Book Review

Probabilistic sets : fuzzy and stochastic approach to decision, control and recognition processes

by E. Czogala and K. Hirota, Verlag TÜV Rheinland, ISR n° 91, 1986

This book is a revised and augmented version of another one (Probabilistic Sets in Decision and Control, by E. Czogala, ISR n° 81, 1984, same publisher) already reviewed (see Fuzzy Sets and Systems, vol. 15, n° 3, p.315-316). The main modifications are a more extensive treatment of chapter one devoted to the basic notions, and an additional chapter devoted to pattern recognition. Theoretical addenda deal with probabilistic relations, the representation of a probabilistic set in terms of an expected membership function an a standard deviation function called the vagueness function. An attempt to relate probabilistic sets to other "fuzzy" concepts such as type 2 fuzzy sets of Mizumoto and Tanaka is made. However these discussions remain at a rather superficial level. Chapter 5 about pattern recognition tries to motivate the use of probabilistic sets as a good model of ambiguity and subjectivity in the evaluation of perceptive phenomena. The random parameter w involved in the membership function $\mu(.,w)$ is viewed as the observer's view point. The approach is applied to the performance evaluation of character readers. Other applications to pattern matching, Gaussian noise pattern estimation, and texture analysis are discussed, but they rather look like reinterpretations of existing techniques in the terminology of probabilistic sets.

Many remarks appearing in the review of the original shorter version still apply, especially regarding the status of probabilistic sets as compared to other attempts to mix probabilistic and fuzzy information. A striking difference is between probabilistic sets and random fuzzy sets (also called fuzzy random variables by Puri and Ralescu, 1986, and Kwakernaak, 1979). This difference has its roots in the existence of two basic points of view on a fuzzy set. One is the vertical point of view : a fuzzy set is a membership function, i.e. a set of elements each being attached a positive membership grade. The other one is the horizontal point of view : a fuzzy set is a weighted set of level-cuts, i.e. a family of nested regular sets. These two points of view are often equivalent, and at least related most of the time. For example the vertical view leads to a scalar definition of cardinality, while the horizontal view leads to fuzzy integer-valued cardinality (e.g. Dubois, Prade, 1985). When a random element is introduced in the fuzzy set, the horizontal and vertical points of view apparently diverge. Probabilistic sets seem to adopt a vertical point of view, since they can be represented as random vectors of membership

grades. The horizontal view leads to the idea of a random fuzzy set where each random trial produces a different fuzzy set. In the first approach, membership grades fluctuate in the valuation set [0,1]. In the second approach, a fuzzy set is allowed to move randomly on its support, the moves involving possible distortion of the membership function. The theory of fuzzy random variables is an extension of random set theory (Matheron, 1975) based on the concept of level-cuts. Namely the level-cut of a fuzzy random variable is a random set. Contrastedly, the concept of level-cut is not used in the book.

A basic question is to understand what is the most natural view of a fuzzy set, in terms of elicitation. The probabilistic set approach tends to focus on the idea of membership grades, and underlies the existence of a precise membership grade of 1.70 meters to the set of tall sizes for a given individual. However this membership grade is out of reach, as observed through fluctuations when people are asked for membership grades. Under the horizontal view, people have in their minds intervals with ill-located boundaries. Elicitation procedures based on the horizontal view would never ask for membership grades explicitly, but would rather try to locate various intervals with grades of confidence attached to them. A probabilistic set can be derived when asking a group of people for membership grades (see chapter 5 section 5.3.1.), while a random fuzzy set is obtained by asking each person for a few level-cuts likely to be proper representatives of a fuzzy concept. It is may be too early to definitely choose between these two views (I wonder whether people can supply membership grades in a direct way). But it seems timely that some investigations be made to relate probabilistic sets and random fuzzy sets at the mathematical level ; one would like to know if the horizontal and vertical view remain equivalent under the addition of randomness. A question which this book unfortunately does not consider. However this book offers a valuable introduction to the mathematical concept of probabilistic set, by gathering many results scattered in the literature.

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References

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