APPLICATION OF FUZZY INTEGRAL IN PREDETERMINATION OF DIFFRACTION EFFICIENCY OF REFLECTION HOLOGRAM

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Based on classical statistics, this paper determines the weights of three factors——exposure time, development time, and dilution of developer, which play an important role for diffraction efficiency of reflection hologram. And then, applying the operation of fuzzy integral, this paper gives out fuzzy predetermination results of diffraction efficiency of reflection hologram grating on the certain condition, which coincide with practical case.

Keywords: Application of Fuzzy Sets Theory;
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1. Introduction

The key index of judging the quality of hologram is that has a superior diffraction efficiency. Whereas there are many factors which influence diffraction efficiency of hologram. For example, exposure time, development time and dilution of developer solution all influence the diffraction efficiency of hologram.*1 Therefore, in course of manufacture of hologram, it is difficult to know the numerical value of diffraction efficiency of hologram on the certain condition. It is very difficult how to select the factor condition , if wanting to manufacture the hologram with given diffraction efficiency.

This paper gives out the comprehensive predetermination of diffraction efficiency of reflection holographic grating, taken the factor region weight of key factors which influence diffraction efficiency of hologram, based on classical statistics, as important measure U(.) of fuzzy integral, applying fuzzy integral operation.

2. The Application of Fuzzy Integral Operation for the Diffraction Efficiency Predetermination of Hologram

Definition: If $f(X_i)$ is a measurable function of measurable space $(X, P(x))$, then

$$E = \sup[a \wedge \mu(F_a)] \qquad a \in [0, 1]$$
$$= \sup[\mu(A) \wedge \max f(X_i)] \qquad A \in p(x) \quad X_i \in A \tag{1}$$

is the fuzzy integral of measurable function $f(X_i)$ for fuzzy measure μ in X[*2,3]. It is written as

$$E = \oint f(X_i) \circ \mu(.) \tag{1'}$$

Here,

$$F_a = \{X_i \mid f(X_i) \geqslant a\} \qquad a \in [0, 1]$$

In order to get relation among the diffraction efficiency of reflection hologram and exposure time, development time, and dilution of developer, we have made a series of reflection holographic phase gratings, and determine its diffraction efficiency on different experimental condition. The results of determination are shown in table 1.

Our hope is that if some certain numerical value of exposure time, development time, and dilution of developer are given out, is it possible to pre-estimate the possibility of numerical value of diffraction efficiency of holographic grating to be made? At least, the possible grade of its diffraction efficiency will be given out? For the purpose, we divide the diffraction efficiency of holographic grating samples into three grades according to the numerical value.

Application of Fuzzy Integral in Diffraction Efficiency

1 Grade	$b_1 = \{ b \mid b \geqslant 0.32 \}$	
2 Grade	$b_2 = \{ b \mid 0.24 \leqslant b < 0.32 \}$	(2)
3 Grade	$b_3 = \{ b \mid b < 0.24 \}$	

All the same, we divide exposure time T_1, development time T_2 and dilution of developer T_3 into three region respectively according to certain standard.

Table 1.

number	diffraction efficiency	exposure time(minu)	development time(hours)	dilution of developer	coincidence
1	0.38	1	1.5	0.13	√
2	0.38	1	3.0	0.08	√
3	0.37	2	0.5	0.33	√
4	0.36	2	0.5	0.13	√
5	0.35	0.17	3.0	0.08	×
6	o.32	1	3.0	0.33	√
7	0.32	2	1.5	0.33	√
8	0.30	1	3.0	0.13	×
9	0.30	2	1.5	0.13	√
10	0.29	1	1.5	0.33	√
11	0.29	0.17	1.5	0.08	√
12	0.26	0.17	1.5	0.13	√
13	0.24	0.17	1.5	0.33	√
14	0.24	0.17	0.5	0.13	√
15	0.22	0.17	0.5	0.33	√
16	0.22	0.17	0.5	0.08	√
17	0.21	0.17	3.0	0.33	√
18	0.21	0.17	3.0	0.13	√
19	0.18	1	0.5	0.33	×
20	0.18	2	3.0	0.08	×
	0.34	1.4	2.3	0.10	√
	0.29	0.4	2.0	0.11	√
	0.26	1.2	0.8	0.14	√
	0.23	1.8	1.6	0.08	√

$$T_{11} = \{ T_1 \mid T_1 \leqslant 0.8 \}$$
$$T_{12} = \{ T_1 \mid 0.8 < T_1 \leqslant 1.8 \} \qquad (3)$$
$$T_{13} = \{ T_1 \mid T_1 > 1.8 \}$$

$$\begin{aligned} T_{21} &= \{ T_2 \mid T_2 \leqslant 1 \} \\ T_{22} &= \{ T_2 \mid 1 < T_2 \leqslant 2 \} \\ T_{23} &= \{ T_2 \mid T_2 > 2 \} \end{aligned} \tag{4}$$

$$\begin{aligned} T_{31} &= \{ T_3 \mid T_3 \leqslant 0.1 \} \\ T_{32} &= \{ T_3 \mid 0.1 < T_3 \leqslant 0.2 \} \\ T_{33} &= \{ T_3 \mid 0.2 < T_3 < 1 \} \end{aligned} \tag{5}$$

In order to determine the weights of region factor of any factor for the effect of diffraction efficiency, we use the classical statistics method and take the ratio of sample number in any factor region over total number of samples as the weight of region factor for the effect of diffraction efficiency. For example, combining table 1 and formula (3), the sample number in factor region T_{12} are 6, and total number of samples are 20. Therefore, the weight of factor region T_{12} for the effect of diffraction efficiency is

$$G_{T_{12}} = \frac{6}{20} = 0.30$$

It is the same to the other factor region. Then, we take the ratio of sample number of the certain factor region T_{12} in grade region b_1 of diffraction efficiency over total numbers of samples in factor region T_{12} as the contribution of region factor for the 1st grade diffraction efficiency. For example, in region T_{12}, the sample numbers in region b_1 of diffraction efficiency grade is 3, so the contribution of factor in region T_{12} for 1st grade of diffraction efficiency is

$$H_{b1} = \frac{3}{6} = 0.50$$

It is the same to the other factor regions. The results of all are shown in table 2.

Table 2.

factor	factor region	weight of factor region	the contribution of factor in factor region for each grade of diffraction efficiency
T_1	T_{11}	$G_{T_{11}} = \frac{9}{20} = 0.45$	$H_{b1}(T_{11}) = 1/9 = 0.12$ $H_{b2}(T_{11}) = 4/9 = 0.44$ $H_{b3}(T_{11}) = 4/9 = 0.44$
	T_{12}	$G_{T_{12}} = \frac{6}{20} = 0.30$	$H_{b1}(T_{12}) = 3/6 = 0.50$ $H_{b2}(T_{12}) = 2/6 = 0.33$ $H_{b3}(T_{12}) = 1/6 = 0.17$
	T_{13}	$G_{T_{12}} = \frac{5}{20} = 0.25$	$H_{b1}(T_{13}) = 3/5 = 0.60$ $H_{b2}(T_{13}) = 1/5 = 0.20$ $H_{b3}(T_{13}) = 1/5 = 0.20$
T_2	T_{21}	$G_{T_{21}} = \frac{6}{20} = 0.30$	$H_{b1}(T_{21}) = 2/6 = 0.33$ $H_{b2}(T_{21}) = 1/6 = 0.17$ $H_{b3}(T_{21}) = 3/6 = 0.50$
	T_{22}	$G_{T_{22}} = \frac{7}{20} = 0.35$	$H_{b1}(T_{22}) = 2/7 = 0.29$ $H_{b2}(T_{22}) = 5/7 = 0.71$ $H_{b3}(T_{22}) = 0/7 = 0$
	T_{23}	$G_{T_{23}} = \frac{7}{20} = 0.35$	$H_{b1}(T_{23}) = 3/7 = 0.43$ $H_{b2}(T_{23}) = 1/7 = 0.14$ $H_{b3}(T_{23}) = 3/7 = 0.43$
T_3	T_{31}	$G_{T_{31}} = \frac{5}{20} = 0.25$	$H_{b1}(T_{31}) = 2/5 = 0.40$ $H_{b2}(T_{31}) = 1/5 = 0.20$ $H_{b3}(T_{31}) = 2/5 = 0.40$
	T_{32}	$G_{T_{32}} = \frac{7}{20} = 0.35$	$H_{b1}(T_{32}) = 2/7 = 0.29$ $H_{b2}(T_{32}) = 4/7 = 0.57$ $H_{b3}(T_{32}) = 1/7 = 0.14$
	T_{33}	$G_{T_{33}} = \frac{8}{20} = 0.40$	$H_{b1}(T_{33}) = 3/8 = 0.38$ $H_{b2}(T_{33}) = 2/8 = 0.24$ $H_{b3}(T_{33}) = 3/8 = 0.38$

Importance measure plays a very important role in process of predetermination. It should be the objective judgement criterion based on experimental fact. After the proportional weight of factors for the effect of diffraction efficiency is acquired, we can take $G_{T_{ij}}$ as fuzzy important measure $\mu(\cdot)$ of fuzzy integral predetermination and take the contribution H_{bi} of region factor for each diffraction grade as measurable function $f(x)$ in fuzzy integral predetermination. Then, the predetermination of effect of diffraction efficiency is derived applying fuzzy integral operation (1) .

3. Example of Predetermination of Diffraction Efficiency

With a D-type holographic plate and the formulae of recommended developer, if exposure time is 1.2 minute , development time is 0.8 hours, dilution of developer is 0.14, this numerical value just fall into three regions T_{12}, T_{21}, and T_{32} respectively. According to table 2, the proportional weights of three factor regions for diffraction efficiency are

$$G_{T_{12}} = 0.30, \qquad G_{T_{21}} = 0.30, \qquad G_{T_{32}} = 0.35 \,. \tag{6}$$

And the contribution of factors in this three factor region for 1st grade of diffraction efficiency respectively are

$$H_{b1}(T_{12}) = 0.50, \quad H_{b1}(T_{21}) = 0.33, \quad H_{b1}(T_{32}) = 0.29 \,. \tag{7}$$

Apparently, the membership degree of diffraction efficiency of hologram for the 1st grade diffraction efficiency can be calculated through fuzzy integral by formula (1). Whereas, in order to take account of the role of all factors averagely, we substitute weighted and sumed type $M(\cdot\,\oplus)$ for "sup" and "$\wedge$" operation in fuzzy integral formula (1). Here, "$\cdot$" represents ordinary multiplication, "$\oplus$" operation is definited as $a \oplus b = \mathrm{Min}(1,\ a+b)$.

From

$f(X_i)$: $(H_{b1}(T_{ij}))$ 0.50, 0.33, 0.29

$\mu(X_i)$: $(G_{T_{ij}})$ 0.30, 0.30, 0.35

We can get table 3.

Table 3.

a	$\mu(F_a)$	$a.\mu(F_a)$	$\sum a.\mu(F_a)$
0.29	Max{0.30, 0.30, 0.35}	0.10	
0.33	Max{0.30, 0.30}	0.10	0.35
0.50	0.30	0.15	

All the same, the membership degrees of diffraction efficiency of 2nd and 3rd grade are calculated as 0.38 and 0.25. Among the three data, 0.38 is biggest. Therefore, the possibility appertaining to general case of the diffraction efficiency of hologram gained accoeding to the above condition is most apparent. In fact, determined diffraction efficiency is 0.26. It is come true that the predetermination is correspond with practical result. The results of predetermination of diffraction efficiency for other three holograms are coincidence with practical determination results. (See table 1.)

4. Conclusion

Applying this method, the coincident rate of the results of diffraction efficiency predetermination of 24 holograms given in table 1 with practical determination is about 80%. It will be seen this paper gives out a practical and appliable method of diffraction efficiency predetermination of hologram.

The key of this method is selecting factors which play an important role for diffraction efficiency . At the same time, correct division of factor region is also important, only in this manner, the statistics weights are more objectable.

In this paper, the operation to weight and sum in the process of fuzzy integral are used. Of course, operation "$\vee$" and "$\wedge$" definited by Sugneo or $(\wedge \oplus)$, $(\cdot \vee)$ operation also can be used according to demand of question.[*4]

This paper only solves the predetermination question of diffraction efficiency grade of hologram on condition that exporse time, development time and dilution of developer are given out. Contrary to this question, namely, if it is need to manufacture a holographic grating with certain diffraction efficiency, how can we select the three factor numerical value. That is a rather difficult problem, and it need to make a thorough investigation and study.

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Reference

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