FUZZY CATEGORIES IN APPLIED SOCIOLOGY

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As a robust tool for modelling social systems the apparatus of fuzzy categories was examined. In a simple case, a social system (e.g. a small group) can be defined as the category Set, where objects are sets and morphisms are correspondences between pairs of sets. When we wish to express the uncertainty and the potentiality inside and around the social system, we define this system as the category SET, where morphisms are fuzzy correspondences between nonfuzzy sets.

In this way, a social system is considered as a set of social objects (groups, individuals) together with a set of relations between these individuals or groups. Each relation is combined with a measure as expression of "realization prospect".

So, social systems can be regarded as (fuzzy) binary relations of sets. Further following M. Mesarovič, morphisms of systems and categories of systems are useful for characterization of the "family" of alternative social systems.

A more detailed description of social systems can be obtained by deriving some special cases of categories. One interesting example is the ordered category with involution (OI-category) in application to hierarchical and controlled social systems.

The category Kat is called ordered category with involution (OI-category), if over the class of their morphisms the correspondence $a \mapsto a^x$ is defined (i.e. the involution) and each set of morphisms Kat (A,B) is ordered by the relation $\subset$ so, that following conditions are fulfilled:

1/ if $a \in$ Kat (A,B), then $a^x \in$ Kat (B,A) ;

2/ $a^{xx} = a$ for each morphism a ;

3/ $(ab)^x = b^x a^x$ for each morphisms a, b ;

4/ if $a \subset b$, then $a^x \subset b^x$;

5/ if $a \subset b$ and compositions fa and ag are defined, then fa $\subset$ fb and ag $\subset$ bg.

Special cases of the OI-category are:

a/ category of binary relations between sets (or: category of systems above the category of sets)

b/ category of binary relations between moduls (or: category of linear systems)

c/ category of fuzzy correspondences between sets (here the class of objects is the class of sets and the morphism of the set X into the set Y is defined as a fuzzy set in X x Y)

d/ category of bayesian systems (the objects are probabilistic spaces with all possible events and morphisms are functions of the type $f: X \times Y \rightarrow [0;1]$)

The study of duality for OI-categories gives more insight to different types of morphisms according to their modularity and decomposability.

In the social context the application of OI-categories and their duals is proper in problems with binary relations - as network analysis, classification problems or decision analysis. Our attention was oriented first of all on social design, when we leave some uncertainty in new or reconstructed social systems in order to stress and to enlarge their potential power in future.

We used following fuzzification of the category "alternative social system". As objects of the OI-category some social groups and individuals were characterized by sets of social indicators. So, X is the set of social costs, Y is the set of social effects, when effects of one social object become costs of the second etc.

Morphisms are here binary relations between sets of social indicators, which we call social normatives or social efficiency indicators.

The attachment of the degree of "realization prospect" to each morphism reflects the potentiality (possibility) of fulfilling this social normative. It is defined by the membership function u_a of the fuzzy correspondence $a: X \rightarrow Y$, where $a \in Kat\ (X,Y)$.

The fuzzy category represents a "family" of normatives (efficiency measures), which are reachable by a given set of social groups or individuals.

This interpretation can expand the scope of mathematical methods in social analysis, planning and control.