MATHEMATICAL MODEL OF FUZZY DISCRIMINANT ANALYSIS FOR LOGICAL CHARACTER

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This paper presents a model of fuzzy discrimination, where the discriminant factors are linguistic variables. We treat discriminant factor levels as the fuzzy subsets on the corresponding universe discussed. We use the maximum membership principle as the criterion in order to establish the discriminant functions. Hence, we may base the study of classification of an object by means of natural language analysis.

KEYWORDS: Linguistic variable, Fuzzy discriminant function, Fuzzy population.

1. Introduction

It is well known that in the theory of classical discriminant analysis, the discriminant factors are all real variables. However, in many practical problems, the discriminant factors possess only a qualitative difference not a quantitative differece. For instance, the sexuality and the occupation etc. Because the qualitativevariables play an important role in many discriminant problems, the theory of qualitative that changes qualitative analysis to quantitative analysis based on two-valued logic has been presented. In fact, many qualitative variables are usually the fuzzy concepts, for example, "fine", "cloudy", "overcast" for the weather and "high", "low", "normal" for the blood pressure etc. It is not suitable that we use methods of general discriminant analysis and the theory of qualitative.

The theory of fuzzy subsets established by L.A. Zadeh provides a suitable mathematical method to research the fuzzy objects. With the aid of the theory of fuzzy subsets, we suggest a discriminant model of linguistic analysis. Using this model, we may classify the samples according to different populations.

2. General description of fuzzy discriminant analysis

Let discriminant factors $\alpha_1, \alpha_2, \ldots, \alpha_p$ be p linguistic variables, $X_i=$

$\{\underset{\sim}{X}_{i1}, \underset{\sim}{X}_{i2}, \dots, \underset{\sim}{X}_{ir_i}\}$ a finite "value domains" of α_i, where $\underset{\sim}{X}_{ij}$ is a fuzzy subset, it is called a level of factor α_i, $i=1,2,\dots,p$; $j=1,2,\dots,r_i$. We consider the ordinal set which consists of all levels of factors $\alpha_1, \alpha_2, \dots, \alpha_p$ as a universe discussed

$$U=\{ \underset{\sim}{X}_{11},\underset{\sim}{X}_{12},\dots,\underset{\sim}{X}_{1r_1},\underset{\sim}{X}_{21},\underset{\sim}{X}_{22},\dots,\underset{\sim}{X}_{2r_2},\dots,\underset{\sim}{X}_{p1},\underset{\sim}{X}_{p2},\dots,\underset{\sim}{X}_{pr_p}\} \tag{1}$$

The observational sample t can represent as a fuzzy subset on U.

$$\underset{\sim}{t}=\{\mu(1,1), \mu(1,2),\dots, \mu(1,r_1),\dots, \mu(p,1),\dots, \mu(p,r_p)\} , \tag{2}$$

where $\mu(i,j)$ is the membership grade of $\underset{\sim}{t}$ to level $\underset{\sim}{X}_{ij}$. The fuzzy subset $\underset{\sim}{t}$ is called a response vector of sample t.

Suppose that there are k populations $S_1, S_2,\dots,S_k$, and there are m_v known samples from the population S_v ($v=1,2,\dots,k$), where the membership relations of each sample to populations is all clear and $\sum_{v=1}^{K} m_v=m$. Now we consider p discriminant function $\alpha_1, \alpha_2,\dots, \alpha_p$ about there samples. For each α_i ($i=1,2,\dots, p$) there are r_i levels $\underset{\sim}{X}_{i1}, \underset{\sim}{X}_{i2},\dots, \underset{\sim}{X}_{ir_i}$. They are respectively some fuzzy subsets on the corresponding universe discussed, and $\sum_{i=1}^{p} r_i = n$.

After observation and analysis of these m sample, we obtain m response vectors, and put them inorder as Table 1. It is called the response matrix of factor levels, there $\mu_l^v(i,j)$ denotes a membership grade of the lth sample t_l^v of population S_v to the jth level $\underset{\sim}{X}_{ij}$ of factor α_i, obviously $\mu_l^v(i,j) \in [\,0,1\,]$. We denote $R= [\,\mu_l^v(i,j)\,]_{m\times n}$.

We have two explanations for the level $\underset{\sim}{X}_{ij}$ of factor.

(1). $\underset{\sim}{X}_{ij}$ is a "value" of the linguistic variable α_i. It is a fuzzy subset on the corresponing universe discussed.

(2). Suppose that $\mathcal{T}$ is the samples space and T is a set of m observational samples t_l^v ($v= 1,2,\dots,k$; $l= 1,2,\dots,m_v$) which are from k known populations. We take

$$T= \{ t_1^1, t_2^1,\dots, t_{m_1}^1,t_1^2,t_2^2,\dots,t_{m_2}^2,\dots,t_1^k,t_2^k,\dots,t_{m_k}^k \} , \tag{3}$$

Table 1. Level response matrix

$$
\begin{array}{cccccccc}
 & X_{11} & X_{12}\cdots\cdots & X_{1r_1} & X_{21}\cdots\cdots & X_{2r_2}\cdots\cdots & X_{p1}\cdots\cdots & X_{pr_p} \\
 & \mu_1^1(1,1) & \mu_1^1(1,2)\cdots & \mu_1^1(1,r_1) & \mu_1^1(2,1)\cdots & \mu_1^1(2,r_2)\cdots & \mu_1^1(p,1)\cdots & \mu_1^1(p,r_p) \\
S_1 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & \mu_{m_1}^1(1,1) & \mu_{m_1}^1(1,2)\cdots & \mu_{m_1}^1(1,r_1) & \mu_{m_1}^1(2,1)\cdots & \mu_{m_1}^1(2,r_2)\cdots & \mu_{m_1}^1(p,1)\cdots & \mu_{m_1}^1(p,r_p) \\
 & \mu_1^2(1,1) & \mu_1^2(1,2)\cdots & \mu_1^2(1,r_1) & \mu_1^2(2,1)\cdots & \mu_1^2(2,r_2)\cdots & \mu_1^2(p,1)\cdots & \mu_1^2(p,r_p) \\
S_2 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & \mu_{m_2}^2(1,1) & \mu_{m_2}^2(1,2)\cdots & \mu_{m_2}^2(1,r_1) & \mu_{m_2}^2(2,1)\cdots & \mu_{m_2}^2(2,r_2)\cdots & \mu_{m_2}^2(p,1)\cdots & \mu_{m_2}^2(p,r_p) \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & \mu_1^k(1,1) & \mu_1^k(1,2)\cdots & \mu_1^k(1,r_1) & \mu_1^k(2,1)\cdots & \mu_1^k(2,r_2)\cdots & \mu_1^k(p,1)\cdots & \mu_1^k(p,r_p) \\
S_k & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & \mu_m^k(1,1) & \mu_m^k(1,2)\cdots & \mu_m^k(1,r_1) & \mu_m^k(2,1)\cdots & \mu_m^k(2,r_2)\cdots & \mu_m^k(p,1)\cdots & \mu_m^k(p,r_p)
\end{array}
$$

$T\in\mathcal{T}$, and $\underset{\sim}{X}_{ij}$ may be consider as a fuzzy subset on T.

For a given sample $t\in\mathcal{T}$, the membership grade $\mu(i,j)$ expresesses a true value of the proposition equivalent to a assignment equation $\alpha_i=\underset{\sim}{X}_{ij}$ where $\underset{\sim}{X}_{ij}(t)$ is also called thefuzzy variable.

The purpose of the fuzzy discriminant analysis is that on the basis of k populations and n levels of factors establish k satisfatory diacriminant functions.

$$y_v(t)=f_v[\ \underset{\sim}{X}_{11}(t),\ \underset{\sim}{X}_{12}(t),\cdots\cdots,\underset{\sim}{X}_{pr_p}(t)\ ] \tag{4}$$

v=1,2,...,k. The discriminant function f_v is a fuzzy logical function formed by fuzzy variables $\underset{\sim}{X}_{ij}(t)$ through finite eimes $\vee$, $\wedge$, c, where

$$
\begin{aligned}
&\underset{\sim}{X}_i(t)\vee\underset{\sim}{X}_j(t)\triangleq \mathrm{Max}\ [\ \underset{\sim}{X}_i(t),\ \underset{\sim}{X}_j(t)\ ]\triangleq \underset{\sim}{X}_i(t)+\underset{\sim}{X}_j(t)\\
&\underset{\sim}{X}_i(t)\wedge\underset{\sim}{X}_j(t)\triangleq \mathrm{Min}\ [\ \underset{\sim}{X}_i(t),\ \underset{\sim}{X}_j(t)\ ]\triangleq \underset{\sim}{X}_i(t).\underset{\sim}{X}_j(t)\\
&\underset{\sim}{X}_i^c(t)\triangleq 1-\underset{\sim}{X}_i(t)
\end{aligned}
\tag{5}
$$

Suppose that f_v shows a objective state D_v, then $y_v(t)$ is a true value of the population "the state of t is D_v".

For the response vector $\underset{\sim}{t}$ of a given sample, we can find out corresponding $y_v(t)$, v= 1,2,...,k, and we take the maximum membership orinciple as discriminatory criterion. For sample t, if

$$y_v(t) = \mathrm{Max}\{\, y_1(t),\ y_2(t), \ldots,\ y_k(t)\} \tag{6}$$

then t is classified into population S_v.

3. Establishment of the discriminant functions

In this section, we will give a general method of esablishing discriminant functions.

Definition 3.1 We take thelevels of factor as logical variables, f is called a fuzzy distinguished disjunctive normal form, if f is a disjunctive normal form and for all (i,j), either $\underset{\sim}{X}_{ij}(t)$ or $\underset{\sim}{X}^c_{ij}(t)$ must occur in each disjunct.

Definition 3.2 If $f(t)\geq\frac{1}{2}$ for all value assignment of the fuzzy variables, then fuzzy logical function (4) is called the fuzzy always true formula, if $f(t)<\frac{1}{2}$ then the fuzzy contradictory.

Proposition 3.1 A fuzzy logical function f is the fuzzy always true formula, if and only if f is a always true formula in two-valued logic.

Proposition 3.1 Explains the relation between the fuzzy logic and the two-valued logic. Using this Proposition, we easily obtain a method to establish the fuzzy discriminant functions which satisfy formula (6).

Suppose that we take m samples from k populations, Table 1 is their response matrix R. We replace the elements $\mu_1^v(i,j)$ of R by $\delta_1^v(i,j)$, where

$$\delta_1^v(i,j) = \begin{cases} 1, & (\ \mu_1^v(i,j)\geq\frac{1}{2}\ ) \\ 0 & (\ \mu_1^v(i,j)<\frac{1}{2}\ ) \end{cases} \tag{7}$$

The matrix $R^* = [\,\delta_1^v(i,j)\,]_{m\times n}$ is called the truth table of the fuzzy disciminant functions. In the truth table, by $\delta_1^v(i,j)=1$ we mean that the proposition "$\alpha_i = X_{ij}$" is fuzzy true to t_1^v ; and $\delta_1^v(i,j)=0$ is fuzzy false to t_1^v. Thus, the fuzzy propositional calculus in the fuzzy logic may be changed as the propositional calculus of the two-valued logic. It is easy to see that the submatrix from 1th row to m_1th row of R^* is a truth table of discriminant function f_1, from (m_1+1)th to (m_1+m_2)th row is a truth table of $f_2, \ldots, m_k$ rows at rear of the matrix R^* is a truth table of f_k.

We mssume that there are no similar samples, i.e., $\forall\, t_a^w \in S_w$, $t_b^v \in S_v$, then exist at least a pair of i,j so that $\delta_a^w(i,j) \neq \delta_b^v(i,j)$. If there are similar samples, then reject them from the samples set.

On the basis of Proposition 3.1, the discriminant functions $f_1, f_2, \ldots, f_k$ of each populations can be eatablished by corresponding truth tables, all of them are the distinguished disjunctive normal form. It is easy to see that for all $v=1,2,\ldots,k$ and $l=1,2,\ldots,m_v$, we have

$$y_v(t_l^v)=\mathrm{Max}\{y_1(t_l^v),\ y_2(t_l^v),\ldots,\ y_k(t_l^v)\} \quad . \tag{8}$$

Each disjunct of the discriminant function f_i is called a minor term of the distinguished disjunctive normal form, it described a possible state which is shown by sample t. In the truth table of f_i, each row vector is called a assignmen of a sample state.

4. The completeness of the discriminant function group

Definition 4.1 Let the population $S_1, S_2, \ldots, S_k$ be a partition of the samples space, f_i be the discriminant function of S_i ($i=1,2,\ldots,k$). $f_1, f_2, \ldots, f_k$ is called a complete group of discriminant functions for the samples space $\mathcal{T}$, if for an arbitrary sample $t \in \mathcal{T}$. there exists a discriminant function f_v $(v=1,2,\ldots,k)$, so that

$$f_v(t) > \mathop{\mathrm{Max}}_{\substack{i=1,2,\ldots,k \\ i \neq v}} \{ f_i(t) \} . \tag{9}$$

It is well known that n proposition variables can form 2^n minor terms. Because each minor term which is constituted by n fuzzy variables $\underset{\sim}{X}_{ij}(t)$ is a possible state of the sample t, all possible states of samples have 2^n kinds when we consider n factor levels. To sum up the above mentioned, we easily see that $f_1, f_2, \ldots, f_k$ is a complete group of discriminant functions for $\mathcal{T}$, if and only if there exist 2^n different assignments of the sample states, where n is a number of factor levels. In some pratical problems, because the discriminant factors and the factor levels are so many and observational datas are not adequate, it is difficult to establish a complete group of discriminant functions.

In this paper, we consider a kind of particular case — the discriminant factors are ordered linguistic variables statistically correlative with the populations, a method of establishment of the discriminant function is given, and the misjudgment of the discriminant functions can be decreased.

Definition 4.2 Let $\underset{\sim}{A}$ be a fuzzy subset on X. The nucleus of fuzzy subset $\underset{\sim}{A}$ is a classical subset on X, denoted by

$$H(\underset{\sim}{A}) \triangleq \{x \mid x \in X \text{ and } \underset{\sim}{A}(x) = 1\} \tag{10}$$

Definition 4.3 Let X be a ordered set by order relation $\prec$, $\underset{\sim}{A}_1$, $\underset{\sim}{A}_2$ be two normal convex fuzzy subsets on X, x_1 be a maximal element of $H(\underset{\sim}{A}_1)$ and x_2 minimal element of $H(\underset{\sim}{A}_2)$. If (1)、$x_1 \prec x_2$, (2)、there exists a element $x_0 \in X$, so that $\forall x \in X$, when $x \prec x_0$, we have $\underset{\sim}{A}_1(x) \geq \underset{\sim}{A}_2(x)$; when $x_0 \prec x$, we have $A_1(x) \leq A_2(x)$, then we say $\underset{\sim}{A}_1$ is anterior to $\underset{\sim}{A}_2$, denoted by $\underset{\sim}{A}_1 \prec \underset{\sim}{A}_2$.

Definition 4.4 Let $\underset{\sim}{A}_1, \underset{\sim}{A}_2, \ldots, \underset{\sim}{A}_n$ be n normal convex fuzzy subsets on X and for the relation $\prec$, and $\underset{\sim}{A}_1 \prec \underset{\sim}{A}_2 \prec \ldots \prec \underset{\sim}{A}_n$, then $\{\underset{\sim}{A}_1, \underset{\sim}{A}_2, \ldots, \underset{\sim}{A}_n\}$ is called a group of the ordered fuzzy subsets on X.

We take usually the real number field as X, thus the relation $\prec$ may be denoted as $\leq$.

Definition 4.5 If a set of levels of the factor α_i can be expressed as a group of ordered fuzzy subsets on a ordered universe discussed, then the set of levels of the factor is called the ordered, and α_i is called an ordered linguistic variable.

Definition 4.6 Let $\mathcal{S} \triangleq \{S_1, S_2, \ldots, S_k\}$ be a set of all populations, we define an order relation $\prec$ on $\mathcal{S}$ such that

$$S_1 \prec S_2 \prec \ldots \prec S_k .$$

If there is a statistical correlation between the populations and the variables of ordered set X, then we say that the ordered linguistic variables are correlative with the set $\mathcal{S}$ of populations.

(1). The establishment of a complete group of discriminant functions for the two-class discrimination.

What is called the two-class discrimination is that the set of population $\mathcal{S}$ contains only two population, i.e., $\mathcal{S} = \{S_1, S_2\}$. Suppose that all discriminant factors α_i (i=1,2,...,p) are the ordered linguistic variables on the corresponding universe discussed. Let $\{\underset{\sim}{X}_{i1}, \underset{\sim}{X}_{i2}, \ldots, \underset{\sim}{X}_{ir_i}\}$ be a set of the levels of α_i, and for order relation $\underset{\sim}{X}_{i1} \prec \underset{\sim}{X}_{i2} \prec \ldots \prec \underset{\sim}{X}_{ir_i}$ and $S_1 \prec S_2$, the relation between α_i and β is the positive statistical correlation.

Let $\underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p}$ and $\underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p}$ be respectively the different states of two samples and $l_k \geq j_k$ ($k=1,2,\ldots,p$), where $\underset{\sim}{X}_i\cdot\underset{\sim}{X}_j \triangleq \underset{\sim}{X}_i \wedge \underset{\sim}{X}_j$. Because the relation between α_i and β is the positive correlation for any $i=1,2,\ldots,p$. Thus, if $\underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p}$ is a minor term of discriminant function f_1, then $\underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p}$ is also a minor term of f_1. We say that state $\underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p}$ includes the state $\underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p}$ for S_1. denoted as follows

$$\underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p} \overset{S_1}{\supset} \underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p} \tag{11}$$

If $\underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p}$ is a minor term of f_2, then $\underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p}$ is also a minor term of f_2, and we say that state $\underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p}$ includes state $\underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p}$ for S_2. denoted as follows

$$\underset{\sim}{X}_{1j_1}\cdot\underset{\sim}{X}_{2j_2}\cdots\underset{\sim}{X}_{pj_p} \overset{S_2}{\supset} \underset{\sim}{X}_{1l_1}\cdot\underset{\sim}{X}_{2l_2}\cdots\underset{\sim}{X}_{pl_p} \tag{12}$$

Definition 4.7 Let D_i and D_j be two states of the samples, we say that state D_j is a sub-state of D_i for S_k, if $D_i \overset{S_k}{\supset} D_j$.

Definition 4.8 Let $\mathcal{D}_k$ be the collection of the states of all observational samples of population S_k. State D_i is called a maximal state of S_k, if $D_i \in \mathcal{D}_k$ and for any $D_j \in \mathcal{D}_k$, have $D_j \overset{S_k}{\not\supset} D_i$.

State D_l is called a maximal separated state, if D_l is a maximal state, and for any D_i, if $D_i \overset{S_k}{\supset} D_l$, have $D_i \notin S_k$.

If the number of observational samples is not adequate, in the two-class discrimination, it is usually impossible to find all maximal separated states of the populations. However, when the number of observational samples is larger, the maximal separted states of the populations may be replaced by maximal states.

Suppose that $D_1, D_2, \ldots, D_q$ are q maximal states in m sample states of population S_1, and $\hat{\mathcal{D}}_1$ is collection of all sub-states of $D_1, D_2, \ldots, D_q$. Then the set of all possible states of S_1 is as follows

$$\mathcal{D}_1 = \{D_1, D_2, \ldots, D_q\} \cup \hat{\mathcal{D}}_1 \tag{13}$$

This way is analogous to replenish artificially some sampled datas in order to make up the lack of samples. We take all states $\underset{\sim}{X}_{1u_1} \cdot \underset{\sim}{X}_{2u_2} \cdots \underset{\sim}{X}_{pu_p}$ in $\mathcal{D}_1$ as the minor terms of the distingnished disjunctive normal form, discriminant function f_1 is established consequently, and take $f_2 = f_1^c$.

(2). **The establishment of a complete group of discriminant functions for the k-class discrimination.**

Let $\mathcal{S} = \{S_1, S_2, \ldots, S_k\}$, α_i be the ordered linguistic variables on the corresponding universe discussed ($i=1,2,\ldots,k$), and α_i is correlative with $\mathcal{S}$ for the order relation $X_{i1} \prec X_{i2} \prec \cdots \prec X_{ir_i}$ and $S_1 \prec S_2 \prec \cdots \prec S_k$.

First we partition $\mathcal{S}$ so that $\mathcal{S} \triangleq \{S_1^{(1)}, S_2^{(1)}\}$, where $S_1^{(1)} = S_1$, $S^{(1)} = \{S_2, S_3, \ldots, S_k\}$, to make two-class discrimination to $\mathcal{S}$. We apply the maximal states of $S_1^{(1)}$ to establish discriminant function $f_1^{(1)}$, which is the discriminant function of S_1, i.e., $f_1 = f_1^{(1)}$.

After that we partition $S_2^{(1)}$ so that $S_2^{(1)} = \{S_1^{(2)}, S_2^{(2)}\}$, where $S_1^{(2)} = S_2$, $S_2^{(2)} = \{S_3, S_4, \ldots, S_k\}$ to make the two-class discrimination to $S_2^{(1)}$. We apply the maximal states of $S_1^{(2)}$ to establish function $f_1^{(2)}$ now the discriminant function of S_2 is $f_2 = f_1^{(2)} - f_1$ ($f_1^{(2)} - f_1$ means that all disjuncts of f_1 are cut out from the formula $f_1^{(2)}$).

On the analogy of this, we can obtain discriminant functions $f_1, f_2, \ldots, f_{k-1}$, then take $f_k = (f_1 + f_2 + \cdots + f_{k-1})^c$.

5. Fuzzy discrimination for fuzzy populations

Population S is called the fuzzy population, if the membership relations of samples for the population S are not clearcut, and S can bexpressed by a fuzzy subset on certain universe discussed.

Definition 5.1 Let $\underset{\sim}{S}_1, \underset{\sim}{S}_2, \ldots, \underset{\sim}{S}_k$ be a group of the fuzzy subsets on the universe discussed U, $\underset{\sim}{S}_i(u)$ be membership function of $\underset{\sim}{S}_i$ (i=1,2,...,k). If we have $\sum_{i=1}^{k} S_i(u) = 1$, for $\forall\ u \in U$, then $\{\underset{\sim}{S}_1, \underset{\sim}{S}_2, \ldots, \underset{\sim}{S}_k\}$ is called a fuzzy partition of U.

In the fuzzy discrimination for the fuzzy populations, $\underset{\sim}{S}_1, \underset{\sim}{S}_2, \ldots, \underset{\sim}{S}_k$ are a fuzzy partition of the universe discussed U. For any sample $t_j \in \mathcal{J}$. there exists only a $u_j \in U$ corresponding to t_j, thus we can obtain respectively all membership grades of samples to each population $\underset{\sim}{S}_i$. we write that $\underset{\sim}{S}_i(t) \triangleq \underset{\sim}{S}_i(u_j)$, i=1,2,...,k, and $\sum_{i=1}^{k} S_i(t_j) = 1$.

Suppose that $t_1, t_2, \ldots, t_m$ are m konwn samples, we consider p discriminant factors $\alpha_1, \alpha_2, \ldots, \alpha_p$, there α_i contains r_i levels $\underset{\sim}{X}_{i1}$, $\underset{\sim}{X}_{i2}, \ldots, \underset{\sim}{X}_{ir_i}$ (i=1,2,...,p), which are some fuzzy subsets on the corresponding universe discussed. Our purpose is to give a mathematical model of the discrimination on the basis of the response vectors of observational samples $t_1, t_2, \ldots, t_m$ and the membership grades of the samples to population S_i (i=1,2,...,k).

First, using maximum membership principle, we divide the set of the samples $\mathcal{J}' = \{t_1, t_2, \ldots, t_m\}$ into k-classes if

$$\underset{\sim}{S}_v(t_j) = \max_{i=1,2,\ldots,k} \{\underset{\sim}{S}_i(t_j)\} \ , \tag{14}$$

then t_j belongs to populayion $\underset{\sim}{S}_v$. Then according the section 4, we can establish a complete group of discriminant functions $f_1, f_2, \ldots, f_k$.

Denote

$$y_{ij} = f_i(t_j), \quad s_{ij} = \underset{\sim}{S}_j(t_i), \quad i=1,2,\ldots,k;\ j=1,2,\ldots,m\ , \tag{15}$$

there y_{ij} is the true value of the proposition "t belonging to the state $\underset{\sim}{D}_{S_i}$" to sample t_j.

Let

$$\underset{\sim}{R}_1(\mathcal{D}_S, \mathcal{J}) = \begin{bmatrix} y_{11} & y_{12} & \cdots y_{1m} \\ y_{21} & y_{22} & \cdots y_{2m} \\ \vdots & \vdots & \vdots \\ y_{k1} & y_{k2} & \cdots y_{km} \end{bmatrix}, \text{and} \quad \underset{\sim}{R}_2(\mathcal{J}, \mathcal{S}) = \begin{bmatrix} s_{11} & s_{12} & \cdots s_{1k} \\ s_{21} & s_{22} & \cdots s_{2k} \\ \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & \cdots s_{mk} \end{bmatrix} \tag{16}$$

there $\mathcal{D}_S = \{ D_{S_1}, D_{S_2}, \ldots, D_{S_k} \}$, D_{S_i} is a set of the sample states of the population $\underset{\sim}{S}_i$ (i=1,2,...,k). R_1 is a fuzzy relation on $\mathcal{D}_S \times \mathcal{J}'$, and R_2 on $\mathcal{J}' \times \mathcal{S}$. Using the cosposition product of the fuzzy relation matrix, we can obtain a fuzzy relation on $\mathcal{D}_S \times \mathcal{S}$.

$$\underset{\sim}{Q}(\mathcal{D}_S, \mathcal{S}) = \underset{\sim}{R}_1 \circ \underset{\sim}{R}_2 = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1k} \\ q_{21} & q_{22} & \cdots & q_{2k} \\ \vdots & \vdots & & \vdots \\ q_{k1} & q_{k2} & \cdots & q_{kk} \end{bmatrix} \tag{17}$$

where $\underset{\sim}{Q} = \underset{\sim}{R}_1 \circ \underset{\sim}{R}_2 \Leftrightarrow q_{ij} = \underset{l=1,2,\ldots,m}{\mathrm{Max}} [\, \mathrm{Min} (y_{il}, s_{lj}) \,]$, $\underset{\sim}{Q}$ is called a fuzzy transformation from the state set of samples $\mathcal{D}_S = \{ D_{S_1}, D_{S_2}, \ldots, D_{S_k} \} \triangleq \{ f_1, f_2, \ldots, f_k \}$ to the set of populations $\mathcal{S} = \{ S_1, S_2, \ldots, S_k \}$.

For an undetermined sample t, we compute

$$\underset{\sim}{Y}^T = (y_1, y_2, \ldots, y_k)$$

where $y_l = f_l(t)$, l=1,2,...,k. Compute

$$\underset{\sim}{Q}^T \circ \underset{\sim}{Y} = \begin{bmatrix} q_{11} & q_{21} & \cdots & q_{k1} \\ q_{12} & q_{22} & \cdots & q_{k2} \\ \vdots & \vdots & & \vdots \\ q_{1k} & q_{2k} & \cdots & q_{kk} \end{bmatrix} \circ \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix} \tag{18}$$

where $c_i = \underset{l=1,2,\ldots,k}{\mathrm{Max}} [\, \mathrm{Min} (q_{li}, y_l)]$ denotes the possibility degree of sample t belonging to population S_i.

The fuzzy discriminant analysis is not only used to classify samples which are expressed as name of compound linguistic variable, and to describe the character of researched populations by means natural languages.

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