GENERALIZED RELIABILITY OF ENGINEERING SYSTEMS

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ABSTRACT

It is pointed out in this paper that any uncertain factor (not only random factor) in an engineering system will lead to the uncertainty of safety degree of the system. Thus the concept of reliability should be extended. The reliability taking account of other uncertain factors besides random one may be called generalized reliability.

At present, people have gradually recognized that in the uncertainties of a system there often exist fuzziness besides randomness. Starting from the general case, this paper presents a method of fuzzy-random reliability analysis of complex engineering systems with multiple failure modes. For this purpose, the concepts such as fuzzy response, fuzzy safe criterion, satisfaction degree of the response to the fuzzy safe criterion and fuzzy safe region of the system are defined. As the special cases of fuzzy-random reliability, "random reliability" and "fuzzy reliability" are obtained respectively under conditions that in the system there exists only randomness or only fuzziness. The former is the reliability defined in the current reliability theory.

Key Words: engineering system, generalized reliability, fuzzy-random safe region, fuzzy-random reliability.

I. INTRODUCTION

Engineering systems are generally subjected to some external excitations and have some responses. In order to judge whether a system works normally, some limits are often set to its responses. The relative relationship of any response and its limit forms one of constraints of the safe criterion of the system.

The uncertainty of systems which has been cosidered in the current reliability theory is only randomness. In this case, the reliability of a system is naturally defined as the probability for the system to work normally in certain service life "T" under some conditions. At present, people have gradually recognized that a system itself and its external excitations may have not only randomness but also fuzziness. In addition, the allowable intervals of the responses should not be determined by distinct boundaries, i.e. the limits on responses should be fuzzy, or there should be an intermediary transition between absolute permission and absolute impermission for any one response[1]. This kind of uncertainty leads to the fuzziness of the safe criterion. In our previous papers [1-4], we have discussed how to take account of the fuzziness of both the earthquake excitation and the allowable intervals of the responses in optimum design and reliability analysis for aseismic structures.

This paper will start from the general case of simultaneously taking account of the fuzziness and randomness of systems and propose a method of fuzzy-random reliability analysis for complex systems.

II. FUZZY-RANDOM RELIABILITY OF ELEMENTS

Generally, a complex system consists of some subsystems and a subsystem consists of some elements.

At present, the elements of system have not yet consistent definition. From the angle of strictness and convenience of analysis, in this paper, an element is defined as a minimum unit with certain response constrained by corresponding limit. According to this definition, an element needn't be a concrete member of the system. When the action of a member is represented by its several responses with corresponding constraints, this action should be represented by several elements. In this sense, an element is equivalent to a constraint of the safe criterion of the system. For example, if the action of a bar in a truss is constrained by both its strength and stability, then this bar will have two failure modes, and thus, as a member of the truss, it may be represented by two elements.

Assume the engineering system under research has following characteristics:

(1) The N physical quantities $x_n (n=1,2,...,N)$ representing the design scheme of the system constitute a design vector

$$x = \{x_1, ..., x_n, ..., x_N\} .$$

(2) The M independent random parameters $\eta_m (m=1,2,...,M)$ in the system constitute a random vector

$$\eta = \{\eta_1, ..., \eta_m, ..., \eta_M\} .$$

Obviously, every realization of η, $y=\{y_1, ..., y_m, ..., y_M\}$, is a vector in Euclidean space R^M.

(3) There are L elements in the system. The behaviour response of the lth element is denoted by r_l $(l=1,2,...,L)$ and the maximum value of r_l by S_l. The maximum response vector

$$S = \{S_1, ..., S_l, ..., S_L\}$$

decides the safety of the system. Every response S_l is the function of x and η.

In the reliability analysis, it is generally assumed that the design vector x and the probability density $p_\eta(y)$ of the random vector η are known. When only randomness in the system is considered, the safe criterion of the lth element can be expressed by following relationship:

$$S_l(\eta) < R_l(\eta) \tag{1}$$

or

$$g_l(\eta) = R_l(\eta) - S_l(\eta) > 0 \tag{2}$$

in which $S_l(\eta)$ is the maximum behaviour response of the lth element of the system due to external excitation, $R_l(\eta)$ is the threshold limiting the response $S_l(\eta)$, $g_l(\eta)$ is called the behaviour function of the lth element.

Thus, the safe region of the lth element according to the safe criterion (2) is

$$\Omega_l = \{y \mid g_l(y)>0\} \tag{3}$$

It is a set which is made up of those realizations y of the random vector η satisfying $g_l(y)>0$. In fact, Ω_l is a random safe event.

If η_m is a random variable in the probability space (E_m, A, P) $(m=1, 2, \ldots, M)$, then $g_l(\eta)$ will be a random variable in the derived probability space (R^n, B, P_η)[4], i.e.

$$\Omega_l = \{y \mid g_l(y)>0\} \in B \tag{4}$$

Therefore, the random reliability of the lth element is

$$P_s^{(l)} = \Pr\{g_l(\eta)>0\} = \int_{\Omega_l} p_\eta(y)dy \tag{5}$$

Generally speaking, since the external excitation and the system itself may have not only randomness but also fuzziness, sometimes the fuzziness is even preponderant, these two kinds of uncertainty should be taken into consideration in the reliability analysis. Besides, the threshold $R_l(\eta)$ is generally fuzzy, i.e. there should be a fuzzy boundary (an intermediary transition) between absolute permission and absolute impermission of the behaviour response.

As a general case, when the randomness and fuzziness of the excitation and the system and the fuzziness of the limits on the behaviour responses are simultaneously taken account of, the fuzzy-random maximum responses and their allowable intervals can be respectively expressed by $\underset{\sim}{S}_l(\eta)$ and $\underset{\sim}{R}_l(\eta)$ $(l=1, 2, \ldots, L)$. For any realization y of the random vector η, $\underset{\sim}{S}_l(y)$ and $\underset{\sim}{R}_l(y)$ are two fuzzy subsets on the Euclidean space R^1 and have the membership functions similar to the shapes shown in Fig.1. The horizontal coordinate r_l is the behaviour response of the lth element.

Obviously, $\underset{\sim}{S}_l(\eta)$ and $\underset{\sim}{R}_l(\eta)$ are fuzzy-random variables[5] in the derived probability space (R^n, B, P_η).

The membership function of the maximum response $\underset{\sim}{S}_l(y)$ as shown in Fig.1(a) can be derived from the membership functions of the fuzzy parameters of the system and excitation by means of the method of system analysis and the extension principle of fuzzy mathematics. For example, the membership functions of the fuzzy responses of a structure due to an earthquake with certain fuzzy seismic intensity can be derived from the membership function of the fuzzy intensity by means of the earthquake response spectrum method and extension principle[2-3].

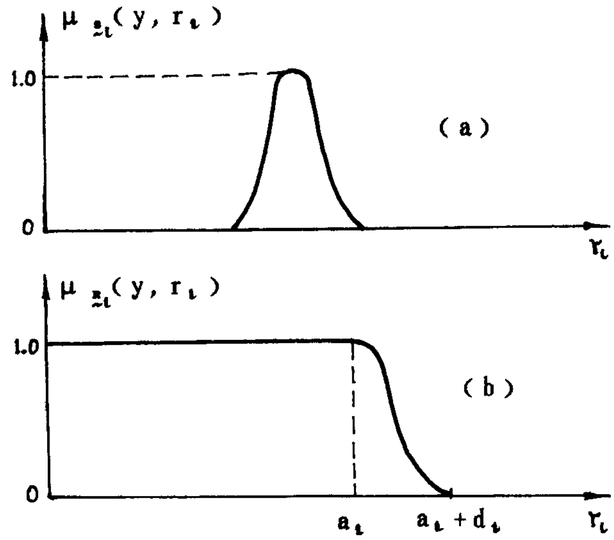

Fig.1 The membership functions of the fuzzy maximum response of the element and its fuzzy allowable interval

According to the membership function of the fuzzy allowable interval $\underset{\sim}{R}_l(y)$ as shown in Fig.1(b), the response $r_l \leqslant a_l$ is absolutely allowable ($\mu_{\underset{\sim}{R}_l}=1$), the response

$r_l > a_l + d_l$ is absolutely unallowable ($\mu_{\tilde{R}_l}=0$) and the response between above two extreme cases, $a_l < r_l \leqslant a_l + d_l$, is allowable to a certain extent (corresponding to the value of membership degree $\mu_{\tilde{R}_l}$). This means that the allowable interval $\tilde{R}_l(y)$ has a fuzzy boundary on the real axis, or there is an intermediary transition between the absolute permission and absolute impermission. The length of the transition d_l may be called permissible deviation. A proper curve of the transition can be chosen according to the character of the element.

Therefore, for the system having both fuzziness and randomness, the fuzzy-random safe criterion of the lth element can be exressed as

$$\tilde{S}_l(\eta) \subseteq \tilde{R}_l(\eta) \qquad (6)$$

Using the notation of L.A.Zadeh, the fuzzy safe region corresponding to the safe criterion (6) can be expressed by

$$\tilde{\Omega}_l = \int \mu_{\tilde{\Omega}_l}(y) / \{y \mid \tilde{S}_l(y) \subseteq \tilde{R}_l(y)\} \qquad (7)$$

It is a fuzzy set which is made up of those realizations y of the random vector η which make the maximum response $\tilde{S}_l(y)$ satisfy the safe criterion (6) to a certain extent (i.e. with membership degree $\mu_{\tilde{\Omega}_l}(y)$). This set, in fact, is a fuzzy-random safe event.

For a given realization y of η, the fuzzy-random safe criterion (6) becomes following fuzzy safe criterion,

$$\tilde{S}_l(y) \subseteq \tilde{R}_l(y) \qquad (8)$$

which, in fact, is a constraint to fuzzy response $\tilde{S}_l(y)$. For different realization y of η, the satisfaction degree of $\tilde{S}_l(y)$ to this constraint is different. This satisfaction degree is also the membership degree $\mu_{\tilde{\Omega}_l}(y)$ for the realization y to the fuzzy safe region $\tilde{\Omega}_l$.

The relative positions (Fig.2) of the membership function curves of fuzzy maximum response $\tilde{S}_l(y)$ and its fuzzy allowable interval $\tilde{R}_l(y)$ vividly show the satisfaction degree of $\tilde{S}_l(y)$ to fuzzy constraint (8). We have suggested[2-3] that this satisfaction degree could be defined as

$$\beta_l(y) = \mu_{\tilde{\Omega}_l}(y) = \frac{\int_{-\infty}^{\infty} \mu_{\tilde{S}_l}(y, r_l)\, \mu_{\tilde{R}_l}(y, r_l)\, dr_l}{\int_{-\infty}^{\infty} \mu_{\tilde{S}_l}(y, r_l)\, dr_l} \qquad (9)$$

When $\mu_{\tilde{S}_l}$ is covered entirely by the interval of $\mu_{\tilde{R}_l}=1$ (Fig.2a), the constraint (8) is satisfied completely, $\beta_l=1$; when $\mu_{\tilde{S}_l}$ is located entirely out of $\mu_{\tilde{R}_l}$ (Fig. 2c), the constraint (8) is not satisfied absolutely, $\beta_l=0$; while $\mu_{\tilde{S}_l}$ and $\mu_{\tilde{R}_l}$ overlap each other

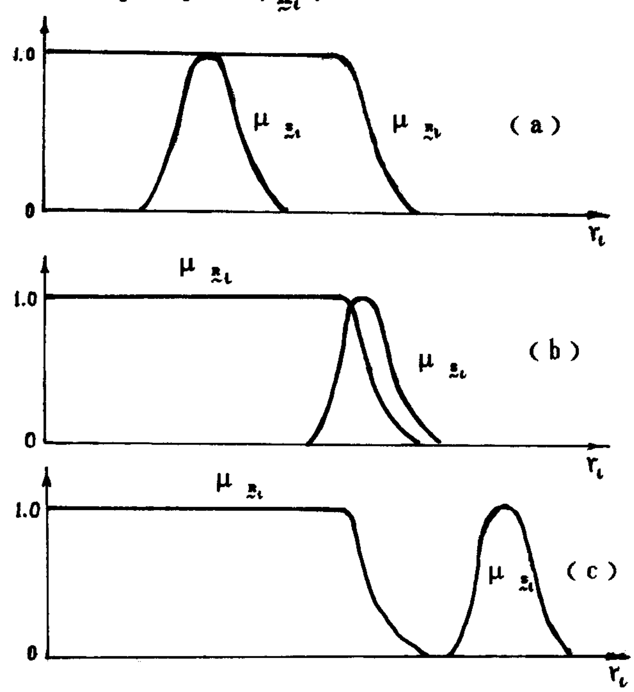

Fig.2 Relative positions of the membership functions of fuzzy maximum response and its fuzzy allowable interval

(Fig. 2b), the constraint (8) is satisfied to a certain extent, $\beta_l \in (0,1)$. Therefore, the set consisting of all y which make $\beta_l(y)=\mu_{\underset{\sim}{\Omega}_l}(y)>0$ is just the support set of the fuzzy safe region $\underset{\sim}{\Omega}_l$ expressed by Eq. (7).

Thus, according to the probability formula of the fuzzy-random event, the fuzzy-random reliability of the lth element is

$$\Psi_l = \Pr(\underset{\sim}{\Omega}_l) = \int_{-\infty}^{\infty} p_\eta(y)\,\mu_{\underset{\sim}{\Omega}_l}(y)dy \tag{10}$$

in which $\mu_{\underset{\sim}{\Omega}_l}(y)$ can be calculated by Eq. (9).

Ⅲ. FUZZY-RANDOM RELIABILITY OF SUBSYSTEMS

A complex system is often composed of some subsystems. A subsystem is a part of the system, its safe region can be easily determined. Such as series, parallel, and compound series or parallel subsystems are often used in complex systems. Of course, sometimes they may be used as independent systems also.

1. Series Subsystems

A series subsystem is schematically shown in Fig. 3. Assume the ith subsystem has l_i elements in all ($l_i \leqslant L$). According to the definition of a series subsystem, the subsystem can normally work only when all of its elements are normally working. So, the fuzzy safe region of a series subsystem is

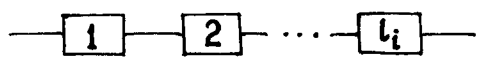

Fig. 3 Scheme of the series subsystem

$$\underset{\sim}{\Omega}_i = \bigcap_{l=1}^{l_i} \underset{\sim}{\Omega}_l \tag{11}$$

in which $\underset{\sim}{\Omega}_l$ is the fuzzy safe region of the lth element. $\underset{\sim}{\Omega}_l$ is expressed by Eq. (7) and its membership function $\mu_{\underset{\sim}{\Omega}_l}(y)$ can be determined by Eq. (9). According to the basic operation laws of fuzzy sets, the membership function of the fuzzy safe region of the series subsystem can be yielded as

$$\mu_{\underset{\sim}{\Omega}_i}(y) = \min_{l=1}^{l_i} \mu_{\underset{\sim}{\Omega}_l}(y) \tag{12}$$

Then, according to the probability formula of the fuzzy-random event, the fuzzy-random reliability of the series subsystem is

$$\Psi^{(i)} = \Pr(\underset{\sim}{\Omega}_i) = \int_{-\infty}^{\infty} p_\eta(y)\,\mu_{\underset{\sim}{\Omega}_i}(y)dy \tag{13}$$

in which $\mu_{\underset{\sim}{\Omega}_i}$ can be determined by Eq. (12).

2. Parallel Subsystems

A parallel subsystem is schematically shown in Fig. 4. Assume the ith subsystem has l_i elements in all ($l_i \leqslant L$). According to the definition of a parallel subsystem, the subsystem will fail only when all of its elements fail. In other words, if any one element is normally working, the subsystem will normally work. So, the fuzzy safe region

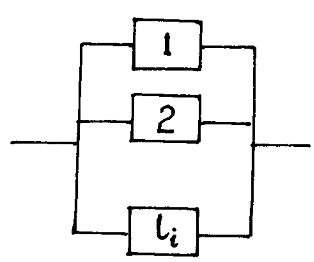

Fig. 4 Scheme of the parallel subsystem

of the parallel subsystem is

$$\underset{\sim}{\Omega}_i = \bigcup_{l=1}^{l_i} \underset{\sim}{\Omega}_l \qquad (14)$$

and its membership function is

$$\mu_{\underset{\sim}{\Omega}_i} = \max_{l=1}^{l_i} \mu_{\underset{\sim}{\Omega}_l}(y) \qquad (15)$$

Then, the fuzzy-random reliability of the parallel subsystem can be obtained as

$$\Psi^{(i)} = \Pr(\underset{\sim}{\Omega}_i) = \int_{-\infty}^{\infty} p_\eta(y)\,\mu_{\underset{\sim}{\Omega}_i}(y)dy \qquad (16)$$

in which $\mu_{\underset{\sim}{\Omega}_i}$ can be determined by (15).

3. Compound Series and Parallel Subsystems

When some elements themselves in Fig.3 or 4 are series or parallel sub-sub-systems, the subsystem shown in that figure becomes compound series or parallel subsystem. In this case, the sub-subsystems should be analyzed beforehand and considered as simple elements afterward in analysis of the whole subsystem. Then, the methods mentioned above can be used also to determine the fuzzy safe region, membership function and fuzzy-random reliability of the compound series and parallel subsystems or systems.

IV. FUZZY-RANDOM RELIABILITY OF COMPLEX SYSTEMS

In analyzing the reliability of a complex system, first of all, the system should be decomposed into several subsystems (including compound subsystems and individual elements to be considered as subsystems). Suppose, there are I subsystems in all. Then, find individually the fuzzy safe regions $\underset{\sim}{\Omega}_i$ and the membership functions $\mu_{\underset{\sim}{\Omega}_i}(y)$ for all subsystems $(i=1,2,\ldots,I)$. After that, in the reliability analysis for the whole system, all subsystems can be considered as elements of the system.

In order to find out the fuzzy safe region of the whole system, the concept of "minimum safe set"[8] E_k $(k=1,2,\ldots,K)$ can be used to express all of the K safe modes of the system. Each E_k is a subset of the set $\{1,2,\ldots,I\}$ consisting of all ordinal numbers of the subsystems, i.e.

$$E_k \subset \{1,2,\ldots,I\} \qquad (k=1,2,\ldots,K)$$

Since each ordinal number in $\{1,2,\ldots,I\}$ corresponds to a subsystem, E_k can be also regarded as a subset of the set consisting of all I subsystems. The minimum safe set E_k has such a property that the system is safe when all of the subsystems in E_k are safe. And at this time, other subsystems which don't belong to E_k, whether they are safe or not, have no effect on the safety of the whole system. So, E_k represents the kth safe mode and the corresponding fuzzy safe subregion of the system is the intersection of all $\underset{\sim}{\Omega}_i$ $(i \in E_k)$, i.e.

$$\underset{\sim}{\Omega}^{(k)} = \bigcap_{i \in E_k} \underset{\sim}{\Omega}_i \qquad (17)$$

in which $\underset{\sim}{\Omega}_i$ is the fuzzy safe region of the ith subsystem.

As for complex systems, their minimum safe sets may be determined by using system analysis techniques, such as the fault-tree method.

Since the safety of the system must at least include a certain minimum safe set in which all subsystems are safe and corresponding to every minimum safe set E_k there is a fuzzy safe subregion $\Omega^{(k)}$ which can secure the safety of the system, the entire fuzzy safe region of the system is the union of all K fuzzy safe subregions, i.e.

$$\underset{\sim}{\Omega} = \bigcup_{k=1}^{K} \underset{\sim}{\Omega}^{(k)} = \bigcup_{k=1}^{K} \bigcap_{i \in E_k} \underset{\sim}{\Omega}_i \quad (18)$$

Therefore, the membership function of the entire fuzzy safe region $\underset{\sim}{\Omega}$ of the system is

$$\mu_{\underset{\sim}{\Omega}}(y) = \max_{k=1}^{K} \min_{i \in E_k} \mu_{\underset{\sim}{\Omega}_i}(y) \quad (19)$$

in which $\mu_{\underset{\sim}{\Omega}_i}(y)$ is the membership function of the fuzzy safe region of the ith subsystem and can be determined by the methods mentioned in Section Ⅲ.

Finally, in the light of the probability formula of the fuzzy-random event, the fuzzy-random reliability of the system can be obtained by

$$\Psi = \Pr(\underset{\sim}{\Omega}) = \int_{-\infty}^{\infty} p_\eta(y)\,\mu_{\underset{\sim}{\Omega}}(y)dy \quad (20)$$

in which $p_\eta(y)$, as abovementioned, is the probability density of the random vector η and is generally known beforehand. It is easily seen from Eq.(20) that the reliability Ψ of the system is also the expectation value of the membership function $\mu_{\underset{\sim}{\Omega}}(y)$ of the fuzzy safe region $\underset{\sim}{\Omega}$ of the system.

Ⅳ. TWO SPECIAL CASES —— FUZZY RELIABILITY AND RANDOM RELIABILITY

In general, the maximum response S_l of any element and its allowable interval R_l may possess respectively one of the following four possible characteristics, deterministic, random, fuzzy and fuzzy-random. The combination of all their possible characteristics has 16 kinds of cases in all. We will only discuss the following two important special cases.

1. Random Reliability

When the maximum responses S_l $(l=1,2,\dots,L)$ of all elements and their allowable intervals R_l have only randomness without fuzziness, the safe region Ω of the system will have distinct boundary. For any realization y of the random vector η which belongs to Ω, $\mu_\Omega(y)=1$, otherwise $\mu_\Omega(y)=0$. In this case, Eq.(20) becomes

$$\Psi = \Pr(\Omega) = \int_\Omega p_\eta(y)dy \quad (21)$$

which is just the random reliability defined in current reliability theory, i.e. the reliability taking account only of the randomness in the system.

2. Fuzzy Reliability

When the maximum responses S_l $(l=1,2,\dots,L)$ of all elements and their allowable intervals R_l have only fuzziness without randomness, and hence there is no random vector in the problem, the membership function of the safe region of the system will become constant $\mu_{\underset{\sim}{\Omega}}$. In this case, since

$$\int_{-\infty}^{\infty} p_\eta(y)dy = 1$$

Eq.(20) becomes

$$\Psi = \mu_{\underset{\sim}{\Omega}} \tag{22}$$

which is the membership degree of the system response to the fuzzy safe region and may be named as fuzzy reliability of the system. It is seen from this case that even there is no randomness in the system, the fuzziness will also lead to some uncertainty in the safety of the system. So, for this case, the reliability analysis of the system must be made also.

V. CONCLUSIONS

In this paper, the concept of reliability is extended and the method of fuzzy-random reliability analysis for the complex system with multiple failure modes is presented. The system under discussion may have various kinds of members and one or more than one constraints may be imposed on each member. Each constraint corresponds to one element. The random vector η under consideration represents all the random parameters in both the external excitation and the system itself.

When some uncertain factors in the system become deterministic ones, the corresponding reliability is only a special case of the fuzzy-random reliability. Specially, for the system which only include fuzzy factors without random ones, the safety of the system should be measured also by a reliability, i.e. fuzzy reliability.

In our previous papers [3] and [9], the fuzzy-random reliability analysis of aseismic structures with single failure mode[3] and with multiple failure modes[9] are respectively made. They will serve as examples of the application of the theory presented in this paper.

The reliability analysis of a system may serve as the basis of the decision of the system. So, this paper provides a possibility for the decision of systems taking full account of various uncertainties in the problem.

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