FUZZY CUBIC MATRIX ALGEBRA

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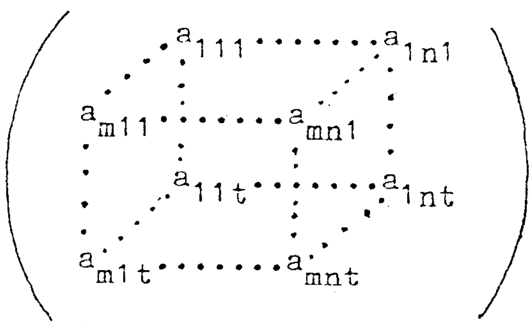
Abstract

In this paper a fuzzy cubic matrix is definited, and some operations of fuzzy cubic matrix are introduced. Applying them, we discussed some problems of fuzzy cubic matrix .

Keywords: Fuzzy cubic matrix.

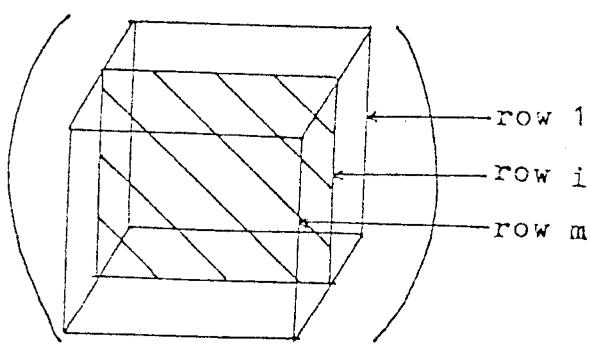
I. A Definition of Fuzzy Cubic Matrix

Definition 1.1 An arrangement of mxnxt elements of (o,1) in m rows, n columns and t storeys (see (1)) is called a mxnxt fuzzy cubic matrix, denoted by $A=(a_{ijk})_{m\times n\times t}$.

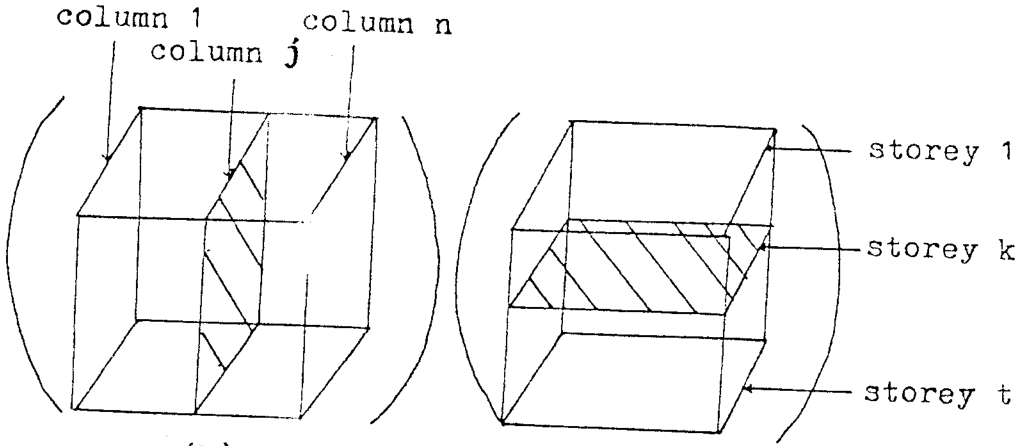


(1)

Its row i, column j, storey k are respectively shown by shadow part of picture 1 (a), (b), (c).



(a)



(b) (c)

Picture 1

As see row i of mxnxt fuzzy cubic matrix A is a txn fuzzy matrix

$$\begin{pmatrix} a_{i11} \cdots\cdots a_{in1} \\ \vdots \qquad\qquad \vdots \\ a_{i1t} \cdots\cdots a_{int} \end{pmatrix}$$

and is called row i matrix of A. Similarly column j and storey k of mxnxt fuzzy cubic matrix A are respectively txm matrix and mxn matrix

$$\begin{pmatrix} a_{1j1} \cdots\cdots a_{mj1} \\ \vdots \qquad\qquad \vdots \\ a_{1jt} \cdots\cdots a_{mjt} \end{pmatrix} \quad \begin{pmatrix} a_{11k} \cdots\cdots a_{1nk} \\ \vdots \qquad\qquad \vdots \\ a_{m1k} \cdots\cdots a_{mnk} \end{pmatrix}$$

and are repectively called column j matrix and storey k matrix of A.

As see the fuzzy cubic matrix is a naturural extantion of the fuzzy matrix.

We definition the particular cubic matrix as follows :

1) $\theta=(0)_{m\times n\times t}$ is called a fuzzy null cubic matrix.

2) $I=(a_{ijk})_{m\times m\times m}$ where $a_{ijk}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}$ is called a

fuzzy identity cubic matrix.

3) $E=(1)_{m\times n\times t}$ is called a fuzzy universal cubic matrix.

II. The Intersection, Union And Complement Of Fuzzy Cubic Matrix

In this section let $V_{m\times n\times t}$ be the set of all $m\times n\times t$ fuzzy cubic matrices. For arbitrary A, B ,$C\in V_{m\times n\times t}$ that is $A=(a_{ijk})_{m\times n\times t}$, $B=(b_{ijk})_{m\times n\times t}$, $C=(c_{ijk})_{m\times n\times t}$.

Definition 2.1 $A\cup B\triangleq(a_{ijk}\vee b_{ijk})_{m\times n\times t}$,$A\cap B\triangleq(a_{ijk}\wedge b_{ijk})_{m\times n\times t}$ is respectively called the intersection and union of A and B. $A^c\triangleq(1-a_{ijk})_{m\times n\times t}$ is called the complement of A.

The corresponding operations and properties of fuzzy matries [1] is extanded as follows:

Proposition 2.1

1) commutative laws $A\cup B=B\cup A$, $A\cap B=B\cap A$
2) associative laws $(A\cup B)\cup C=A\cup(B\cup C)$, $(A\cap B)\cap C=A\cap(B\cap C)$
3) distributive laws $(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$ $(A\cap B)\cup C=(A\cup C)\cap(B\cup C)$
4) idempotent laws $A\cup A=A$, $A\cap A=A$
5) absorption laws $(A\cup B)\cap B=B$, $(A\cap B)\cup B=B$
6) involution law $(A^c)^c=A$
7) $\theta\cup A=A$, $\theta\cap A=\theta$, $E\cup A=E$, $E\cap A=A$

Theorem 2.1

1) $(V_{m\times n\times t};\cup)$ is a commutative semigroup where θ is a indentity element.

2) $(V_{m\times n\times t};\cup)$ is a semilattice.

3) $(V_{m\times n\times t};\cap)$ is a commutative semigroup where E is a identity element.

4) $(V_{m\times n\times t};\cap)$ is a semilattice.

Theorem 2.2 $(V_{m\times n\times t};\cap,\cup)$ is a incline.

The definitions of commutative semigroup, semilattice and incline see (2).

III. The Multiplying of Fuzzy Cubic Matrix

Definition 3.1 Let $A=(a_{ijk})_{m\times n\times t}$ and $B=(b_{ijk})_{m\times q\times n}$ then $C=(c_{ijk})_{m\times q\times t}$ is called product of A and B, it denoted by C=AB, where $c_{ijk}=\bigvee_{\lambda=1}^{n}(a_{i\lambda k}\wedge b_{ij\lambda}),(i=1,\ldots,m;j=1,\ldots,q,\ k=1,\ldots,t)$.

We denote "$\cup$" of definition 2.1 by "+", then we have:

Proposition 3.1 In $V_{m\times m\times m}$ there stand:

1) associative law: (AB)C=A(BC)

2) distributive laws: (A+B)C=AC+BC, A(B+C)=AB+AC

3) AI=IA=A

Theorem 3.1 $(V_{m\times m\times m};\cdot)$ forms a semigroup that I is identity element.

Notice: Multiplying of fuzzy matrices has not commutative law, and so does multiplying of fuzzy cubic matrices.

IV. The Scalar Product Of Fuzzy Cubic Matrices

Definition 4.1 Let $A=(a_{ijk})_{m\times n\times t}$, $h\in[0,1]$ then $B=(h\wedge a_{ijk})_{m\times n\times t}$ is called a scalar product of a scalar h and a cubic matrix A and denoted by B=hA.

Proposition 4.1 Let $A=(a_{ijk})_{m\times n\times t}$, $B=(b_{ijk})_{m\times n\times t}$ and $h,p\in[0,1]$, then

1) h(A+B)=hA+hB

2) (h+p)A=hA+pA

3) (hp)A=h(pA)

4) $1\cdot A=A$

Proposition 4.2 For $h\in[0,1]$, $A=(a_{ijk})_{m\times n\times t}$, $B=(b_{ijk})_{m\times q\times n}$, then $h(AB)=(hA)B=A(hB)$.

V. Fuzzy Cubic Matrix Space

Similar to fuzzy matrix space we define the concepts of linear dependent, linear independent and linear independent group and so on (see [3]).

From theorem 2.1 and theorem 3.1 we abtain:

Theorem 5.1 Under the operation of the addition and multiplication all $m\times m\times m$ fuzzy cubic matrices form a semiring.

The definition of semiring see [4].

From theorem 2.1 and proposition 4.1 we obtain

Theorem 5.2 Under the operation of the addition and scalar product all $m\times n\times t$ fuzzy cubic matrices form a fuzzy semilinear space, we denote it by $V_{m\times n\times t}$.

The definition of fuzzy semilinear space see [5].

Definition 5.1 A non-vacuous subset W of $V_{m\times n\times t}$ is a subspace of $V_{m\times n\times t}$ iff for arbitrary $A,B\in W$ and $h,p\in[0,1]$ there is $hA+pB\in W$.

Proposition 5.1 Intersection set W of arbitrary finite subspaces $W_1,\ldots,W_s$ of $V_{m\times n\times t}$ is still a subspace of $V_{m\times n\times t}$, we write $W=\bigcap_{i=1}^{t} W_i$.

Proposition 5.2 Let $W_1,\ldots,W_s$ be subspace of $V_{m\times n\times t}$ if $W=\{A\mid A=h_1A^{(1)}+\ldots+h_sA^{(s)},\ h_i\in[0,1],\ A^{(i)}\in W_i,\ i=1,\ldots,s\}$ then W is a subspace of $V_{m\times n\times t}$ and is called a sum space of $W_1,\ldots,W_s$.

Proposition 5.3 Let $A^{(1)},\ldots,A^{(s)}\in V_{m\times n\times t}$, then all linear combination $h_1A^{(1)}+\dots+h_sA^{(s)}$ where $h_1,\dots,h_s \in [0,1]$ forms a subspace of $V_{m\times n\times t}$, we write $W=L(A^{(1)},\dots,A^{(s)})$. W is called a subspace generated by $A^{(1)},\dots,A^{(s)}$. $A^{(1)},\dots,A^{(s)}$ is called a generating set of W.

Proposition 5.4 Let $A^{(1)},\dots,A^{(s)} \in V_{m\times n\times t}$ and $B^{(1)},\dots,B^{(p)} \in V_{m\times n\times t}$. If $A^{(1)},\dots,A^{(s)}$ are all linearly represented by $B^{(1)},\dots,B^{(p)}$ then $L(A^{(1)},\dots,A^{(s)}) \subseteq L(B^{(1)},\dots,B^{(p)})$.

VI. The Rank Of Fuzzy Cubic Matrix

Definition 6.1 For a fuzzy cubic matrix $A=(a_{ijk})_{m\times n\times t}$. The set of all linear combination of its row matrices form a subspace of $V_{t\times n}$, we write $R(A)$. Row rank $\rho_r(A)$ of A is the number of matrices that linear independent generating set contain . Column space $C(A)$ and column rank $\rho_c(A)$ can be similarly definited and so do storey space $S(A)$ and storey rank $\rho_s(A)$. If $\rho_r(A)=\rho_c(A)=\rho_s(A)$ then $\rho_c(A)$ is definited a rank of A. If $A=\theta$ then we definite $\rho_r(A)=\rho_c(A)=\rho_s(A)=\rho(A)=0$. If $A\neq\theta$ we use the method given in [3] and [6] to find $\rho_r(A)$, $\rho_c(A)$ and $\rho_s(A)$.

Let $A=(a_{ijk})_{m\times n\times t}\neq\theta$, It is lumped in row , column, storey as follows:

$$A=\begin{pmatrix} & & A_1 \\ & \cdot^{\cdot^{\cdot}} & \\ A_m & & \end{pmatrix} =(B_1,\dots,B_n) = \begin{pmatrix} C_1 \\ \vdots \\ C_t \end{pmatrix}$$

The subspace generated by row $1,\dots,m$ of A is symboled of $L(A_1,\dots,A_m)$ then $L(A_1,\dots,A_m)=R(A)$. The method of finding linear independent basis of $L(A_1,\dots,A_m)$ given in [3] and [6] is as follows :

We observe the first cyclic equations of $A_1,\dots,A_m$:

$$(A_1,\dots,A_{i-1},A_{i+1},\dots,A_m)\begin{pmatrix}x_1\\ \vdots\\ x_{i-1}\\ x_{i+1}\\ \vdots\\ x_m\end{pmatrix}=A_i,\quad (i=1,\dots,m)$$

If they have not solution then $A_1,\dots,A_m$ is a linear independent basis of $R(A)$, thus $\rho_r(A)=m$.

If some equation , for example m-th equation , have solution then $L(A_1,\dots,A_m)=L(A_1,\dots,A_{m-1})$. We study the first cyclic equation of $A_1,\dots,A_{m-1}$ again.

Similarly we go on with the above discusses . Such as we have discussed k-th stop. The first cyclic equations of $A_1,\dots,A_{m-k}$ have not solution then $\rho_r(A)=m-k$. There is such k, otherwise contradict that $A_1,\dots,A_m$ are all non-zero.

VII. The Standard Fuzzy Cubic Matrix

Definition 7.1 For fuzzy cubic matrices $A=(a_{ijk})_{m\times n\times t}$ and $B=(b_{ijk})_{m\times n\times t}$.

1) $A\leq B$ iff for arbitrary i,j,k there is $a_{ijk}\leq b_{ijk}$.

2) $A=B$ iff for arbitrary i,j,k there is $a_{ijk}=b_{ijk}$.

Proposition 7.1 If to fuzzy cubic matrix there is $A=B^{(1)}+\dots+B^{(s)}$ then $A\geq B^{(i)}$, $(i=1,\dots,s)$.

For the following discusses we suppose $A^{(1)},\dots,A^{(s)}\in V_{m\times n\times t}$ and they are lumped in the row as follows

$$A^{(p)}=\begin{pmatrix} & & A_{p1}\\ & \cdot^{\cdot^{\cdot}} & \\ A_{pm} & & \end{pmatrix}\qquad (p=1,\dots,s)$$

then A_{ij} $(i=1,\dots,s;\ j=1,\dots,m)$ are all $t\times n$ fuzzy matrices.

Definition 7.2 If there is $A^{(p)}$ in $A^{(1)},\dots,A^{(s)}$ such that when $A^{(p)}=\sum_{j=1}^{s} k_{pj}A^{(j)}$ there always is $A^{(p)}=k_{pp}A^{(p)}$ then $A^{(p)}$ is called the standard fuzzy cubic matrix in $W=L(A^{(1)},\ldots,A^{(s)})$. If every fuzzy cubic matrix of spanning set of W is a standard cubic matrix, then $A^{(1)},\ldots,A^{(s)}$ is called a standard fuzzy cubic matrix group in W.

We observe the following equation

$$x_1 A^{(1)}+\ldots+ x_s A^{(s)} = A^{(p)} \quad (p=1,\ldots,s) \qquad (2)$$

(2) is written in form

$$(A^{(1)},\ldots,A^{(s)}) \begin{pmatrix} x_1 \\ \vdots \\ x_s \end{pmatrix} = A^{(p)} \quad (p=1,\ldots,s) \qquad (3)$$

(3) is called the second cyclie equation of fuzzy cubic matrix group $A^{(1)},\ldots,A^{(s)}$. In (3) for given p $(1\leqslant p\leqslant s)$ then

$$(A^{(1)},\ldots,A^{(s)}) \begin{pmatrix} X_1 \\ \vdots \\ X_s \end{pmatrix} = A^{(p)} \qquad (4)$$

is equivalent to the equation group

$$\begin{cases} (A_{11},\ldots,A_{s1}) \begin{pmatrix} x_1 \\ \vdots \\ x_s \end{pmatrix} == A_{p1} \\ \cdots\cdots\cdots\cdots \\ (A_{1m},\ldots,A_{sm}) \begin{pmatrix} x_1 \\ \vdots \\ x_s \end{pmatrix} == A_{pm} \end{cases} \qquad (5)$$

As arbitrary solutions of (4) all satisfy (5) and arbitrary solutions of (5) all satisfy (4), thus we solve fuzzy ralation equation group (5) and so obtain all solutions of (4) . In particular the greatest solution of fuzzy relation equation group (5) is also the greatest solution of (4), the smallest solution of fuzzy relation equation group (5) is also the smallest solution of (4), and vice versal.

Theorem 7.1 $A^{(p)}$ $(1 \le p \le s)$ is a standard fuzzy cubic matrix of $W=L(A^{(1)},\dots,A^{(s)})$ iff for an arbitrary solution $(x_1^o,\dots,x_t^o)^T$ of (4) there is $A^{(p)}=x_p^o A^{(p)}$.

Corollary $A^{(p)}$ $(1 \le p \le s)$ is a standard fuzzy cubic matrix of $W=L(A^{(1)},\dots,A^{(s)})$ iff for an arbitrary greatest solution or smallest solution $(\bar{x}_1,\dots,\bar{x}_n)^T$ of (4) there is $A^{(p)}=\bar{x}_p A^{(p)}$.

VIII. The Standard Basis Of Finite Generating Subspace Of $V_{m\times n\times t}$

Definition 8.1 The linear independent basis $A^{(1)},\dots,A^{(s)}$ of finite generating subspace W of $V_{m\times n\times t}$ is called a standard basis of W, if $A^{(1)},\dots,A^{(s)}$ are all standard cubic matrices of W.

Theorem 8.1 The linear independent cubic matrix group $A^{(1)}$, $\dots,A^{(s)}$ of $V_{m\times n\times t}$ is a standard basis of $W=L(A^{(1)},\dots,A^{(s)})$ iff to arbitrary solution $(x_{p1}^o,\dots,x_{ps}^o)^T$ of the p-th (p=1, ...,s) equation of the second cyclic equation of $A^{(1)},\dots,$ $A^{(s)}$ there are $A^{(p)}=x_{pp}^o A^{(p)}$.

Lemma 8.1 Let $A^{(1)},\dots,A^{(s)}$ be linear independent and $h\in[0,1]$. If $A^{(2)}$ is a standard cubic matrix of $L(A^{(1)},\dots,A^{(s)})$ then $A^{(2)}$ is also a standard cubic matrix of $L(hA^{(1)},\dots,A^{(s)})$.

Theorem 8.2 The linear independent basis of a finite generating subspace W of $V_{m\times n\times t}$ is changed into a standard basis of W .

Proof. We observe p-th equation (4) of the second equation of $A^{(1)},\dots,A^{(s)}$. Let $A^{(p)}$ be not standard cubic matrix and the smallest solution of the equation (4) be $(x_1^{(11)},\dots,x_s^{(11)})^T$ $,\dots,(x_1^{(1i_1)},\dots,x_s^{(1i_1)})^T$.

For $x_p^1=\min\{x_p^{(11)},\dots,x_p^{(1i_1)}\}$ We construct another equation:

$$(A^{(1)},\dots,A^{(p-1)},x_p^1A^{(p)},A^{(p+1)},\dots,A^{(s)})\begin{pmatrix}x_1\\ \vdots\\ x_s\end{pmatrix}=x_p^1A^{(p)} \qquad (6)$$

i) From the supposition that $A^{(p)}$ is not a standard cubic matrix we easy infer $x_p^1A^{(p)}\neq A^{(p)}$ and there is some a_{ijk} in $m\times n\times t$ elements of $A^{(p)}$ such that $x_p^1 < a_{ijk}$.

ii) We infer that $A^{(1)},\dots,A^{(p-1)},x_p^1A^{(p)}, A^{(p+1)},\dots,A^{(s)}$ are linear independent. In fact so long as we prove that $x_p^1\ A^{(p)}$ can not be represented the linear combination of other cubic matrices. We use proof by contradiction. If

$$x_p^1A^{(p)}=y_1A^{(1)}+\dots+y_{p-1}A^{(p-1)}+y_{p+1}A^{(p+1)}+\dots+y_sA^{(s)} \qquad (7)$$

and we suppose that $(\bar{x}_1,\dots,\bar{x}_{p-1},x_p^1,\bar{x}_{p+1},\dots,\bar{x}_s)^T$ is a solution of (4) . Then

$$A^{(p)}=\bar{x}_1A^{(1)}+\dots+\bar{x}_{p-1}A^{(p-1)}+x_p^1A^{(p)}+\bar{x}_{p+1}A^{(p+1)}+\dots+\bar{x}_sA^{(s)} \qquad \dots\dots\dots\dots(8)$$

(7) is substituted into (8) and so there is

$$A^{(p)}=(\bar{x}_1+y_1)A^{(1)}+\dots+(\bar{x}_{p-1}+y_{p-1})A^{(p-1)}+ +(\bar{x}_{p+1}+y_{p+1})A^{(p+1)}+\dots+(\bar{x}_s+y_s)A^{(s)}.$$

The result contradicd that $A^{(1)},\dots,A^{(s)}$ are linear independent.

iii) Futher we infer

$$L(A^{(1)},\dots,A^{(p-1)},x_p^1A^{(p)},A^{(p+1)},\dots,A^{(s)}) =L(A^{(1)},\dots\dots,A^{(s)})$$

First as seen

$$L(A^{(1)},\dots,A^{(p-1)},x_p^1A^{(p)},A^{(p+1)},\dots,A^{(s)})\subseteq L(A^{(1)},\dots,A^{(s)})$$

second for any $A\in L(A^{(1)},\dots,A^{(s)})$ we suppose

$$A{=}{=}y_1A^{(1)}+\dots\dots+y_sA^{(s)} \qquad (9)$$

and suppose that $(z_1,\dots,z_{p-1},x_p^1,z_{p+1},\dots,z_s)^T$ is a solution of (4) then

$$A^{(p)}{=}{=}z_1A^{(1)}+\dots+z_{p-1}A^{(p-1)}+x_p^1A^{(p)}+z_{p+1}A^{(p+1)}+\dots+z_sA^{(s)} \quad \dots\dots\dots \quad (10)$$

(10) is substituted into (9) and so there is

$$A{=}{=}(y_1+y_pz_1)A^{(1)}+\dots+(y_{p-1}+y_pz_{p-1})A^{(p-1)}+y_px_p^1A^{(p)}+$$
$$+(y_{p+1}+y_pz_{p+1})A^{(p+1)}+\dots+(y_s+y_pz_s)A^{(s)}$$

thus $L(A^{(1)},\dots,A^{(s)})\subseteq L(A^{(1)},\dots,A^{(p-1)},x_p^1A^{(p)},A^{(p+1)},\dots,A^{(s)})$. So

$$L(A^{(1)},\dots,A^{(s)}){=}{=}L(A^{(1)},\dots,A^{(p-1)},x_p^1A^{(p)},A^{(p+1)},\dots,A^{(s)})$$

iv) If $x_p^1A^{(p)}$ is still not a standard cubic matrix of W. Let $(x_1^{(21)},\dots,x_s^{(21)})^T,\dots,(x_1^{(2i_2)},\dots,x_s^{(2i_2)})^T$ is the smallest solution of (6). From $x_p^2=\min\{x_p^{(21)},\dots,x_p^{(2i_2)}\}$ we infere $x_p^2\leq x_p^1$. Otherwise if

$$x_p^2>x_p^1 \qquad (11)$$

We suppose that $(x_1^o,\dots,x_{p-1}^o,x_p^2,x_{p+1}^o,\dots,x_s^o)^T$ is a solution of (6) then

$$x_p^1A^{(p)}=x_1^oA^{(1)}+\dots+x_{p-1}^oA^{(p-1)}+x_p^2x_p^1A^{(p)}+x_{p+1}^o\ A^{(p+1)}+\dots+x_s^oA^{(s)}$$

From (11) we know

$$x_p^1A^{(p)}=x_1^oA^{(1)}+\dots+x_{p-1}^oA^{(p-1)}+x_p^1A^{(p)}+x_{p+1}^oA^{(p+1)}+\dots+x_s^oA^{(s)}$$

Thus $(x_1^o,\dots,x_{p-1}^o,x_p^1,x_{p+1}^o\ ,\dots,x_s^o)^T$ is also a solution of (6) . Its p-th coordinate contradicts the given of x_p^2.

v) Futher from $x_p^1 A^{(p)}$ is not standard we know $x_p^2 x_p^1 A^{(p)} \neq x_p^1 A^{(p)}$. Thus $x_p^2 < x_p^1$. We construct the second cyclic equation of $A^{(1)},\ldots,A^{(p-1)}, x_p^2 A^{(p)}, A^{(p+1)},\ldots,A^{(s)}$ and so on and so forth. Since $A^{(p)}$ has only $m\times n\times t$ elements we discussed finite times such as k_p times and obtain a descending sequence $x_p^1, x_p^2, \ldots, x_p^{k_p}$ where $x_p^{k_p-1} = x_p^{k_p}$. Here $x_p^{k_p} A^{(p)}$ is a standard cubic matrix of $L(A^{(1)},\ldots,A^{(p-1)}, x_p^{k_p} A^{(p)}, A^{(p+1)},\ldots,A^{(s)})$. From iii) we know that $L(A^{(1)},\ldots,A^{(p-1)}, x_p^{k_p} A^{(p)}, A^{(p+1)},\ldots, A^{(s)}) == L(A^{(1)},\ldots,A^{(s)})$ and $x_p^{k_p} A^{(p)}$ is a standard cubic matrix of W .

vi) $A^{(1)},\ldots,A^{(s)}$ can be changed into $x_1^{k_1} A^{(1)},\ldots,x_s^{k_s} A^{(s)}$ where $x_p^{k_p} A^{(p)}$ ($p=1,\ldots,s$) are a standard cubic matrix of W and $W=L(x_1^{k_1} A^{(1)},\ldots,x_s^{k_s} A^{(s)})$.

This theorem is proved.

Definition 8.2 The number of fuzzy cubic matrices contained in the standard basis of a finite generating subspace of W of $V_{m\times n\times t}$ is called the dimension of W, we write dim(w).

Proposition 8.2 For fuzzy cubic matrix A there are $\dim(R(A)) = \rho_r(A)$; $\dim(C(A)) = \rho_c(A)$; $\dim(S(A)) = \rho_s(A)$.

Lemma 8.2 Let $A^{(1)},\ldots,A^{(s)}$ be a standard basis of a finite generating subspa W of $V_{m\times n\times t}$. If there is a cubic matrix group $B^{(1)},\ldots,B^{(p)}$ ($p \geqslant s$) of W such that $A^{(j)} = \sum_{\lambda=1}^{p} B^{(\lambda)}$, ($j=1,\ldots,s$) then there is some $B^{(i)}$ ($1 \leq i \leq p$) in the cubic matrix group such that $B^{(i)} = A^{(j)}$.

Theorem 8.3 There is only a standard basis in a finite generating subspace of $V_{m\times n\times t}$.

REFERENCE

〔1〕 Wang Peizhuang, Theory And Applycation Of Fuzzy Sets, Shanghai Science and Technology Pubication Company, Shanghai China, 1983, pp59-61.

〔2〕 Cao Zhiqiang, On Algebraic System Incline, Systems Engineering-Theory & Practice, VoL.IV.No.4 ,1984,pp1-7

〔3〕 Wang Hongxu & Liang Baoqi.,Fuzzy Matrix Space, Learned Journalof of Shenyang Teachers Training College,1(1984), pp31-41

〔4〕 Kim.K.H & Roush.F.W.,Generalized Fuzzy Matrices , Int.J. Fuzzy Sets And System, Vol.3. No.4. pp293-315 ,1980

〔5〕 Wang Hongxu & He Zhong xiong., Solution Of Fuzzy Matrix Rank, Fuzzy Mathematics , 4(1984), pp35-44

〔6〕 Zha Jianlu., The Construction Of Set Solution By General Fuzzy Relation Equation, Fuzzy Mathematics, 4(1984), pp71-80