Fuzzy Orthogonal Experiment Design and Synthetic Judgment

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Abstract

Fuzzy orthogonal experiment design is the fuzzinization of the resultant experimental information corresponding to the experimental objects. A fuzzy set is then formed in the domain of objects, and furthermore, items of information k_1, $k_2 \ldots k_n$ of different levels of various factors attained. With the corresponding extreme differences calculated, we are able to select more desinable technological condition and optimal experiment. Thus, we can get more information with synthetic analysis by employing the approach and theory of fuzzy mathematics than with normal means.

In this paper we are to introduce a orthogonal layout of fuzzy experiment design, to define Cartesian product and Cartesian outer product, projection and inner projection, and n-ary superior (inferior) fuzzy-relation-network-matrix. Besides we are to show that fuzzy-relation-network-matrix are symmetric. Finally we'll take an example to illustrate the analytical approch to fuzzy orthogonal experiment design, to see what new information could be drawn, and to analyse the orthogonal experiment comprehensively with fuzzy relation equation.

1. Introduction

The method of orthogonal experiment design is mostly harnessed in scientific experiments and productive processes. However, with the increasing system complexity of experimental objects, the Characteristic capability of accurate description will be accordingly reducing; sometimes there being little information differences amongst the results of various experiments puts it in a dilemma to decide which is better; even more, indefiniteness of information corros ponding to a experimental result emerges casually. Not only can we make the subject discussed much closer to its reality, but also more information can be expected be get by using the analytical method to fuzzy orthogonal experiment design, which is built upon the theory and approach of fuzzy mathematics, than by using normal method. The newly-got information is as follows:

1. The influences on experimental results by the matings of diverse levels of two arbitrary factors in the experiment can be got.
2. The factors which exert most or least influence on experimental results can be sifted from mating two arbitrary levels.
3. The varying tendencies of influences on experimental results by the matings of various levels, through applying the fuzzy-relation-network-matrix defined in the essay.

So in case of little resultant experimental differences among each object, we either optimize some peculiar factors or levels, or search for the clues for further experiment according to the varying tendencies of the matings from among diverse levels.

All the Symbols and some relevant conceptions coincide with reference [1], and in this paper, the numerical interpretation of experimental results is called index.

2. The Orthogonal Layout Of Fuzzy Orthogonal Experiment

Let A_1, A_2......A_n be n factors, U_1, U_2......U_n be respective domains. Select m levels each factor, we have

$U_1 = (u_{11}, u_{12}, \ldots\ldots u_{1m})$

$U_2 = (u_{21}, u_{22}, \ldots\ldots u_{2m})$

....

$U_n = (u_{n1}, u_{n2}, \ldots\ldots u_{nm})$.

Opt for an appropriate orthogonal layout

Levels / Factors / Experimental Object	A_1, A_2...... A_n	Indexes
1		
2		
⋮		
l		

(Fig. 1)

Consider the experimental objects $M = \{1, 2, \ldots\ldots \ell\}$, a domain. Turn the experimental results of each experimental objects into percentages, which after being normalized, is considered a fuzzy set on M, demoted as $\underset{\sim}{D} = (d_1, d_2\ldots\ldots d_\ell)$. $\underset{\sim}{D}$ then reflects the influences on indexes by the experiment of each object. In order to distinguish the differences among the levels of various factors, we add the indexes of j-th ($j = 1\ldots m$) level corresponding to A_i ($i = 1\ldots n$) in $\underset{\sim}{D}$. $\underset{\sim}{D}$ then generates n fuzzy sets

$$\underset{\sim}{A_1} = (l_{11}, l_{12}, \cdots\cdots l_{1m})$$
$$\underset{\sim}{A_2} = (l_{21}, l_{22}, \cdots\cdots l_{2m})$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\underset{\sim}{A_n} = (l_{n1}, l_{n2}, \cdots\cdots l_{nm})$$

Among which lij are the sum of the indexes in $\underset{\sim}{D}$ corresponding to j-th level of factor A_i.

Consider the fuzzy sets $\underset{\sim}{D}$, $\underset{\sim}{A_1}$, $\underset{\sim}{A_2}$, ... $\underset{\sim}{A_n}$ in an orthogonal layout. Thus we have

Factors / Levels / Experimental Objects	$A_1.\ A_2 \cdots\cdots A_n$	$\underset{\sim}{D}$
1 2 $\vdots$ l		d_1 d_2 $\vdots$ d_e
k_1 k_2 $\vdots$ k_m	$l_{11}\ l_{12} \cdots\cdots l_{n1}$ $l_{12}\ l_{22} \cdots\cdots l_{n2}$ $\vdots \quad \vdots \quad \vdots$ $l_{1m}\ l_{2m} \cdots\cdots l_{nm}$	
Extremal Differences		

(Fig. 2)

We can get the optimal combination of levels and the primary or secondary of all the factors by analysing the fuzzy orthogoual layout with usual approach. In order to get more information, we introduce the following conceptions.

Let Mnxm be the whole fuzzy matrices of n rows, m columns.

Definition 1: Suppose $\underset{\sim}{a} \in m_{1\times n}$, $\underset{\sim}{b} \in m_{1\times m}$

$$\underset{\sim}{a} \times \underset{\sim}{b} \triangleq \underset{\sim}{a}^T \circ \underset{\sim}{b} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \circ (b_1, b_2, \cdots b_m) = \begin{pmatrix} a_1\wedge b_1, & a_1\wedge b_2, & \cdots\cdots & a_1\wedge b_m \\ a_2\wedge b_1, & a_2\wedge b_2, & \cdots\cdots & a_2\wedge b_m \\ \cdots & \cdots & \cdots & \cdots \\ a_n\wedge b_1 & a_n\wedge b_2, & \cdots\cdots & a_n\wedge b_m \end{pmatrix}$$

is called Cartesian product of fuzzy vectors, and

$$\underset{\sim}{a} \otimes \underset{\sim}{b} \triangleq \begin{pmatrix} a_1\vee b_1, & a_1\vee b_2, & \cdots\cdots & a_1\vee b_m \\ a_2\vee b_1, & a_2\vee b_2, & \cdots\cdots & a_2\vee b_m \\ \cdots & \cdots & \cdots & \cdots \\ a_n\vee b_1, & a_n\vee b_2, & \cdots\cdots & a_n\vee b_m \end{pmatrix}$$

called Cartesian outer product of fuzzy vectors.

If $\underset{\sim}{a}$ is a fuzzy set on U, $\underset{\sim}{b}$ is a fuzzy set on V, then $\underset{\sim}{a} \times \underset{\sim}{b}$ and $\underset{\sim}{a} \otimes \underset{\sim}{b}$ reflect the transformation (relation) between the two domains. When the same fuzzy conception is reprecented by different domains, the elements of the two domains have to do with each other to certain extent. $\underset{\sim}{a} \otimes \underset{\sim}{b}$ and $\underset{\sim}{a} \times \underset{\sim}{b}$ just reflect that relationship.

Definition 2: Let U, V be finite sets, Suppose $\underset{\sim}{R} \in M_{n\times m} \subset F(U\times V)$. The projection of $\underset{\sim}{R}$ in U is a fuzzy subset of U, denoted by $\underset{\sim}{R}_U$, which has a membership function

$$\mu_{\underset{\sim}{R}_U}(u) \triangleq \bigvee_{v\in V} \mu_{\underset{\sim}{R}}(u, v)$$

The projection of $\underset{\sim}{R}$ in V is a fuzzy set of V, denoted by $\underset{\sim}{R}_V$, which has a membership function

$$\mu_{\underset{\sim}{R}_V}(v) \triangleq \bigvee_{u\in U} \mu_{\underset{\sim}{R}}(u, v)$$

The inner projection of $\underset{\sim}{R}$ in U is a fuzzy subset of U, denoted by $\underset{\cdot}{R}_U$, which has a membership function

$$\mu_{\underset{\cdot}{R}_U}(u) \triangleq \bigwedge_{v\in V} \mu_{\underset{\sim}{R}}(u, v)$$

The inner projection of $\underset{\sim}{R}$ in V is a fuzzy subset of V, denoted by $\underset{\cdot}{R}_V$, which has a membership function

$$\mu_{\underset{\cdot}{R}_V}(v) \triangleq \bigwedge_{u\in U} \mu_{\underset{\sim}{R}}(u, v)$$

The projections present the row maximum and column maximum of a fuzzy matrix, the inner projections present the row minimum and column minimum of a fuzzy matrix.

Definition 3: Let

$$\underset{\sim}{R}_1=\begin{pmatrix} a_{11}, a_{12}, \cdots\cdots a_{1m} \\ \cdots\cdots\cdots\cdots \\ a_{n1}, a_{n2} \cdots\cdots a_{nm} \end{pmatrix} \cdots\cdots \underset{\sim}{R}_n=\begin{pmatrix} r_{11}, r_{12} \cdots\cdots r_{1m} \\ \cdots\cdots\cdots\cdots \\ r_{n1}, r_{n2}, \cdots\cdots r_{nm} \end{pmatrix}$$

be all fuzzy relation matrices, we call

$$\underset{\sim}{R}_1 \cup \underset{\sim}{R}_2 \cup \cdots\cdots \cup \underset{\sim}{R}_n \triangleq \begin{pmatrix} a_{11}\vee\cdots\cdots\vee r_{11}, a_{12}\vee\cdots\cdots\vee r_{12}, \cdots\cdots, a_{1m}\vee\cdots\cdots\vee r_{1m} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}\vee\cdots\cdots\vee r_{n1}, a_{n2}\vee\cdots\cdots\vee r_{n2}, \cdots\cdots a_{nm}\vee\cdots\cdots\vee r_{nm} \end{pmatrix}$$

a n-ary superior fuzzy-relation-network-matrix, and call

$$\underset{\sim}{R}_1 \cap \underset{\sim}{R}_2 \cap \cdots\cdots \cap \underset{\sim}{R}_n \triangleq \begin{pmatrix} a_{11}\wedge\cdots\cdots\wedge r_{11}, a_{12}\wedge\cdots\cdots\wedge r_{12}, \cdots\cdots a_{1m}\wedge\cdots\cdots\wedge r_{1m} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}\wedge\cdots\cdots\wedge r_{n1}, a_{n2}\wedge\cdots\cdots\wedge r_{n2}, \cdots\cdots a_{nm}\wedge\cdots\cdots\wedge r_{nm} \end{pmatrix}$$

a n-ary inferior fuzzy-relation-network-matrix.

The n-ary superior (inferior) fuzzy-relation-network-matrix is a fuzzy nxm-matrix, in which each element r'_{ij} $(i=1\ldots n,\ j=1\ldots m)$ is called mesh element of the network. If the mesh elements apply to $r'_{ij}=r'_{ji}$, the fuzzy-relation-network-matrix is called Symmetric.

The influences on indexes by the matings of diverse levels of two arbitrary elements in fuzzy orthogonal experiment can be analysed by using Cartesian (outer) product of fuzzy vectors, while projection and inner projection can be used to analyse the influences on indexes by the matings of a fixed level of one factor with the diverse level of all the other factors. And fuzzy-relation-network can be used to comprehensively survey the results of our experiment, that is, the mesh elements in the fuzzy relation network see the influences on indexes by the matings of a arbitrary level of one arbitrary factor with a arbitrary level of all the other factors in our experiment. In addition, the superior (inferior) fuzzy-relation-network-matrix formed by Cartesian (outer) product in fuzzy orthogonal experiment design is a symmetric fuzzy matrix —— the theorem in this section. Hence we are in the way which makes it more convinent for us to analyse the results of our experiment, and in the meantime, illuminates the routes of further experiments.

Theorem: The inferior (superior) fuzzy-relation-network-matrix of a fuzzy orthogonal experiment is a symmetric fuzzy matrix.

Proof: Let

$$\underset{\sim}{A}_1 = (l_{11}, l_{12}, \cdots\cdots l_{1m})$$
$$\underset{\sim}{A}_2 = (l_{21}, l_{22}, \cdots\cdots l_{2m})$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\underset{\sim}{A}_n = (l_{n1}, l_{n2}, \cdots\cdots l_{nm})$$

be the n fuzzy sets generated by $\underset{\sim}{D}$ in the fuzzy orthogonal layout.

It is obvious that

$$\underset{\sim}{R} = \underset{\sim}{R}_1 \cap \underset{\sim}{R}_2 \cap \cdots\cdots \cap \underset{\sim}{R}_n = \begin{pmatrix} r_{11}, r_{12} \cdots\cdots r_{1m} \\ \cdots\cdots\cdots\cdots \\ r_{n1}, r_{n2} \cdots\cdots r_{nm} \end{pmatrix}$$

the inferior fuzzy-relation-network of

$$\underset{\sim}{R}_1 = \underset{\sim}{A}_1 \times \underset{\sim}{A}_2$$
$$\underset{\sim}{R}_2 = \underset{\sim}{A}_2 \times \underset{\sim}{A}_3$$
$$\cdots\cdots\cdots\cdots$$
$$\underset{\sim}{R}_n = \underset{\sim}{A}_n \times \underset{\sim}{A}_1$$

is a fuzzy matrix, we need only to show its Symmetry.

When $i = 1, 2, \ldots n$, $j = 1, 2, \ldots\ldots m$.

$$r_{ij} = (l_{1i} \wedge l_{2j}) \wedge (l_{2i} \wedge l_{3j}) \wedge \cdots \wedge (l_{ni} \wedge l_{1j})$$
$$= (l_{1i} \wedge l_{2i} \wedge \cdots\cdots \wedge l_{ni}) \wedge (l_{1j} \wedge l_{2j} \wedge \cdots \wedge l_{nj})$$
$$r_{ji} = (l_{1j} \wedge l_{2i}) \wedge (l_{2j} \wedge l_{3i}) \wedge \cdots\cdots \wedge (l_{nj} \wedge l_{1i})$$
$$= (l_{1j} \wedge l_{2j} \wedge \cdots\cdots \wedge l_{nj}) \wedge (l_{1i} \wedge l_{2i} \wedge \cdots\cdots \wedge l_{ni})$$

Then $r_{ij} = r_{ji}$

Hence $\underset{\sim}{R}$ is a Symmetry fuzzy matrix.

3. The Analytical Method to A Fuzzy Orthogonal Layout

We are to take an example to illustrate the analytical method to a fuzzy orthogonal layout.

Example: To increase transformability of a chemicals, we choose three relevant factors —— reaction temperature (A), reaction time (B), alkalies applied (C), and select three levels each factor:

A: 80°C, 85°C, 90°C.

B: 90', 120', 150'.

C: 5%, 6%, 7%

Arranging the experiment with the orthogonal layout $L_9(3^4)$, and by using the approach in section 2, we therefore gain a fuzzy orthogonal layout

Factors / Levels / Experimental Objects	A	B	C	Transformabilities
1	80°	90'	5%	0.070
2	80°	120'	6%	0.121
3	80°	150'	7%	0.084
4	85°	90'	6%	0.117
5	85°	120'	7%	0.109
6	85°	150'	5%	0.093
7	90°	90'	7%	0.126
8	90°	120'	5%	0.138
9	90°	150'	6%	0.142
k_1	0.275	0.313	0.301	
k_2	0.319	0.368	0.380	
k_3	0.406	0.319	0.319	
Extreme Differences	0.131	0.055	0.079	

Let

$U = \{80°, 85°, 90°\}$

$V = \{90', 120', 150'\}$

$W = \{5\%, 6\%, 7\%\}$

$M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Then

$\underset{\sim}{A} = (0.275, 0.319, 0.406)$

$\underset{\sim}{B} = (0.313, 0.368, 0.319)$

$\underset{\sim}{C} = (0.301, 0.380, 0.319)$

$\underset{\sim}{D} = (0.070, 0.121, 0.084, 0.117, 0.109, 0.093, 0.126, 0.138, 0.142)$

are fuzzy sets on the domains U, V, W and M respectively.

By calculation, the Cartesian outer products amongst the fuzzy sets $\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underset{\sim}{C}$, are respectively

$$\underset{\sim}{A} \otimes \underset{\sim}{B} = \begin{pmatrix} 0.313 & 0.368 & 0.319 \\ 0.319 & 0.368 & 0.319 \\ 0.406 & 0.406 & 0.406 \end{pmatrix} = \underset{\sim}{R_1}$$

$$\underset{\sim}{B} \otimes \underset{\sim}{C} = \begin{pmatrix} 0.313 & 0.380 & 0.319 \\ 0.368 & 0.380 & 0.368 \\ 0.319 & 0.380 & 0.319 \end{pmatrix} = \underset{\sim}{R_2}$$

$$\underset{\sim}{C} \otimes \underset{\sim}{A} = \begin{pmatrix} 0.301 & 0.319 & 0.406 \\ 0.380 & 0.380 & 0.406 \\ 0.319 & 0.319 & 0.406 \end{pmatrix} = \underset{\sim}{R_3}$$

and the projections of $\underset{\sim}{R_1}$, $\underset{\sim}{R_2}$, $\underset{\sim}{R_3}$ in U, V, W are

$$\mu_{\underset{\sim}{R_1} U}(u) = (0.368, 0.368, 0.406)$$

$$\mu_{\underset{\sim}{R_2} V}(v) = (0.380, 0.380, 0.380)$$

$$\mu_{\underset{\sim}{R_3} W}(w) = (0.406, 0.406, 0.406)$$

$$\mu_{\underset{\sim}{R_1} V}(v) = (0.406, 0.406, 0.406)$$

$$\mu_{\underset{\sim}{R_2} W}(w) = (0.368, 0.380, 0.368)$$

$$\mu_{\underset{\sim}{R_3} U}(u) = (0.380, 0.380, 0.406)$$

the superior fuzzy-relation-network of $\underset{\sim}{R}_1$, $\underset{\sim}{R}_2$, $\underset{\sim}{R}_3$ is

$$\underset{\sim}{R} = \underset{\sim}{R}_1 \cup \underset{\sim}{R}_2 \cup \underset{\sim}{R}_3 = \begin{pmatrix} 0.313, & 0.380, & 0.406 \\ 0.380, & 0.380, & 0.406 \\ 0.406, & 0.406, & 0.406 \end{pmatrix}$$

From the definitions 1, 2, 3 and the above-calculations come the following points:

1. In the above case the indexes directly proportional to transformability and the quality of the chemicals. Yet $\underset{\sim}{R}_1$ connotes the fuzzy conception 'high transformability' to the relationship between reaction temperature and reaction time, $\underset{\sim}{R}_{1u} = \begin{pmatrix} 0.368 \\ 0.368 \\ 0.406 \end{pmatrix}$ being its row maximum of $\underset{\sim}{R}_1$, therefore, from $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_{1u}$, and in the case of considering only the two factors — reaction temperature and reaction time — we can see that if $A_1 = 80°$, $B_2 = 120'$, the influence on transformability by that relationship is interpreted as 0.368; if $A_2 = 85°$, $B_2 = 120'$, it remains the same; if $A_3 = 90°$, and the reaction time is whatever B_1, B_2 or B_3, it is interpreted as 0.406, when $B_2 = 120'$, the transformability of factors B is the highest, and consequently, if A_3B_2 is chosen, their relationship has the strongest influence on the transformability. Therefore, when only the level matings of factor A and B taken into account, the higher the reaction temperature is, the higher the transformability, provided that the reaction time is fit.

$\underset{\sim}{R}_{1v} = (0.406, 0.406, 0.406)$ the column maximum of $\underset{\sim}{R}_1$, sees that whether the reaction time is B_1, or B_2, or B_3, the influences on transformability by their relationship are basically the same if $A = 90°$ — the same conclution as got from $\underset{\sim}{R}_{1u}$.

By similar analyses of $\underset{\sim}{R}_{2v}$, $\underset{\sim}{R}_{2w}$, $\underset{\sim}{R}_{3w}$, $\underset{\sim}{R}_{3u}$, we also come to the conclusion that when only the two factors — reaction time and alkalies applied — are considered, B_2C_2 is the best alternative of their level matings; while when only alkalies applied and reaction temperature considered, C_2A_3 is the best alternative.

2. The mesh element r_{ij} in fuzzy relation-network represents the numeral showing the influence=on transformability by the matings of the i-th levels of A, B, C with the j-th levels of A, B, C. Cite the third column of the above network in illustration of it.

r_{13} connotes that when A_1B_3, B_1C_3, C_1A_3, the level matings of A, B, C, is conoerned, that relationship has the strongest influence on the ransformability, i.e. 0.406.

r_{23} connotes that when A_2B_3, B_2C_3, C_2A_3, the level matings of A, B, C, is conoerned, that relationship has the strongest influence on the transformability, i.e. 0.406.

r_{33} connotes that when A_3B_3, B_3C_3, C_3A_3 the level matings of A, B, C, is concerned, that relationship also exerts the greatest influence on the transformability i.e. 0.406.

We can gain similar conclusion from r_{31} and r_{32}, considering the symmetry of the fuzzy-relation-network-matrix.

A_1B_3, A_2B_3, A_3B_1, A_3B_2, A_3B_3 indicate that the relation degree of the transformabilities to the domains U and V, under the matings of the five above levels, are basically the same, i.e. 0.406. But, isn't there any difference among the five matings? The membership degrees of $\underset{\sim}{A}$ and $\underset{\sim}{B}$ see that wnen A_3(90°C) is mated with B_2(120'), the relation of transformability to U and V is the strongest.

Similarly B_1C_3, B_2C_3, B_2C_1, B_3C_1, B_3C_3, indicate that the relation degrees of the transformabilities to the domains V and W, under the matings of the five above levels, are basically the same, i.e. 0.406, the membership degrees of $\underset{\sim}{B}$, $\underset{\sim}{C}$ see that the relation of transformabilities to V and W is the strongest, when B_2(120') is mated with C_2(6%).

And C_1A_3, C_2A_3, C_3A_1, C_3A_2, C_3A_3 indicate that the relation degrees of transformabilities to the domains W and U, under the matings of the five above levels, are basically the same, i.e. 0.406, the membership degrees of $\underset{\sim}{C}$, $\underset{\sim}{A}$ see the strongest relation of transformabilities to W and U, when C_2(6%) is mated with A_3(90°C).

From above, we see that when the level mating $A_3B_2C_2$ of the three factors is chosen, we get the highest transformability, if interactions disregarded.

3. Because of the Symmetry of relation-network-matrix, mesh elements r_{13} , r_{31} and r_{23}, r_{32} then reflect the matings of the corresponding factors and levels, the relation degrees of transformabilities to different domains are unaltered. The distribution of the strongest relation being in the third column and third row, therefore better experimental design could be traced along the direction, in which the subscripts of level ascend.

While using the fuzzy orthogonal experiment layout, we should notice the following points.

1. To some experiments in which the designs concerned with low indexes are better than those concerned with high indexes, we use Cartesian product, inner projection and inferior fuzzy-relation-network-matrix to analyse the experimental results with the same method as in the above example.

2. If there exist interactions among factors, we consider the influences on interaction on indexes as a fuzzy set, which is equally treated to any factor, then the influences on indexes by interactions among the factors can be attained with the means in the above example.

4. Synthetic Judgment

From experience, we can, in certain technologies approximately know the membership degrees of indexes at all the levels of factors, if the membership degrees of indexes to each factor are supposed to be know. Then, what is the membership degrees of indexes at each level? Here we take the example in secion 3 again. Suppose a fuzzy matrix

$$\begin{array}{cc} & \begin{array}{ccc} \text{1-Level} & \text{2-Level} & \text{3-Level} \end{array} \\ \underset{\sim}{R} = & \begin{pmatrix} 0.275 & 0.319 & 0.406 \\ 0.313 & 0.368 & 0.319 \\ 0.301 & 0.380 & 0.319 \end{pmatrix} \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

and membership degrees of indexes to each factor

$$\underset{\sim}{\alpha} = (0.406,\ 0.368,\ 0.380)$$

are given. Then the membership degrees of indexes at each level is a fuzzy set

$$\underset{\sim}{b} = \underset{\sim}{\alpha} \circ \underset{\sim}{R} = (0.313,\ 0.380,\ 0.406)$$

On the contrary, if the membership degrees of indexes at each level is a fuzzy subset $\underset{\sim}{b}$, to get the interpretation of indexes on the domain of factors, we need only to solve a fuzzy relational equation

$$X \bullet \underset{\sim}{R} = b$$

The former is called the positive problem of synthetic judgment, the latter is called contrary problem of synthetic judgment, both of which have been found their uses in technological reforms.

Reference

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