Studies on Fuzzy PID Controllers: Part I

Xian-Tu Peng*, Xiang-Chu Tu+ and Pei-Zhuang Wang*

* Dept. of Math., Beijing Normal University, Beijing, People's Republic of China

\+ The 3rd Dept., Beijing Polytechnic University, Beijing, People's Republic of China

By introducing the term $\frac{d^2E}{dt^2}$ to fuzzy (logic) controllers and generalizing SRC ([10]), we get a general form of parametric fuzzy PID controllers.

Key Word Index--Computer control, fuzzy controllers, fuzzy PID controllers, linguistic control rules, implication operators, self-regulating parameters.

1. INTRODUCTION

SINCE the introducing of fuzzy (logic) controllers, there have been many applications in various fields, even in industrial processes([8] and [17]). In the theoretical aspects there have been also many advances([2] and [16] etc.). The control policy of SOC([16]) is one that can change with respect to the process it is controlling and the enviroment it is operating in. Another way to change the control policy with respect to the process is that proposed by S.Z.Long and P.Z.Wang ([10]) called self-regulation controllers (SRC) that was studied in more details by [7].

The importance of SRC lies not only in that it can change the control rules on-line but also in that it is the first controller to express the linguistic control rules by a parametric function. Almost all the published fuzzy controllers' rules are obtained by analysing the operators' experiences and composed of usually of the order 10-20 rules. There are some undefined actions so it cannot simulate the continuous thinking and deciding character of human-brain.

In this paper, we consider the integral action based on fuzzy logic in fuzzy controllers and the possibility of generating linguistic control rules by machine. The main topics are:

1. The general form of fuzzy PID controllers.

After analysing the fuzzy controllers and integral actions, we point out that the so-called "fuzzy PD" controllers are, in fact, fuzzy PI controllers and fuzzy PID controllers are of the form $\Delta u = F(\frac{dE}{dt}, E, \frac{d^2E}{dt^2})$ which is equivalent to $u = G(E, \int_{t_0}^{t} E dt, \frac{dE}{dt})$.

2. Digital simulations for fuzzy PID controllers.

3. Self-regulating of parameters on-line.

4. Accuracy of implication operators.

2. FUZZY PID CONTROLLERS

2.1 Parametric fuzzy controllers

Fuzzy controllers are defined by N individual linguistic control rules in the following form:

IF error is α and diffrential of error is β

THEN process-input-change is γ.

where α, β and γ are liguistic values, such as positive big, zero, negetive small.

Throughout this paper we suppose error E, differential of error $\frac{dE}{dt}$, process-input-change C and other linguistic variables take seven linguistic values: Negetive Big (NB), Negetive Medium (NM), Negetive Small (NS), ZEro (ZE), Positive Small (PS), Positive Medium (PM) and Positive Big (PB). Of course, those linguistic values have different meanings depending on different variables and processes.

The seven linguistic values are denoted by L_1= NB, L_2= NM, L_3= NS, L_4= ZE, L_5= PS, L_6= PM and L_7= PB. The SRM of 10 is expressed in the following form:

$$\text{If } E \text{ is } L_i \text{ and } \frac{dE}{dt} \text{ is } L_j \text{ THEN } C \text{ is } L_{f(i,j)}$$

where $f(i,j) = \langle A(i-4)+(1-A)(j-4)\rangle_0 + 4,\ 1 \leq i,j \leq 7,\ 0 < A < 1$ and

$$\langle a \rangle_0 \triangleq \begin{cases} a & \text{If a is an integer} \\ [a]+\text{sing}(a) & \text{otherwise.} \end{cases}$$

The parameter A may take various values with respect to various processes.

In the following section we will generalize the function f(i,j) to obtain fuzzy PID controllers.

2.2 Fuzzy PID controllers

Before setting up fuzzy PID controllers,let's take a look at the classical PID controller:

$$u(t) = K_p \cdot (e(t) + \frac{1}{T_i}\int_0^t e(t)dt + T_d \cdot \frac{de}{dt})$$

or by difference equation

$$u(n) == u(n-1) + K_p(e(n)-e(n-1) + \frac{T}{T_i} \cdot e(n) + \frac{T_d}{T}(e(n)-2e(n-1)+e(n-2))$$

that is,

$$u(n) = u(n-1) + \Delta u$$

$$\Delta u = K_p(e(n)-e(n-1)) + \frac{K_p T}{T_i} e(n) + \frac{K_p T_d}{T}(e(n)-2e(n-1)+e(n-2))$$

From the last equations we realize why the integral action $\frac{1}{T_i S}$ can reduce steady state error: If $e(n) \doteq e(n-1) \doteq e(n-2)$ and $e(n) \neq 0$, then the next process-input-change is $\Delta u = \frac{K_p T}{T_i} e(n)$, that is, only the integral term acts then.

There have been some studies on how to apply an integral action to fuzzy controller ([24]). But the main ideas are different from the classical integral action as analysed above.

In the classical PID controller, the differential term is acted to Δu with the difference

$$\frac{e(n)-e(n-1)}{T} - \frac{e(n-1)-e(n-2)}{T}$$

not with the differential directly. The above term just likes the term $\frac{d^2 e}{dt^2}$.

Based on the above analysis we give the general forms of liguistic control rules for fuzzy PI, fuzzy PD and fuzzy PID controllers:

Fuzzy PI controller: IF $\frac{dE}{dt}$ is α and E is β THEN C is γ.

Fuzzy PD controller: IF $\frac{dE}{dt}$ is α and $\frac{d^2 E}{dt^2}$ is β THEN C is γ.

Fuzzy PID controller:

IF $\frac{dE}{dt}$ is α and E is β and $\frac{d^2E}{dt^2}$ is γ THEN C is δ.

where α, β, γ and δ are linguistic values.

Fuzzy PD and fuzzy PID controllers are different from all the published fuzzy controllers.There have many studies on fuzzy PI controllers and other ways considering the integral action such as introducing the liguistic control rules:

IF the error-sum ......THEN process-input-change ....

In fact for the process controlled we have no knowledge to control the error-sum at any fixed point.Whereas for all processes we want to control the error,differential of error and the second differential of error at zero. Based this intuitive ideal we point out that it may be more effective to use fuzzy PID controllers introduced here for complex processes.

By generalizing SRM we get parametric fuzzy PID controller (FPID):

$$\text{IF } \frac{dE}{dt} \text{ is } L_i \text{ and } E \text{ is } L_j \text{ and } \frac{d^2E}{dt^2} \text{ is } L_k \text{ THEN } C \text{ is } L_{f(i,j,k)}, \quad 1 \le i,j,k \le 7.$$

where $f(i,j,k)=$

$\langle F_p(i-4)+F_i(j-4)+F_d(k-4) + 4 \rangle$, and $\langle a \rangle = 7 \wedge (\langle a \rangle_0 \vee 1)$. The parameters F_p, F_i and F_d are called FPID parameters.By adjusting those parameters we can get various linguistic control rules.

Similarly,we have FPI and FPD controllers.If we let $F_p = 0.7$ and $F_i = 0.7$ then the linguistic control rules of FPI controllers are the same with A2 of [2]:

ė \ e	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NB	NB	NM	NS	ZE	PS
NS	NB	NB	NM	NS	ZE	PS	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PM	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PB	ZE	PS	PM	PB	PB	PB	PB

2.3 Simplified computation

The seven linguistic values are defined by the following fuzzy sets on [-5,5].

$$L_1(w) = NB(w) = \begin{cases} 1 & \text{If } w \leq -4 \\ e^{-|w+4|} & \text{if } -4 < w < 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$L_2(w) = NM(w) = \begin{cases} e^{-|w+3|} & \text{If } w < -3 \\ 1 & \text{if } -3 \leq w \leq -2 \\ e^{-|w+2|} & \text{if } -2 < w < 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$L_3(w) = NS(w) = \begin{cases} e^{-|w+1|} & \text{If } w < -1 \\ 1 & \text{if } -1 \leq w < 0 \\ 0 & \text{otherwise .} \end{cases}$$

$$L_4(w) = ZE(w) = \begin{cases} 1 & \text{If } w = 0 \\ 0 & \text{otherwise .} \end{cases}$$

$$L_5(w) = PS(w) = NS(-w)$$

$$L_6(w) = PM(w) = NM(-w)$$

$$L_7(w) = PB(w) = NB(-w).$$

The parametric fuzzy PID (FPID) controller is defined by $7^3=343$ individual statements.The following is the computation algorithm.

For a triple input $E, \frac{dE}{dt}, \frac{d^2E}{dt^2}$,the output fuzzy set C is

$$C(u) = \bigvee_{i=1}^{7} \bigvee_{j=1}^{7} \bigvee_{k=1}^{7} \left(L_i(\dot{e}) \wedge L_j(e) \wedge L_k(\ddot{e}) \wedge L_{f(i,j,k)}(u) \right)$$

where $\dot{e} = \frac{dE}{dt}/GD$, $e = E/GE$, $\ddot{e} = \frac{d^2E}{dt^2}/GH$, i.e., they are scaled by scaling factors GD, GE and GH. Non-fuzzifying decision operator is the procedure of finding the "centre of gravity":

$$\Delta c \triangleq D(C) = \frac{\int_{-5}^{5} u \cdot C(u)\, du}{\int_{-5}^{5} C(u)\, du}$$

By multiplying a scaling factor GC we get the process-input-change:

$$\Delta u = \Delta c \cdot GC$$

The above procedure can be illustrated by Fig.1 .

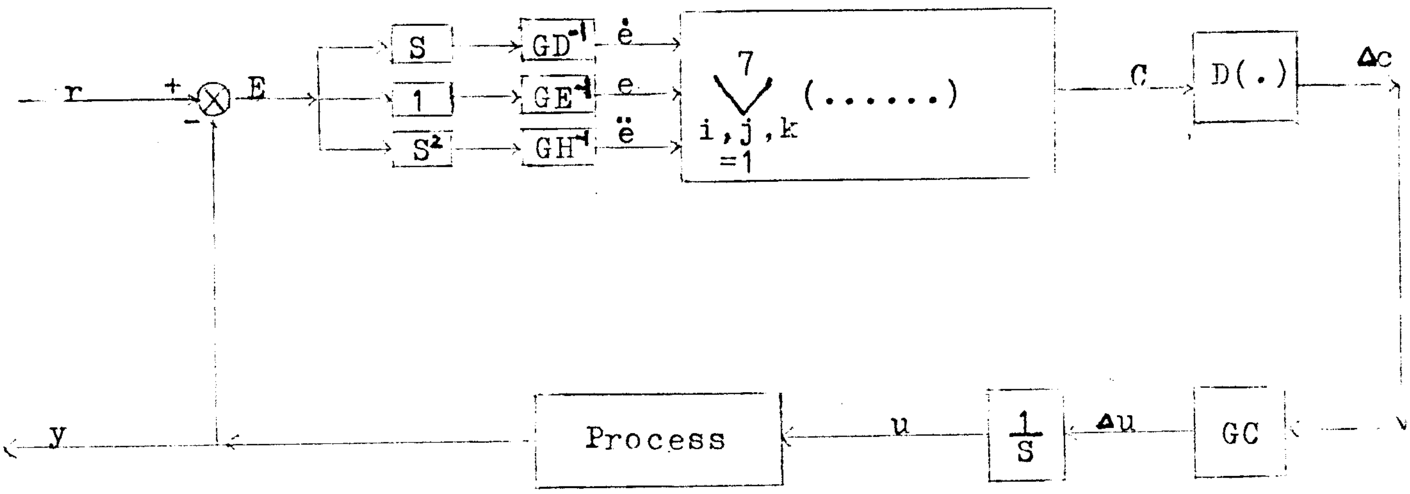

Fig.1 Fuzzy PID controller

It takes 7×7×7×4 = 1372 times min-max operations to compute the membership C(u) for any u by the above formula.Then it will take very long time to compute $D(C) = \int_{-5}^{5} u\cdot C(u)du \ / \int_{-5}^{5} C(u)du$ .It is neccessary to simplify the computation.

Noticing that $L_5(x) = L_6(x) = L_7(x) = 0$ whenever $x \leq 0$,we have a simplified formula for C(u).

$$C(u) = \bigvee_{i=i_0}^{i_0+3} \bigvee_{j=j_0}^{j_0+3} \bigvee_{k=k_0}^{k_0+3} \left(L_i(\dot{e}) \wedge L_j(e) \wedge L_k(\ddot{e}) \wedge L_{f(i,j,k)}(u) \right)$$

where i_0 **equals** 1 or 4 with respect to ė is negetive or not,i.e.,

$$i_0 = \begin{cases} 1 & \text{If } \dot{e} \leq 0 \\ 4 & \text{otherwise.} \end{cases}$$

j_0 and k_0 are defined similarly.

According to the new expression of C(u),it takes only 4×4×4×4 = 256 times min-max operations to compute C(u).

2.4 Digital simulation results

For processes $\frac{e^{-0.4S}}{(0.3S+1)^2}$ and $\frac{e^{-0.8S}}{(S+1)(S+2)}$,let FPID parameters be:

$$F_p = 0.3,\ F_i = 0.45,\ F_d = 0.15$$

and scaling factors be:

$$GD = 1,\ GE = 1.667,\ GH = 1,\ GC = 0.8\ .$$

With the above parameters the responses are sketched in Fig.2 .

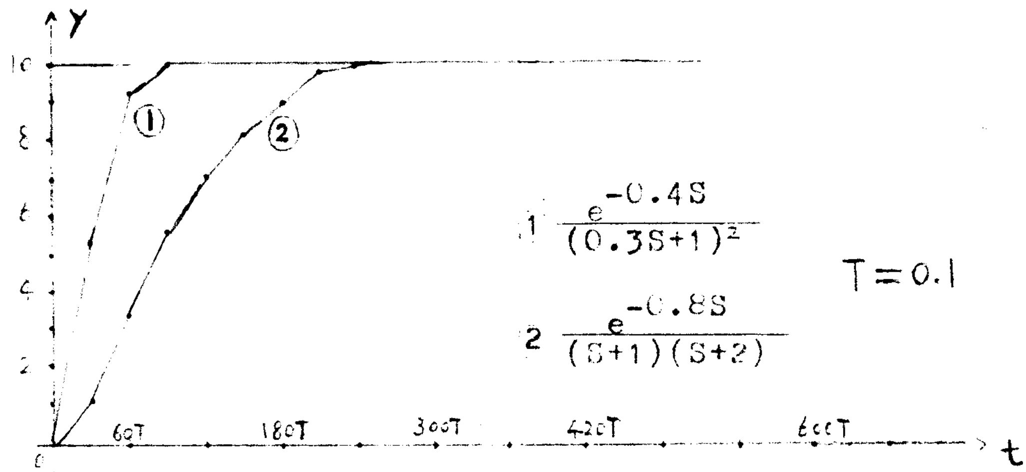

Fig.2 The responses under FPID controlling

In Fig.2 $y_{①} \in [9.97,\ 10.3]$ and $y_{②} \in [9.92,\ 10.08]$ after $t > 12$ and $t > 24$ respectively.

Under classical PID contolling with optimal PID parameters ([23]) $K_p = 0.630517$, $T_i = 0.594813$, $T_d = 0.237036$ the response of process $\frac{e^{-0.4S}}{(0.3S+1)^2}$ is sketched in Fig.3 .

Comparing Fig.2 and Fig.3 we find that the rise-time is shorter under FPID than that under PID controlling.We list the integral square error $\Delta^2 E = \int_0^{250T} |E(t)|^2\,dt$ and $ITAE = \int_0^{250T} t\,|E(t)|\,dt$ as follows:

	$\Delta^2 E$	ITAE
FPID	199.0	68.6
PID	291.9	236.3

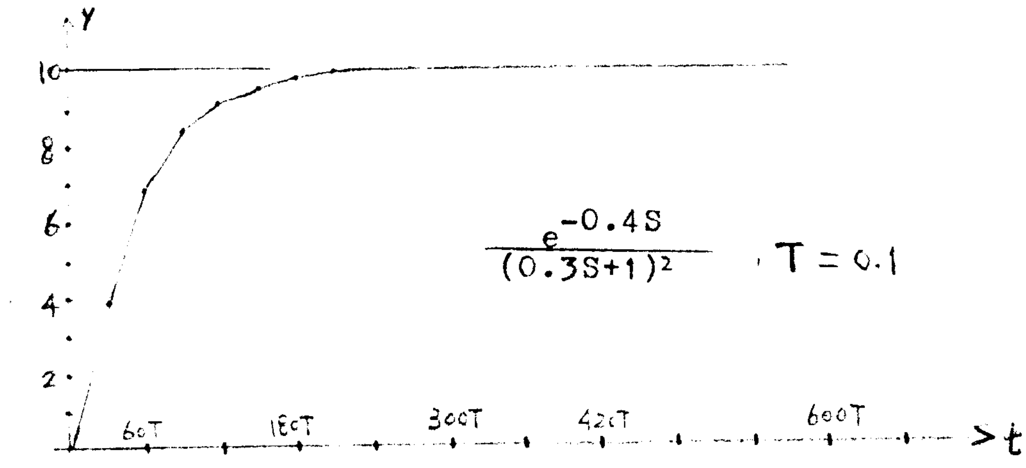

Fig.3 The response under PID controlling

In fuzzy PI controllers there is no direct rules to control the change of $\frac{dE}{dt}$ .Then the error may increase and decrease unregularly.This idea has been proved by digital simulations such as in Fig.4 which sketches the response of the process $\frac{e^{-0.4S}}{(0.3S+1)^2}$ under a fuzzy PI controlling with the same parameters as in the above fuzzy PID controller. In Fig.4 $y \in [9.6, 10.4]$ after $t > 9$.

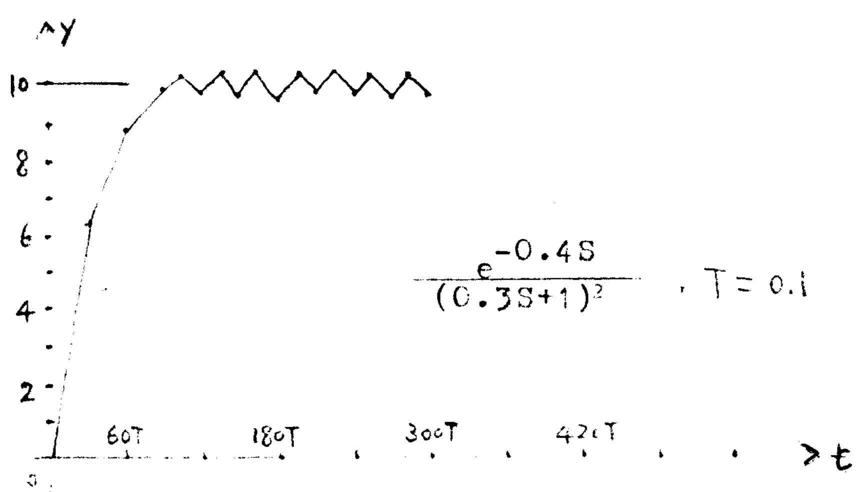

Fig.4 The response under FPI controlling

(to be continued)