ON THE CARDINALITY DEFINITION FOR TWOFOLD FUZZY SETS

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SUMMARY. In this paper we discuss, from the viewpoint of the many-valued logic, the suitability of the cardinality definition proposed by D.Dubois and H.Prade for twofold fuzzy sets (/1/,/2/), and present its important consequences.

1. TWOFOLD FUZZY SETS

The notion of twofold fuzzy sets has been introduced by D.Dubois and H.Prade (see e.g. /1/,/2/) as a generalization of Gentilhomme's flou sets and seems to be one of the most interesting approaches to representing incomplete knowledge. A twofold fuzzy set T is defined as an ordered pair (A,C) of fuzzy subsets $A,C\subset U$ such that $A\subset 1_{C_1}$. T models a set with a fuzzy boundary. A is then a set of elements from U which more or less certainly belong to T whereas C contains (in the sense of many-valued logic) elements which more or less possibly are in T. The intersection $C\cap A'$ is composed of elements whose belonging to T is dubious. The fulfilment of the intuitive postulate 'more or less certain belonging to T implies that its possibility equals 1' is ensured by the assumption C_1 contains A.

Any fuzzy set $F\subset U$ can also be considered as a twofold fuzzy one in two ways, different from the viewpoint of interpretation of membership (see /1/,/2/): either as $(1_{F_1},F)$ or as $(F,\mathrm{supp}(F))$. The pair (D,D) is a "twofold fuzzy" representation of a crisp subset $D\subset U$.

2. DEFINING THE CARDINALITY OF T=(A,C)

Assume A,C are finite fuzzy sets, i.e. their supports are finite. D.Dubois and H.Prade use Zadeh's fuzzy cardinals (see /7/),

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*Seminar on Interval and Fuzzy Mathematics directed by Prof.Dr. Jerzy Albrycht, Technical University of Poznań denoted by FGCount, as a tool for describing the cardinality of twofold fuzzy sets:

$$FGCount_F(k) := \sup\{t \in (0,1] : card(F_t) \geqslant k\}.$$

It is well known that $FGCount_F(k)$ is the possibility of the event 'the fuzzy set F contains at least k elements'. Moreover, $FGCount_F(k)=f_k$, where f_k denotes the k-th element in the nonincreasingly ordered sequence of values F(x) for all $x \in supp(F)$ with $f_o:=1$ and $f_j:=0$ for $j > card(supp(F))$ (see /5/).

So, $FGCount_C(k)$ is the possibility that C (i.e. the set of more or less possible elements of the twofold fuzzy set T=(A,C)) contains at least k elements, and $1-FGCount_A(k+1)$ is the necessity that the number of more or less sure elements of T equals at most k. Then (see /2/) "... in order to have k as a somewhat possible value for the cardinality of T, k must be somewhat certain as an upper bound of the cardinality of the set of the more or less sure elements of T and somewhat possible as a lower bound of the cardinality of the set of the more or less possible elements of T." Thus, conclude the authors of /2/, the cardinality of T should be defined as follows:

(&) $$card_T(k) := \min(c_k, 1-a_{k+1}) ,$$

where the values a_k, c_k (corresponding to A and C, respectively) are defined as f_k for F. In the opinion of the author of this note, that does not suffice to accept formula (&) as a reasonable approach to the important question how to define the cardinality of T. The above cited words could only be a nice explanation or interpretation of (&).

3. FORMAL SOURCE OF THE FORMULA UNDER DISCUSSION

Let $P_k(C)$ denote the family consisting of all the k-element crisp subsets of supp(C). By $[E]$ we shall denote the truth value of an expression E.

<u>Proposition. (/6/)</u> For each T=(A,C) and k=0,1,2,...

$$[\exists Y \in P_k(C): A \subset Y \subset C] = \min(c_k, 1-a_{k+1}) .$$

Proof. Is quite analogous to that of Proposition 3.3 in /5/.

REMARKS AND COROLLARIES

1. Formula (&) seems to be now quite acceptable and clear.
2. The cardinality definition (&) for twofold fuzzy sets is (taking into account the Proposition) in principle a slightly adapted version of Klaua's definition of partial cardinals for partial sets (see /3/,/4/).
3. On the other hand, the definition

   (&&) $\quad card_T(k) := [\exists Y \in P_k(C):\ A \subset Y \subset C]$

   is a simple and natural generalization of the definition of the Cd-cardinals (see /5/) obtained by replacing the condition A=Y (equivalent to $A \subset Y \subset A$) with $A \subset Y \subset C$, where $A \subset C$. Here also

   $$[\exists Y \in P_k(C):\ A \subset Y \subset C] = [\exists Y \in P_k(U):\ A \subset Y \subset C].$$

4. By putting some specially selected sets as A and C (see /6/), formula (&&) can be used as a generator for obtaining fuzzy cardinal numbers $FGCount_F$, Crd_F, Cdt_F, Cd_F (see /5/) proposed by L.A.Zadeh, D.Dubois, E.P.Klement, and by the author, respectively.
5. Using various types of fuzzy cardinal numbers (see /5/) and considering F first as a fuzzy set and then as a twofold fuzzy set, we obtain identical or different information about the cardinality of F.

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REFERENCES

/1/ D.Dubois, H.Prade, Twofold fuzzy sets: An approach to the representation of sets with fuzzy boundaries based on possibility and necessity measures, J. of Fuzzy Mathematics (Wuhan, China), 3(4)(1983)53-76.

/2/ D.Dubois, H.Prade, Twofold fuzzy sets and rough sets - Some issues in knowledge representation, Fuzzy Sets and Systems, to appear.

/3/ D.Klaua, Partielle Mengen mit mehrwertigen Grundbeziehungen, Monatsber. Deut. Akad. Wiss. Berlin 11(1969)573-584.

/4/ D.Klaua, Partielle Mengen und Zahlen, Monatsber. Deut. Akad. Wiss. Berlin 11(1969)585-599.

/5/ M.Wygralak, Fuzzy cardinals based on the generalized equality of fuzzy subsets, Fuzzy Sets and Systems 18(1986)143-158.

/6/ M.Wygralak, Twofold fuzzy sets and their cardinality, a paper prepared for the First IFSA-EC and EURO-WG Workshop on Progress in Fuzzy Sets in Europe, November 25-27, 1986, Warsaw, Poland.

/7/ L.A.Zadeh, Fuzzy probabilities and their role in decision analysis, in: Proc. IFAC Symp. on Theory and Applications of Digital Control, New Dehli (1982), preprint.