APPLICATION OF FUZZY NUMBER IN EVALUATION OF ENVIRONMENTAL QUALITY

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In this paper, a hypothesis that the possible distribution of each contaminative factor for evaluation grade of environmental quality can be represented with a fuzzy number is given, and according to the operation rule of fuzzy number, to calculate the sum for possible distribution of all contaminative factors which belong to evaluation grade of environmental quality, as well as to take the $\lambda=\lambda_\alpha$ as threshold, the maximum possibility of evaluation of environmental quality which belongs to a certain grade in the effect of all contaminative factors can be delivered.

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1. The Principle of Evaluation of Environmental Quality with Fuzzy Number

The evaluation of environmental quality depends on the extent of environmental contamination , the demarcation line of classification of contaminative extent is not apparent. Therefore the extent of contamination is a fuzzy concept, it can be described with fuzzy mathematics. *1

(1) Designing the membership function of each contaminative factor for evaluation grade

If the set of contaminative factors in regional environment is $X: \{x_1, x_2, \ldots, x_n\}$, and the set of evaluation grades is $V: \{1, 2, \ldots, m\}$.

According to classification criterion of each contaminative

factor, the membership functions of contaminative factors for each evaluation grade are designed as follows:

$$\mu_1(x_i),\ \mu_2(x_i),\ \ldots,\ \mu_m(x_i).$$

Here, $\mu_k(x_i)$ represents membership function of contaminative factor i for evaluation grade k, its value is in $[0, 1]$.

(2) Substituting determinate numerical value of each contaminative factor in regional environment into corresponding membership function the corresponding membership degree value of contaminative factor for evaluation grade is acquired, and the possible distribution of membership degree value of contaminative factor for all evaluation grades is represented as follows: *2

$$\underset{\sim}{F}_i = \sum_{k=1}^{m} \frac{\mu_k(x_i)}{K} = \frac{\mu_1(x_i)}{1} + \frac{\mu_2(x_i)}{2} + \ldots + \frac{\mu_m(x_i)}{m} \quad . \tag{1}$$

Here, each item represents the membership degree of contaminative factor x_i for a certain grade. The "+" which links up adjacent two items is Zadeh sign.

(3) To calculate the sum for the possible distribution of contaminative factors for each evaluation grade by operation of fuzzy number ("V", "$\wedge$" or ordinary operation " $\sum$ ", "$\cdot$").

The possible distribution for evaluation grade in the effect of n contaminative factors is delivered as follows:

$$\underset{\sim}{K}(Z) = \bigvee_{Z=x*y} (\underset{\sim}{F}_i(x) \wedge \underset{\sim}{F}_j(y)) \tag{2}$$

(4) To take the $\lambda=\lambda_\alpha$ as threshold, the contaminated extent of maximum possibility in regional environment for a certain evaluation grade can be given.

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4. Example

(1) If the water quality in a certain region is contaminated with five contaminative factors: DO, BOD, COD, PH, CN. The set is represented as X: $\{x_1, x_2, x_3, x_4, x_5\}$. The set of evaluation grade is Very Clean, Clean, A Little Pollution, Moderate Pollution, Serious Pollution . It is represented as V: $\{1, 2, 3, 4, 5\}$. The classification criterions of the five contaminative factors are shown in table 1 .

Table 1 Unit: mg/l .

contaminative factors	the 1st grade	the 2nd grade	the 3rd grade	the 4th grade	the 5th grade
x_1(DO)	7.0	5.0	3.0	2.0	1.0
x_2(BOD)	1.5	2.0	3.0	5.0	8.0
x_3(COD)	2.0	3.0	5.0	8.0	10.0
x_4(PH)	0.002	0.005	0.01	0.02	0.03
x_5(CN)	0.001	0.002	0.005	0.01	0.02

The membership functions of x_1 for five evaluation grades are designed respectively as follows:

$$U_1(x_1) = \begin{cases} 0 & x \leqslant 5 \\ \frac{1}{2}(x-5) & 5 < x < 7 \\ 1 & x \geqslant 7 \end{cases}$$

$$U_2(x_1) = \begin{cases} \frac{1}{2}(x-3) & 3 < x < 5 \\ -\frac{1}{2}(x-7) & 5 < x < 7 \\ 0 & 3 \geqslant x \text{ , } x \geqslant 7 \end{cases}$$

$$U_3(x_1) = \begin{cases} x-2 & 2 \leq x \leq 3 \\ -\frac{1}{2}(x-5) & 3 \leq x \leq 5 \\ 0 & 5 \leqslant x,\ x \leqslant 2 \end{cases}$$

$$U_4(x_1) = \begin{cases} x-1 & 1 \leq x \leq 2 \\ -(x-3) & 2 \leq x \leq 3 \\ 0 & 3 \leqslant x,\ x \leqslant 1 \end{cases}$$

$$U_5(X_1) = \begin{cases} 1 & x \leqslant 1 \\ -(x-2) & 1 \leq x \leq 2 \\ 0 & x \geqslant 2 \end{cases}$$

The membership functions of x_2, x_3, x_4, x_5 can be designed analogously.

(2) Substituting determinate numerical values of x_1, x_2, x_3, x_4, x_5 (They are shown in the 2nd column in table 2) into membership functions of x_1, x_2, x_3, x_4, x_5. The calculated values which belong to each evaluation grade are shown in table 2.

Table 2

contaminative factors	determinate values	U_1	U_2	U_3	U_4	U_5
x_1(DO)	2.69	0	0	0.69	0.31	0
x_2(BOD)	2.50	0	0.5	0.5	0	0
x_3(COD)	7.73	0	0	0.09	0.91	0
x_4(PH)	0.0076	0	0.48	0.52	0	0
x_5(CN)	0.004	0	0.33	0.67	0	0

According to the formula (1), The possible distributions of contaminative factors for evaluation grades are represented with fuzzy member respectively

$$\underset{\sim}{F}_1 = \frac{0.69}{3} + \frac{0.31}{4}$$

$$\underset{\sim}{F}_2 = \frac{0.5}{2} + \frac{0.5}{3}$$

$$\underset{\sim}{F}_3 = \frac{0.09}{3} + \frac{0.91}{4}$$

$$\underset{\sim}{F}_4 = \frac{0.48}{2} + \frac{0.52}{3}$$

$$\underset{\sim}{F}_5 = \frac{0.33}{2} + \frac{0.67}{3}$$

(3) According to the operation rule with formula (2), there are

$$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 = \frac{0.5}{2U3} + \frac{0.31}{2U4} + \frac{0.5}{3U3} + \frac{0.31}{3U4}$$

$$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 = \frac{0.09}{2U3U3} + \frac{0.09}{3U3U3} + \frac{0.5}{2U3U4} + \frac{0.5}{3U3U4} + \frac{0.31}{2U4U4} + \frac{0.31}{3U4U4}$$

Here, abstract operation "*" is taken as (iUj), "$\wedge$" is taken as minimum and the maximum of membership degree is adopted for the same combination, namely " operation.

Because the value of membership degree of combination 2U3U3, 3U3U3 are minim, they may be leaved out. So

$$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 = \frac{0.5}{3U3U4} + \frac{0.31}{2U4U4} + \frac{0.5}{2U3U4} + \frac{0.31}{3U4U4}$$

$$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 + \underset{\sim}{F}_4 = \frac{0.48}{2U2U3U4} + \frac{0.31}{2U2U4U4} + \frac{0.52}{2U3U3U4} +$$

$$+ \frac{0.31}{2U3U4U4} + \frac{0.5}{3U3U3U4} + \frac{0.31}{3U3U4U4} \vdots$$

$$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 + \underset{\sim}{F}_4 + \underset{\sim}{F}_5 = \frac{0.33}{2U2U2U3U4} + \frac{0.31}{2U2U2U4U4} + \frac{0.48}{2U2U3U3U4} +$$

$$+ \frac{0.31}{2U2U3U4U4} + \frac{0.52}{2U3U3U3U4} + \frac{0.31}{2U3U3U4U4} +$$

$$+ \frac{0.5}{3U3U3U3U4} + \frac{0.31}{3U3U3U4U4} \vdots$$

(4) Let $\lambda = 0.48$, it will be seen that the possibility of combination which belongs to the 2nd grade is 3 times, and that which belongs to the 4th grade is 3 times too, but that which belongs to the 3rd grade is 9 times. Therefore the contamination of water quality in the regional environment should belong to the 3rd grade, namely "a little pollution". It corresponds practice.

3. Discussion

It is possible that the times of a certain maximal possible distribution of two grades in combination are the same by this method. In this instance, we can find out the grade which has the times just next to the maximal times, that one which closes to above grade is selected as contaminative grade.

The effect of all factors for contamination is taken into account as equal weight in this way. If it is necessary to take account of weight of each contaminative factor in conprehensive

effect, it should be weighted for representation of fuzzy number of each contaminative factor before the operation of fuzzy number is worken out.

Reference

*1 Li Zuoyong, Zhao Bangjie, ACTA SCIENTIAE CIRCUMSTANTIAE, 4, 3(1984), 204-213.

*2 He Zhongxiong(贺仲雄), FUZZY MATHEMATICS AND ITS APPLICATION, Publish House of Science and Technology in Tian Jin, 1983, 76-95.