RELATIONS BETWEEN BASES OF A FINITE GENERATING N-ARY SUBSPACE

Wang Hongxu
Dept. of Math. Benxi Education's College
Benxi. China

ABSTRACT

In this paper, we study relations between bases of finite generating n-ary subspace, prove that the K-R standard basis and Yang basis of a finite generating n-ary subspace are the same, and also give a concrete method of finding out the solution of its dimension and basis.

I. FUNDAMENTAL CONCEPTS

Let fuzzy matrices $A=(a_{ij})_{m\times n}$, $B=(b_{ij})_{m\times n}$ and $k\in[0,1]$. The sum of the two fuzzy matrices, the scalar product of a number and a fuzzy matrix, and the relation "$\leq$" of two fuzzy matrices are definited respectively as follows:

$$A+B=(a_{ij}+b_{ij})_{m\times n}==(\max\{a_{ij},b_{ij}\})_{m\times n}$$

$$kA==(ka_{ij})_{m\times n}==(\min\{k,a_{ij}\})_{m\times n}$$

$$A\leq B \quad \text{iff} \quad \forall i,j, \quad a_{ij}\leq b_{ij}$$

The product of two fuzzy matrices ($A=(a_{ij})_{m\times s}$ and $B=(b_{ij})_{s\times n}$) is definited as follows:

$$A\cdot B=(c_{ij})_{m\times n}==(\sum_{K=1}^{s} a_{ik}b_{kj})_{m\times n}$$

Under the addition and scalar product the set of all n-ary fuzzy row (column) vectors forms a fuzzy semi-linear space, denoted by $V_n(V^n)$.

A vector set $\{A_1,\ldots,A_t\}\subseteq V_n(V^n)$ is independent if and only if there is not $A_i\in\{A_1,\ldots,A_t\}$ such that it is represented as a linear combination of elements of $\{A_1,\ldots,A_{i-1},$

$A_{i+1},\dots,A_t\}$. If there is some $A_i \in \{A_1,\dots,A_t\}$ such that it is a linear combination of elements of $\{A_1,\dots,A_{i-1},A_{i+1},\dots,A_t\}$, the set $\{A_1,\dots,A_t\}$ is said to be dependent.

Let $\{A_1,\dots,A_t\} \subseteq S \subseteq V_n(V^n)$. If $\{A_1,\dots,A_t\}$ are independent and $\forall A \in S$ can be denoted by a linear combination of $A_1,\dots, A_t$, then $\{A_1,\dots,A_t\}$ is called an independent vector set of S.

Let $\{A_1,\dots,A_t\} \subseteq V_n(V^n)$. The set W of all linear combination of $A_1,\dots,A_t$ is a subspace of $V_n(V^n)$, denoted by $W= \langle A_1,\dots,A_t\rangle$, and W is called a generating subspace by $A_1,\dots,A_t$. W is also called a finite generating subspace of $V_n(V^n)$. $\{A_1,\dots,A_t\}$ is called a generating set of W.

In this paper, we only discussion $V_n(V^n)$ and its subspaces.

II. THE BASIS OF A FINITE GENERATING SUBSPACE

Definition 2.1 Let $\{A_1,\dots,A_t\}$ be a linear independent set of a finite generating subspace W. If $W= \langle A_1,\dots,A_t\rangle$, then $\{A_1,\dots,A_t\}$ is called a linear independent basis of W.

Definition 2.2 In a finite generating subspace W. If W can be generated by $\{A_1,\dots,A_t\}$ and arbitrary vectors less than t of arbitrary vectors in W can not generate W, then $\{A_1,\dots,A_t\}$ is called a minimun generating set of W, or a K-R basis of W. (K-R means Kim & Roush (1)).

Definition 2.3 A K-R basis $\{A_1,\dots,A_t\}$ of a finite generating subspace W is called a K-R standard basis of W if and only if $A_s=\sum_{h=1}^{t} k_{sh} A_h$ $(s=1,\dots,t), \forall k_{sh} \in [0,1]$ then $A_s=k_{ss}A_s$.

Theorem 2.1 (1) Let W be a finite generating subspace of V_n.

(1) two K-R bases have the same cardinal number.

(2) W has an unique K-R standard basis.

Let $\{A_{t_1},\dots,A_{t_p}\}$ $(1 \leq p \leq s)$ subset of $\{A_1,\dots,A_s\}$. If $A_{t_1},\dots,A_{t_p}$ are linear independent and $\forall A \in \langle A_1,\dots,A_s\rangle$ can be given by a linear combinition of $A_{t_1},\dots,A_{t_p}$, then $\{A_{t_1},\dots A_{t_p}\}$ is called a linear independent maximal set of $\langle A_1,\dots,A_s\rangle$

Definition 2.4 A linear independent maximal set of a generating set of a finite generating subspace W is called a Zha basis of the subspace (Zha means Zha Jianlu (3)).

Theorem 2.2 (3) $\{A_1,\ldots,A_t\}$ is a Zha basis of finite generating subspace W if and only if $\{A_1,\ldots,A_t\}$ is a K-R basis of W.

Theorem 2.3 Let $\{A_{t_1},\ldots,A_{t_p}\}\subseteq\{A_1,\ldots,A_s\}$ $(1\leqslant p\leqslant s)$. $\{A_{t_1},\ldots A_{t_p}\}$ is a linear independent basis of $\langle A_1,\ldots,A_s\rangle$ if and only if $\{A_{t_1},\ldots,A_{t_p}\}$ is a Zha basis of $\langle A_1,\ldots,A_s\rangle$.

III. A STANDARD BASIS OF A FINITE GENERATING SUBSPACE W

Definition 3.1 In a finite generating subspace W, for $A\in W$, if there are $B,C\in W$, that are non-ordered relation "$\leq$", such that $A=B+C$, then A is called a compound vector of W, otherwise A is called a simple vector of W.

Definition 3.2 Let W be a finite generating subspace. If (1) $W=\langle A_1,\ldots,A_k\rangle$. (2) $\{A_1,\ldots,A_k\}$ is a linear independent set. (3) $A_1,\ldots,A_k$ are simple vectors of W, then $\{A_1,\ldots,A_k\}$ is called Yang basis of W (Yang means Yang Cailiang (2)).

Theorem 3.1 In a finite generating subspace W the K-R standard basis and a Yang basis are the same.

Corollary. Let W be a finite generating subspace, then W has a unique Yang basis.

Therefore theorem I-1 (2) is also proved in another way.

Theorem 3.2 Let W be a finite generating subspace. Then:

the cardinal number of a linear independent basis of W
= the cardinal number of a K-R basis of W
= the cardinal number of a Zha basis of W
=the cardinal number of the K-R standard basis of W
= the cardinal number of the Yang basis of W

Definition 3.3 The cardinal number of a linear independent basis of a finite generating subspace W is called the dimension of W.

IV. SOLUTION TO A LINEAR INDEPENDENT BASIS

OF A FINITE GENERATING SUBSPACE W .

Let $W=\langle A_1,\ldots,A_m\rangle$ and $A_1,\ldots,A_m$ be non-zero vectors. We cousider fuzzy relational equations

$$(x_1,\ldots,x_{t-1},x_{t+1},\ldots,x_m)\begin{bmatrix}A_1\\ \vdots\\ A_{t-1}\\ A_{t+1}\\ \vdots\\ A_m\end{bmatrix}=A_t,\quad (t=1,\ldots,m)$$

If they have no solution , then $A_1,\ldots,A_m$ is a linear independent basis of W.

Otherwise, if some equation, say the m-th equation, has a solution, then $W=\langle A_1,\ldots,A_{m-1}\rangle$. We continue to cousider the fuzzy relational equations of $A_1,\ldots,A_{m-1}$.

Similarly, we go on with the above discusion . until k-th stop. The fuzzy ralational equations of $A_1,\ldots,A_{m-k}$ have not solutions, then $W=\langle A_1,\ldots,A_{m-k}\rangle$, and $\{A_1,\ldots,A_{m-k}\}$ is a linear independent basis of W and the dimension of W is m-k.

Appling this procedure, we not only can find out the linear independent basis of W, but also can find out row rank $\rho_r(A)$ and the column rank $\rho_c(A)$ of fuzzy matrix A.

REFERRENCE

(1) Kim.K.H & Roush.F.W.,Generalized Fuzzy Matrix, Fuzzy Sets And Systems, 4(1980), pp293--315

(2) Yang Cailiang, On Fuzzy Eigen-equations ,Convegence Of Powers Of A Fuzzy Matrix And Stability Of A Fuzzy System, Fuzzy Mathematics, 4(1982), pp37--48

(3) Zha Jianlu, The Dependent of A System Of Fuzzy Vectors And The Rank Of A Fuzzy Matrix, Fuzzy Mathematics,4(1984) pp71--81