An approach to approximate reasoning based on Dempster rule of combination*

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Extended abstract

The problem of automated reasoning in a knowledge base containing uncertain items of information is addressed. The proposed methodology takes advantage of Dempster-Shafer's theory of evidence to deal with uncertainty.

1 - Knowledge representation

The knowledge base contains facts and rules. Elementary facts are of the form "x is A" where A is a subset of some frame of discernment X, and x is a variable defined on X ; what the fact says is that the (unknown) value of x lies in subset A. Composite facts referring to Cartesian products of frames can be contemplated. An elementary rule is of the form "if x is A, then y is B" and is viewed as a composite fact "(x,y) is $\bar{A}$+B" where $\bar{A}$ is the complement of A in X, + denotes the dual of the Cartesian product, and $\bar{A}$+B is a subset of the Cartesian product XxY. Namely $\bar{A}+B = \overline{A \times \bar{B}}$ and is the set-theoretic counterpart of the ordinary logical implication.

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* Prepared for presentation at the 8th IASTED Inter. Symp. on Robotics and Artificial Intelligence, Toulouse, France, June 18-20, 1986.

A fact F = "x is A" may be uncertain. Uncertainty on F is expressed by means of a grade of credibility $Cr(A) \in [0,1]$ which expresses the extent to which F is held for certain. The following conventions are adopted

$Cr(A) = 1$ means that F is a sure fact.

$Cr(A) = 0$ means that F is a totally uncertain fact.

It is actually assumed that Cr is a belief function (Shafer [6]) on the frame X, deriving from a basic probability assignment m in the sense that

$$Cr(A) = \sum \{m(B) \mid \emptyset \neq B \subseteq A\} \qquad (1)$$

where m(B) is positive and $\sum_{B \subseteq X} m(B) = 1$. The quantity $Cr(\bar{A})$ expressing the credibility of the fact "x is not A", is such that $Cr(A)+Cr(\bar{A}) \leq 1$. The quantity $Pl(A) = 1-Cr(\bar{A}) \geq Cr(A)$ is the grade of plausibility of F, and is 0 as soon as the fact F is a certainly false. The knowledge provided by the expert about fact F is supposedly $Cr(A) = \alpha$. Using a principle of minimum specificity (Dubois and Prade [3]), the less arbitrary basic probability assignment satisfying (1) is given by

$$m_F(A) = \alpha \quad ; \quad m_F(X) = 1-\alpha \qquad (2)$$

when $Cr(A) = \alpha$ is the only available knowledge about F. Similarly an uncertain rule R = "if X is A, then Y is B" is assigned a grade of credibility $\beta = Cr(\overline{A \times \bar{B}})$, and is represented by the basic assignment m_R :

$$m_R(\overline{A \times \bar{B}}) = \beta \quad , \quad m_R(X \times Y) = 1-\beta \qquad (3)$$

2 - Outline of the approximate reasoning methodology

Let $F_1,\ldots,F_m$ be a set of facts and $R_1,\ldots,R_n$ be a set of rules, all of which are allowed to be uncertain. Let F be a fact the truth of which must be established, the approximate reasoning procedure runs as follows :

1. Define a frame Ω as the smallest set containing the frames of F_i, $i = 1,m$, R_j, $i = 1,n$, and F. In other words, if the knowledge base pertains to p variables $x_1,\ldots,x_p$, then $\Omega = X_1 x X_2 x \ldots x X_p$.
2. Calculate the cylindrical extension of the basic assignments m_{F_i} and m_{R_j} of facts and rules on Ω. Combine the extended basic assignments via Dempster rule [6].
3. Project the so-obtained basic assignment m on the frame Z pertaining to F. Compute Cr(F) and Pl(F) by (1).

Dempster rule of combination merges two basic assignments m_1 and m_2 on the same frame Ω into a basic assignment $\hat{m}$ defined by, $\forall A \subseteq \Omega$

$$\hat{m}(A) = \frac{m(A)}{1-m(\emptyset)} \tag{4}$$

and

$$m(A) = \sum\{m_1(A_1).m_2(A_2) \mid A_1 \cap A_2 = A\} = [m_1 \cap m_2](A) \tag{5}$$

(5) is a random set intersection, which assumes that the bodies of evidence defined by m_1 and m_2 are independent [2]. (4) is a normalization operation which gets rid of the conflicts between m_1 and m_2 ; the inconsistency between the two bodies of evidence is measured by $m(\emptyset)$. In the above procedure only (5) is applied in order not to artificially hide the partial inconsistency of the knowledge base. The operations of cylindrical extensions and projections are defined in Shafer [7]. This procedure is very similar to Zadeh's approach to approximate reasoning. Here we use plausibility functions instead of possibility measures, and Dempster rule instead of a fuzzy set-theoretic joint operation. Note that Dempster rule is associative and commutative.

3 - Application to modus ponens under uncertainty

As an example, consider a simple knowledge base containing a fact F expressed as an axiom $\vdash p$ in propositional logic and a rule R = "if F, then G", expressed as another axiom $\vdash p \rightarrow q$. Let $\alpha = Cr_1(p)$, $\beta = Cr_2(p \rightarrow q)$. Let $\mathbb{1}$ denote the tautology. We define the two basic assignments, using (2) and (3)

$$m_F(p) = \alpha \quad ; \quad m_F(\mathbb{1}) = 1-\alpha$$
$$m_R(p\to q) = \beta \quad ; \quad m_R(\mathbb{1}) = 1-\beta$$

Using (5) we get

$$m(p\cap q) = \alpha.\beta \ ; \ m(p\to q) = \beta.(1-\alpha) \ ; \ m(p) = \alpha(1-\beta) \ ; \ m(\mathbb{1}) = (1-\alpha)(1-\beta).$$

Note that no normalization step is required.
The fact to be ascertained is q. Using (1), in the language of logic, we obtain

$$Cr(q) = \sum\{m(s) \mid s \to q = \mathbb{1}\} = \alpha.\beta$$

It is easy to see that Pl(q) = 1-Bel(not q) = 1. Hence we obtain the following generalization of modus ponens

$$Cr_1(p) = \alpha$$
$$Cr_2(p\to q) = \beta$$
$$\overline{\hspace{5cm}}$$
$$Cr(q) = \alpha.\beta \ ; \ Pl(q) = 1$$

Note that classical modus ponens is recovered letting $Cr_1(p) = 1$ and $Cr_2(p\to q) = 1$. Moreover if one is totally ignorant about the truth of p ($Cr_1(p) = 0$) or $p\to q$ ($Cr_2(p\to q) = 0$) then $Cr(q) = 0$. Hence the above pattern of reasoning is consistent with propositional logic.

4 - Towards a distributed reasoning procedure

Contrastedly with usual approximate reasoning schemes in expert systems the procedure outlined in section 2 simultaneously applies all rules to all facts. Especially propagation and combination of uncertainty are not performed in separate steps [1], nor progressively on some inference network [5].

Hence the result obtained by the procedure is exact, in so far as the facts and rules are independent, but the actual computation can be very tedious (Gordon & Shortliffe [4]). To decrease this computational burden one idea is to decompose the knowledge base into a network of sub-bases which reflects the structure of the available information. Each sub-base receives as input the results obtained by applying the general methodology on upstream sub-bases, and produces some results in turn, etc ... This distributed computation scheme is made possible due to the associativity and commutativity of Dempster rule.

However the fact that sub-bases are related via a network implies that the inputs of a sub-base may not be independent. Namely is x_1 and x_2 are two input variables of a sub-base S and m_1, m_2 are the respective basic assignment, then the joint basic assignment may not be $[m_1 \cap m_2]$ for instance if m_1 and m_2 were obtained from some evidence pertaining to the same variable x (and possibly others). This implies that x_1 and x_2 must be outputs of the same sub-base, and that the joint basic assignment m_{12} on $X_1 x X_2$ must be calculated via the procedure of section 2. The reason why $m_{12} \neq m_1 \cap m_2$ is that Dempster rule is not idempotent ($m \cap m \neq m$), and that the basic assignment m describing x appears twice in the expression of $m_1 \cap m_2$, while it appears only once in m_{12}.

An approximate reasoning algorithm involving a knowledge base decomposition step is currently under implementation.

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