Periodic Fuzzy Function

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Abstract

In this paper the essence of classical periodic function concept is analysed through concept principle. And an axiomatized periodic function definition on the group G. From this, the concept of periodic fuzzy function is developed and it's basic property discussed.

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In order to develop the important role of the fuzzy mathematic, we try to exted the application of the classical periodic function concept to the fuzzy mathematics. Therefore, we analyse the essence of classical periodic function concept through the concept principle, thereafter, we extend the use of, the periodic function concept into fuzzy mathematics. And point out the property of periodic fuzzy function.

I. Analysis of the classical periodic function concept

1. Concept principle

In mathematics, the concept is essentially the set of the nature. When the concept is to be established, the concept principle must be followed, i.e. there laws should be followed:

(1) The law of no contradiction

The extension of concept established can't be an emptiness. Otherwise, the concept established would be meaningless.

(2) The law of indenpdence

Any property can't be deduced from any others in the set.

(3) The low of perfectness

It's the prerequisite criterion for establishing the concept. Before the concept is established, it is necessary to assure that everyone belonging to the extension of the established concept has a certain property. After the concept is established, it's necessary to be able to verify if. If so, the concept is perfect related to the property.

2. Analysis of concept of classial periodic function

Up till now, it is universally acknowledged that the following property is prerequisite property of periodic function concept. Namely,

The figure in the neighbouring periodic region is the same;

Let T be period of $f(x)$, $x \in D(f)$. For every $x_0 \in D(f)$, there is the same function figure in the $[x_0, x_0+T] \cap D(f)$, $[x_0+T, x_0+2T] \cap D(f)$, ..., $[x_0+nT, x_0+(n+1)T] \cap D(f)$, etc.. This is perfectness of periodic function concept.

The classical periodic function doesnot have the perfectness. But as it can be read some literature, the classical concept is used and the perfectness is concluded. E.g. in the reference [1] it reads as following:

"DEFINITION (A). Let $f(x)$ be a function, $T>0$ (const.), $f(x)=f(x+T)$. Then $f(x)$ is called a periodic function, and the T is the period of the function.

"The periodic function has the following peculiarity:

When an independent variable is increased or decreased by a fixed T, the graph will appear repeatedly." [1]

Ex. Let $f(x)=n$, When $x=n+m\sqrt{2}$ $(n,m=0,1,2,...)$ the domain $D(f)=\{n+m\sqrt{2} \mid n,m=0,1,2,......\}$.

$\forall\ n+m\sqrt{2} \in D(f)$, there are

$$f(n+m\sqrt{2})=f(n+m\sqrt{2}+\sqrt{2})=f(n+\overline{m+1}\sqrt{2})=n.$$

By the "DEFINITION (A)", the function $f(x)=n$ is periodic function, and its period is $T=\sqrt{2}$. But $\max\{f(x)\}=1$, in the region $[0, 2]\cap D(f)$, $\max\{f(x)\}=2$, in the region $[\sqrt{2}, 2\sqrt{2}]\cap D(f)$. And, every "$\max\{f(x)\}$" is different from the other in the regions $[2\sqrt{2}, 3\sqrt{2}]\cap D(f)$, $[3\sqrt{2}, 4\sqrt{2}]\cap D(f)$, ... Hence, the function $f(x)=n$ hasnot property of perfectness.

This trouble lies in:

Though $x\in D(f)\Rightarrow x+T\in D(f)$ by the "DEFINITION(A)", it can't pledge that $x+T\in D(f)\Rightarrow x\in D(f)$. If condition $x\in D(f)\Leftrightarrow x+T\in D(f)$ is added to "DEFINITION(A)", the problem will be solved.

In addition, there is also a DEFINITION(B) in some References:

DEFINITION(B). Let function $y=f(x)$, it's domain M is a real set. If the constant $T\neq 0$ can satisfy the following conditions:

(i) $x\pm T\in M$ for every $x\in M$.

(ii) $f(x+T)=f(x)$ for every $x\in M$.

Then, $f(x)$ is called the periodic function in the M.

The defect of DEFINITION(B) is that it has no independence, because the condition 2 contained alrcady $x+T \in M$.

In order to study further the periodic function, the axiomaticed definition is given as following:

DEFINITION (1). Let f(x) be a function. The domain $D(f)=\phi$, $D(f)\subseteq G$ (The G is a group resulted from addition in the number field) If $T\in G$, $T\neq 0$ exist, that $f(x+T)=f(x)$, and $x-T \in D(f)$ for every $x\in D(f)$, then f(x) is called the periodic function, and the T is its a period. Where, the 0 is zero element in the G, and -T is negative element of the T.

Evidently the DEFINITION (1) conforms with the concept principle.

In order to make the study of the operation of periodic function eary the DEFINITION (2) is give as follows:

DEFINITION (2). In number field R, Let f(x) be periodic function with g(x). Their domain is denoted by D(f) and D(g) respectively, their function field by $D(f^{-1})$ and $D(g^{-1})$ respectively. When $D(f)\subseteq R$, $D(g)\subseteq R$, $D(f^{-1})\subseteq R$, $D(g^{-1})\subseteq R$, We define

$$D(f\pm g) \triangleq D(f)\cap D(g);$$

$$D(f\cdot g)\triangleq D(f)\cap D(g);$$

$$D(f/g) \triangleq D(f)\cap D(g)-\{x|g(x)=0\};$$

$$D(f(g)) \triangleq D(g)-\{x/g(x) \notin D(f)\}.$$

The study of periodic function from the start point of DEFINITION (1) and DEFINITION(2) is called Periodic algebra.

II. Periodic Fuzzy Function

The main aim in this chapter is to extend the periodic function to the fuzzy mathematics, therefrom the periodic fuzzy function can be deduced. Of course, we might start fram the function of the difinition on the group G, just as the periodic algebra was put forward. But we start only from the function of definition on the real set R. Here after, we shall do it in the way it should, if necessary.

DEFINITION (II-1). Let $f: [a,b] \to \tilde{R}$, $x \mapsto f(x)$. The $\tilde{f}$ is called the fuzzy function in $[a,b]$.

In DEFINITION (II-1). the domain $[a,b]$ is a particular subset of real set —— the closed interval. Let $\mathcal{J}(R)$ be the set of all fuzzy subset in real R. The functional value is particular element of $\mathcal{J}(R)$ —— fuzzy number.

When the $[a,b]$ is changed in to a subset of R, and the fuzzy number is changed in to the element of

$\mathcal{J}(R)$, then that is what we are concern about —— the general fuzzy function. The fuzzy function is a particular condition of the general fuzzy function.

DEFINITION (II-2). Let $\tilde{f}: D(f) \to \mathcal{J}(R),\ x \mapsto \tilde{f}(x)$, then the $\tilde{f}$ called the general fuzzy function of the definition in the domain $D(\tilde{f})$, and $D(\tilde{f}) \subseteq R$, $D(\tilde{f}) \neq \phi$.

To be brief, in the following, the general fuzzy function is called the fuzzy function unless specially mentioned.

If function value $f(x)$ of single variable and single value is considered as fuzzy set $\{f(x)\} \subseteq R$ (membership function $\mu_{\{f(x)\}}(f(x))=1$), then it follows that: All functions of single variable and single are fuzzy function. Hence fuzzy function, we say, is the extension of general function.

DEFINITION (II-3) Let fuzzy function $\tilde{f}$ be defined on the $D(f) \subseteq R$, if the const $T \neq 0$, that

$$\tilde{f}(x+T)=\tilde{f}(x), \quad x-T \in D(\tilde{f})$$

are valid for any $x \in D(\tilde{f})$, then the f is called the periodic fuzzy function, and T is its period.

In order to study the union, intersection and complement of periodic fuzzy function, the definition is given as following:

DEFINITION (II-4). Let $D(\tilde{f})$ be the domain of the $\tilde{f}$, $D(\tilde{g})$ be domain of the g, we define

$$D(\tilde{f} \cup \tilde{g}) \triangleq D(\tilde{f}) \cap D(\tilde{g}).$$

$$D(\tilde{f} \cap \tilde{g}) \triangleq D(\tilde{f}) \cap D(\tilde{g}).$$

$$D(\tilde{f}^c) \triangleq D(\tilde{f}).$$

If the study of periodicity of fuzzy function starts from the definition (3) and (4), it is called fuzzy periodic algebra.

The fundamental properties of periodic fuzzy function are as follows:

PROPERTY (1). If T is a period of periodic fuzzy function $\tilde{f}$, then

$$\tilde{f}(x+nT)=\tilde{f}(x), \qquad x-nT \in D(\tilde{f})$$

for every $x \in D(\tilde{f})$. Namely

The period of periodic fuzzy function are innumerable, end there is inevitably a positive period. The smallest positive period is called the basic period.

PROPERTY (2). Let T_* be basic period of $\tilde{f}(x)$, then the b is period of $\tilde{f}$ if and only the $b=mT_*$ (the m is the integer of non-zero).

PROPERTY (3). Let $T_{\tilde{f}}$ be the period of periodic fuzzy function $\tilde{f}$, the $T_{\tilde{g}}$ be the period of periodic fuzzy function $\tilde{g}$, and $T_{\tilde{f}}/T_{\tilde{g}}$ be the rational number,

$D(\tilde{f}) \cap D(\tilde{g}) \neq \phi$, then $\tilde{f} \cup \tilde{g}$, $\tilde{f} \cap \tilde{g}$, $(\tilde{f})^c$ all are periodic fuzzy function, and non-zero common multiple of the $T_{\tilde{f}}$ and $T_{\tilde{g}}$ all are their period.

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References:

(1) Jiang Dinghua, Higher Mathematics, Beijing University, 1982.

(2) Wang Peizhuang, Fuzzy set theory and its Applications, 1983.

(3) Chen Yiyuan, Fuzzy Mathematics, Publisher: Huazhong University of Science and technology press First edition, 3, 1984.