$  is a leaf  
 Fig. 4.1: consciousness-tree

#### 5. CONCLUSION

For the foregoing analysis, we have considered that human thinking process is complex. The complex thinking process contain three kinds of thinking modes, such as logical thinking, thinking in images and inspirational thinking. Every of them may be divided by following three parts:

1. antecedent, 2. contact by intermediary, or consequents  
 The main difference one another is disparity in contacting intermediary, logical thinking by rules, thinking in images by experience or imagery and inspirational thinking by human creativity, imagination, conjecture and insight and so on non-logical and super-images function. In this paper, use the method to building a STIM (Shell with Thinking in Images) expert system, which has been applied to the expert systems for diagnosis of fault in aircraft and blood diseases.

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