DIAGRAMMATIC PRESENTATION OF FUZZY SETS

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ABSTRACT

It is common knowledge that Cantor sets (naive sets) and their operations can be clearly expressed by Venn diagrams. But, for Fuzzy sets, they can be only expressed by the diagrams of membershio functions, at present. How to expresss Fuzzy sets like Venn diagrams ? In the paper, we put forward the concept of pan-Venn-diagrams, by using this we can clearly express Fuzzy sets and their operations.

KEYWORDS: Sets, Fuzzy sets, Venn diagrams, Pan-Venn-diagrams.

Let U be a universe, the objects in U are called elements in U, the whole of some elements in U, A, is called a set on U. Usually people imagine U to be a "square frame", the elements in U to be some "points", and A to be a "circular ring" in the "square frame", see Figure 1.

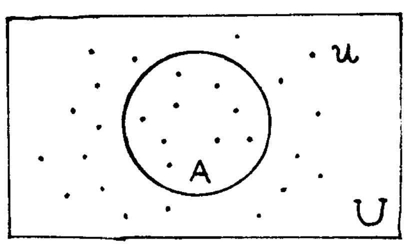

Fig.1. universe, elements, set

For any element $u \in U$, it either belongs to A or doesn't. This means the "circular ring" can not be passed through. Thus the key transforming Cantor sets into Fuzzy set is to exchange " the circular ring which can be passed through" for "the circular ring which can not be passed through".

Now we put forward the concept of pan-Venn-diagrams. First U is regarded as a rectangle in Euclidean plane, the elements in U regarded as some points in the rectangle (they may not necessarily be full of U), see Figure 2.

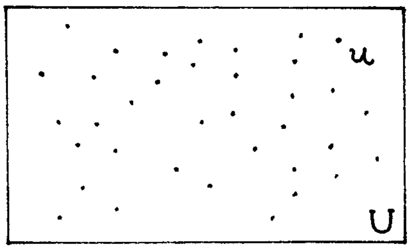

Fig.2. universe,elements

For any point in the rectangle (the point may not necessarily belong to U), all the elements in U must be situated on the rays that the point is regarded starting point, see Figure 3.

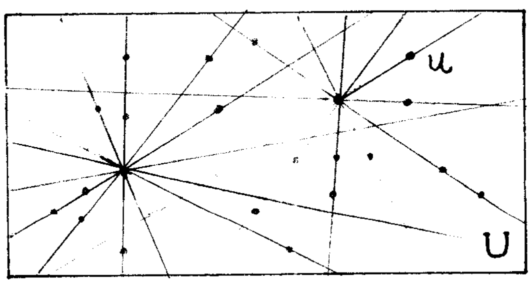

Fig.3. rays,elements

Any Fuzzy set on U can be regarded as a circular which is of annular bound with unit width, that the annular region is regarded as to be passed through. For any $u \in U$, if u is situated in the inside of the inside circle, then $\mu_A(u)=1$ (A is a Fuzzy set); if u in the outside of the outside circle, then $\mu_A(u)=0$; if u in the annular region, then the depth, which u enters along the ray that u is situated on and the starting point is the center of the annular, is just the degree of membership with respect to the Fuzzy set A, see Fig.4.

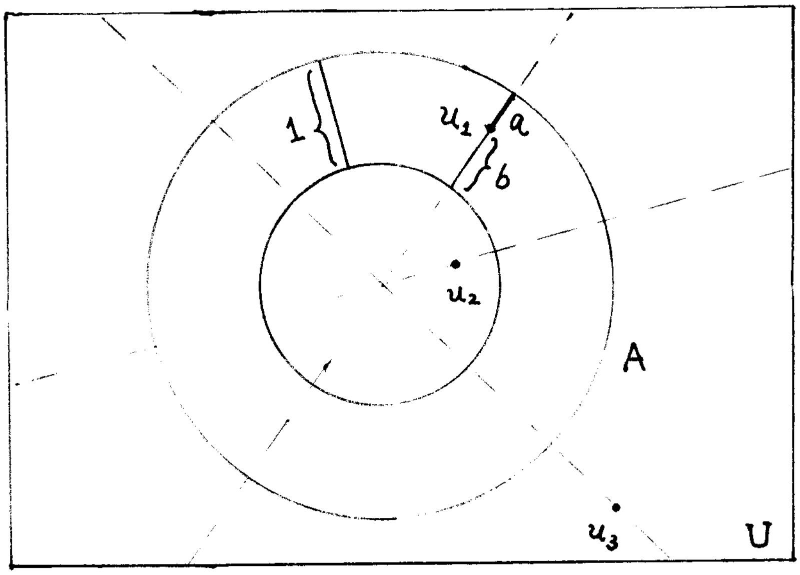

Fig.4. pan-Venn-diagram

The diagrammatic presentation like Fig.4 is called pan-Venn-diagram.

For example, for elements u_1, u_2, u_3, $\mu_A(u_1)=a, \mu_A(u_2)=1$, $\mu_A(u_3)=0$.

The annular region expresses intermediary states of transition about Fuzzy concepts, which is identical with Zadeh's definition of Fuzzy sets.

The illustrated method of complement operation is just opposite. If u is situated in the outside of the outside circle, then $\mu_{A^C}(u)=1$; if u in the inside of the inside circle, then $\mu_{A^C}(u)=0$; if u in the annular region, then the depth, which u exits along the ray, is just the degree of membership, see Fig.4. For example, $\mu_{A^C}(u_1)=b$ (=1-a), $\mu_{A^C}(u_2)=0$ (=1-1), $\mu_{A^C}(u_3)=1$ (1-0).

Fig.5 expresses union operation of Fuzzy sets.

If u is situated in the shodow region of vertical lines, then $\mu_{A\cup B}(u)=1$; if u in the shodow reagion of horizontal

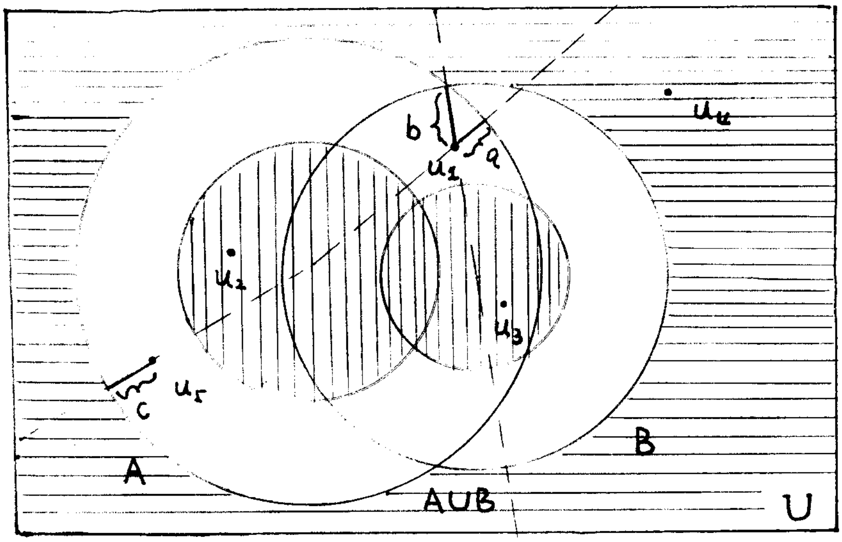

Fig.5. union of Fuzzy sets

lines, then $\mu_{A\cup B}(u)=0$; if u no in two the shodow reagions, then the deeper one of the depthes which enter two the annular regions is taken as the degree of membership about $A\cup B$. For example, $\mu_{A\cup B}(u_1)=a\vee b=b$, $\mu_{A\cup B}(u_2)=1$, $\mu_{A\cup B}(u_3)=1$, $\mu_{A\cup B}(u_4)=0$, $\mu_{A\cup B}(u_5)=c\vee o=c$.

Fig.6 expresses intersection operation of Fuzzy sets.

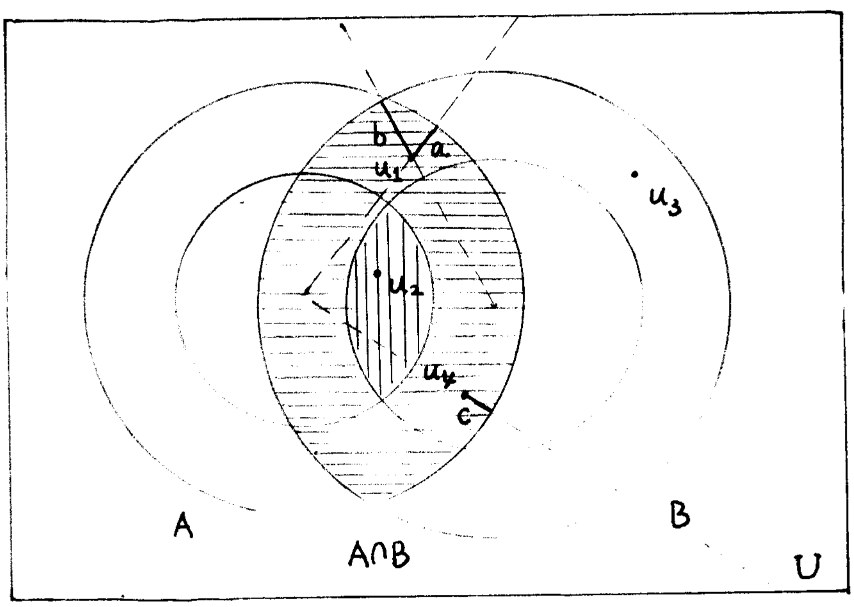

Fig.6. intersection of Fuzzy sets

If u is situated in the shodow region of vertical lines, then $\mu_{A\cap B}(u)=1$; if u in the shodow region of horizontal lines, then shallowed one of the depthes which enter two the annular regions is taken as the degree of membership about $A\cap B$. For example, $\mu_{A\cap B}(u_1)=a\wedge b=a$, $\mu_{A\cap B}(u_2)=1$, $\mu_{A\cap B}(u_3)=0$, $\mu_{A\cap B}(u_4)=c\wedge 1=c$.

Fig.7 expresses contain relation of Fuzzy sets.

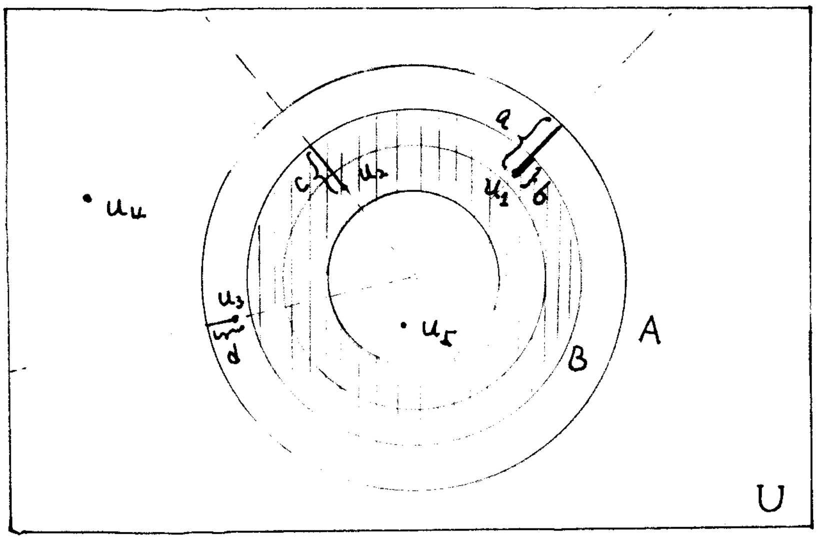

Fig.7. contain relation of Fuzzy sets

For example, $\mu_A(u_1)=a > b=\mu_B(u_1)$, $\mu_A(u_2)=1 > \mu_B(u_2)=c$, $\mu_A(u_3)=d > \mu_B(u_3)=0$, $\mu_A(u_4)=0=\mu_B(u_4)$, $\mu_A(u_5)=1=\mu_B(u_5)$.

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